

ME111
 Instructor: Peter Pinsky
 Class # 19
 November 8, 2000

Today's Topics

- Introduction to static failure of materials.
- Failure of brittle materials under static loading:
 - Maximum normal stress failure theory.
 - Coulomb-Mohr failure theory.

Reading Assignment

Juvinall 6.5, 6.6 and 6.9

19.1 Introduction to Failure of Materials under Static Loading

For static loads:

- Ductile materials ($\epsilon_f > 0.05$) typically limited by their shear strengths
- Brittle materials ($\epsilon_f < 0.05$) typically limited by their tensile strengths

For dynamic or time-varying loads:

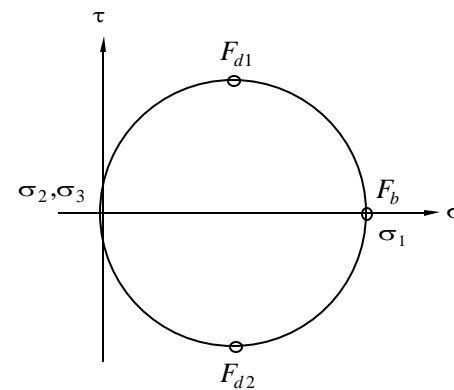
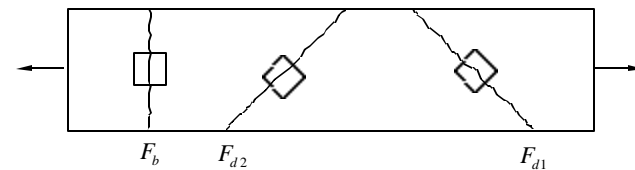
- Ductile materials fail in a brittle manner.
- Need different failure theories -- see later.

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Consider uniaxial tension test:

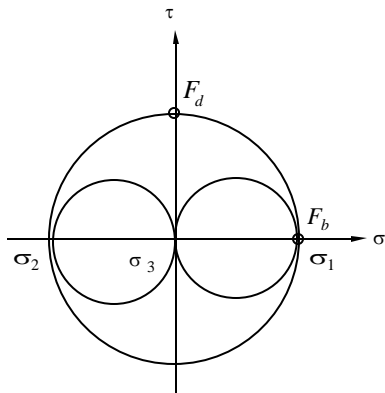
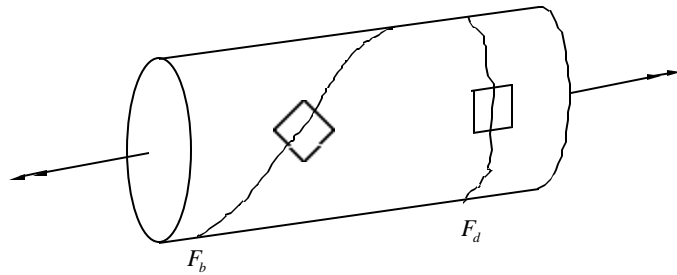


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Consider torsion test:



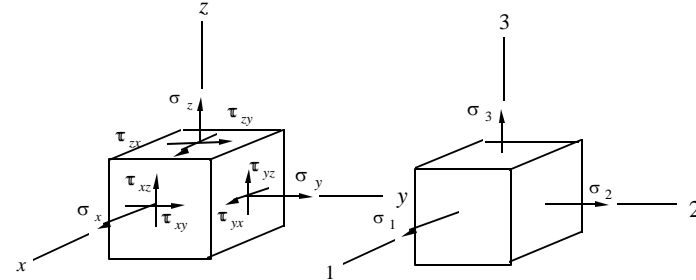
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19.2 Three-Dimensional Stress States

• It's useful to think about 3-d stress states using principal stresses:



Principal stresses

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$$

$$I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2$$

Maximum shear stresses

$$\tau_{13} = \frac{|\sigma_1 - \sigma_3|}{2}$$

$$\tau_{21} = \frac{|\sigma_2 - \sigma_1|}{2}$$

$$\tau_{32} = \frac{|\sigma_3 - \sigma_2|}{2}$$

$$\tau_{max} = \max(\tau_{13}, \tau_{21}, \tau_{32})$$

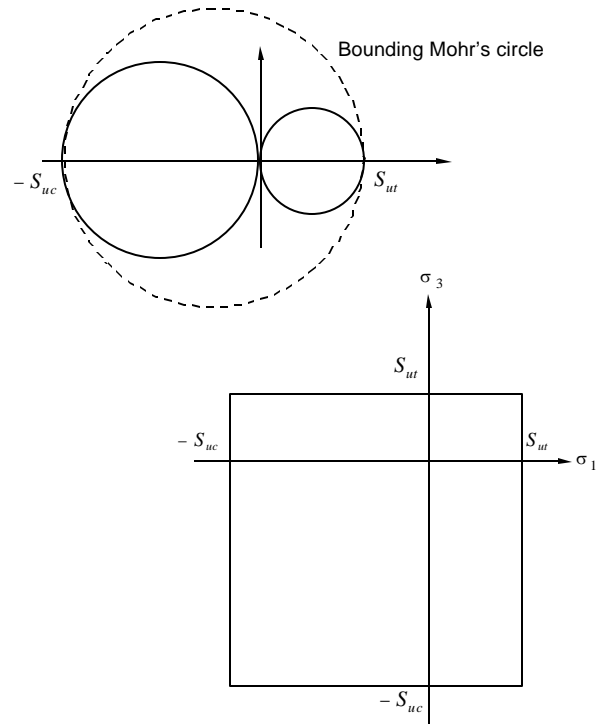
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19.3 Maximum Normal Stress Failure Theory

For biaxial stress state:



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19.4 Brittle Failure Under Static Loads

- Ductile materials fail by yielding
- Brittle materials fail by fracture

• Brittle fracture in tension is due to the normal tensile stress

• Brittle fracture in compression is due to a combination of normal compressive stress and shear stress

Use a failure theory based on the maximum normal tensile stress.

Requires a different failure theory.

- Many cast materials, such as gray cast iron, are brittle but have compressive strengths much higher than their tensile strengths (uneven material)
- Low tensile strength is due to presence of microscopic flaws which nucleate cracks when material is in tension
- Under compression the microscopic flaws are pressed together, increasing resistance and causing failure that depends on the normal compressive and shear stresses.

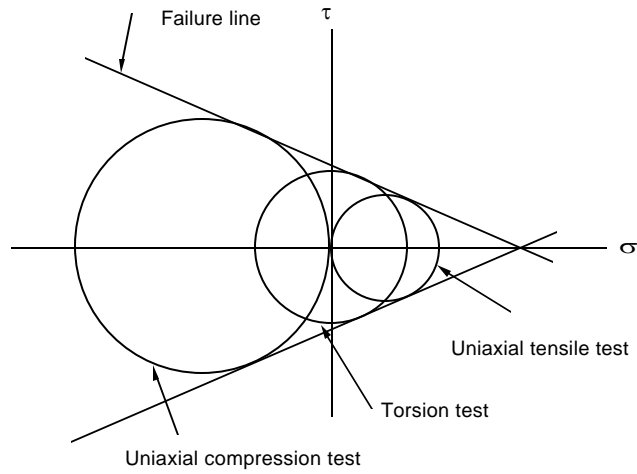
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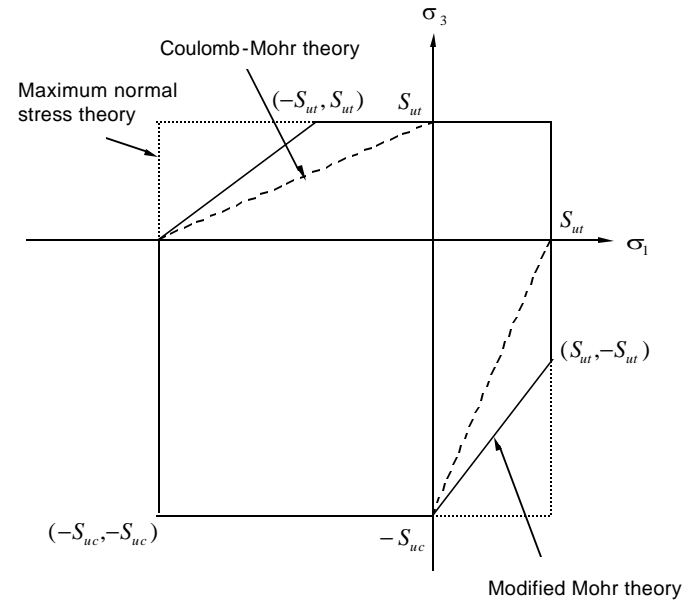
19.5 Coulomb-Mohr Failure Theory

- Ductile materials have a shear strength about one-half of tensile strength.
- Brittle materials have a shear strength which can be greater than their tensile strength.



In the compression region, the material's resistance to shear increases

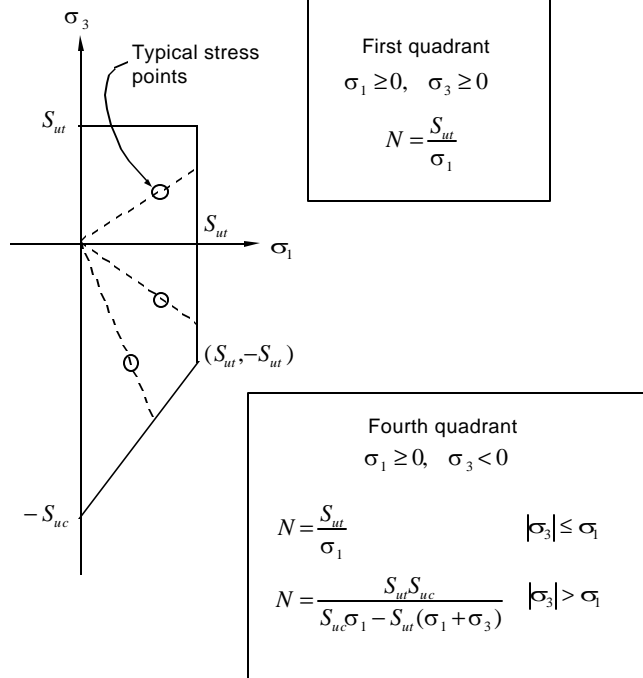
Coulomb-Mohr Failure Theory



Experimental evidence supports the modified Mohr theory

19.6 Factors of Safety in Brittle Failure

Modified Mohr theory:



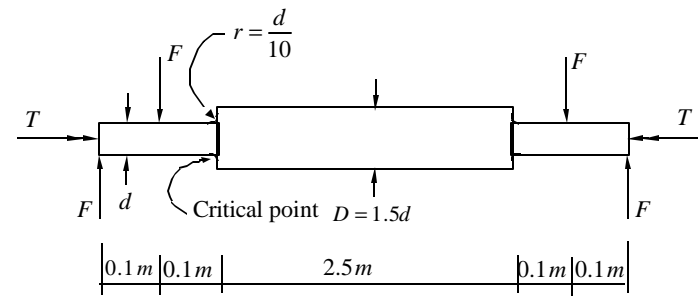
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Example 19.1

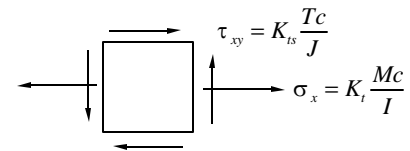
A torsion-bar spring is loaded with $F = 35 \text{ N}$ and $T = 8 \text{ N.m}$. The material is cast iron with $S_{ut} = 207 \text{ MPa}$ and $S_{uc} = 723 \text{ MPa}$. Determine a diameter d using a factor of safety against brittle failure of $N = 4$ using the modified Mohr failure theory.



Stress concentration factors:

Bending: $r/d = 0.1, D/d = 1.5, K_t = 1.68$

Torsion: $r/d = 0.1, D/d = 1.5, K_{ts} = 1.42$



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