

ME111
 Instructor: Peter Pinsky
 Class # 26
 November 27, 2000

Today's Topics

- Introduction to fatigue failure.
- Stages of fatigue failure.
- Creating S-N diagrams for design.
- Overview of approaches for high cycles fatigue (HCF).
- Fully Reversed Uniaxial Stress HCF.
- Fluctuating Uniaxial Stress HCF
- Fully Reversed Multiaxial Stresses HCF.
- Fluctuating Multiaxial Stresses HCF.

Reading Assignment

Juvinal, Chapter 8

Problem Set #8 Due in class Monday, December 4, 2000

*This is the final ME111 problem set and is worth **double** value.*

Ch.8: 5, 6, 7, 17, 19, 21, 23, 26, 28, 30, 33, 38, 46

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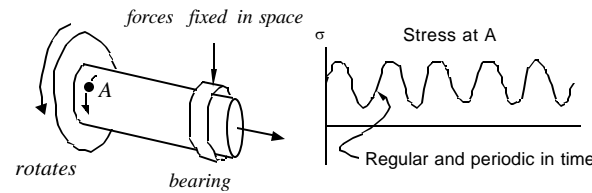
ME111 Lecture 26

1

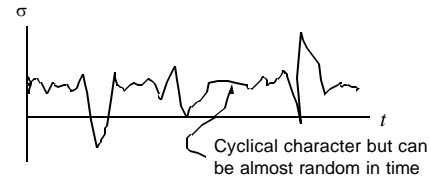
26.1 Failure Due to Time-Varying Loads

- Most failures in machinery are due to time-varying loads

Rotating machinery loads



Service equipment (e.g. ships, automobiles, airplanes)



- With time-varying loads, failure typically will occur at stress levels well below the yield strength.
- Failure under time-varying loads is called fatigue failure.

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2

26.2 Mechanism and Stages of Fatigue Failure

• Crack-Initiation Stage (C-I)

Microscopic cracks form at stress concentrations due to microstructural inhomogeneities such as particles, inclusions and voids

• Crack Propagation Stage (CP)

- Growth of the microcracks due to cyclical tensile stresses causing crack length growth due to plasticity at the crack tip.
- Crack length growth $O(10^{-8})$ to $O(10^{-4})$ inches per cycle.
- Corrosive environment can produce accelerated fatigue failure.

• Fracture (F)

- Fracture occurs when $K = K_C$
- Failure occurs instantaneously

11/28/00

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3

26.3 Fatigue-Failure Models

• Fatigue Regimes

Based on the number of cycles (of stress/strain) that the part must undergo in its lifetime, we consider:

Low-Cycle Fatigue (LCF)

$$0 \leq N \leq 10^3$$

High-Cycle Fatigue (HCF)

$$N > 10^3$$

• Stress-Life Approach

- Stress-based approach is most useful for HCF.
- Works best when stress cycles are regular.
- Seek the **fatigue strength** and **endurance limit**.
- Perform design based on factor of safety applied to the fatigue strength

• Strain-Life Approach

- Strain-based approach is most useful for describing the crack initiation stage.
- Most often applied to LCF.

• Linear Elastic Fracture Mechanics (LEFM)

- LEFM provides the best model for crack propagation stage.
- Most often applied to LCF.
- Especially useful for predicting life of parts with known cracks.

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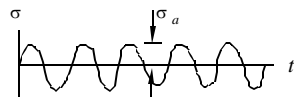
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4

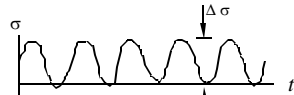
26.4 Characterizing Machine Loads

Two general types:

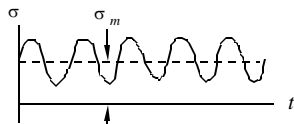
Rotating Machinery Loading



Fully reversed loading



Repeated loading



Fluctuating loading

Service Equipment Loading

Semi-random and does not repeat with any clear period

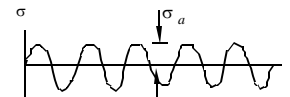


Stress Definitions (for rotating machinery-type loading)

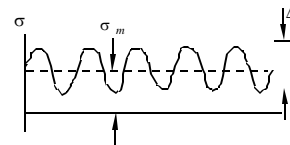
| | | | |
|--------------------|--|-----------------|---|
| Stress range | $\Delta \sigma = \sigma_{\max} - \sigma_{\min}$ | Stress ratio | $R = \frac{\sigma_{\min}}{\sigma_{\max}}$ |
| Alternating stress | $\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$ | Amplitude ratio | $A = \frac{\sigma_a}{\sigma_m}$ |
| Mean stress | $\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$ | | |

26.5 Overview of Approach to HCF Analysis

| | |
|--|--|
| I Fully reversed Uniaxial stress | II Fluctuating Uniaxial stress |
| III Fully reversed Multiaxial stresses | IV Fluctuating Multiaxial stresses |

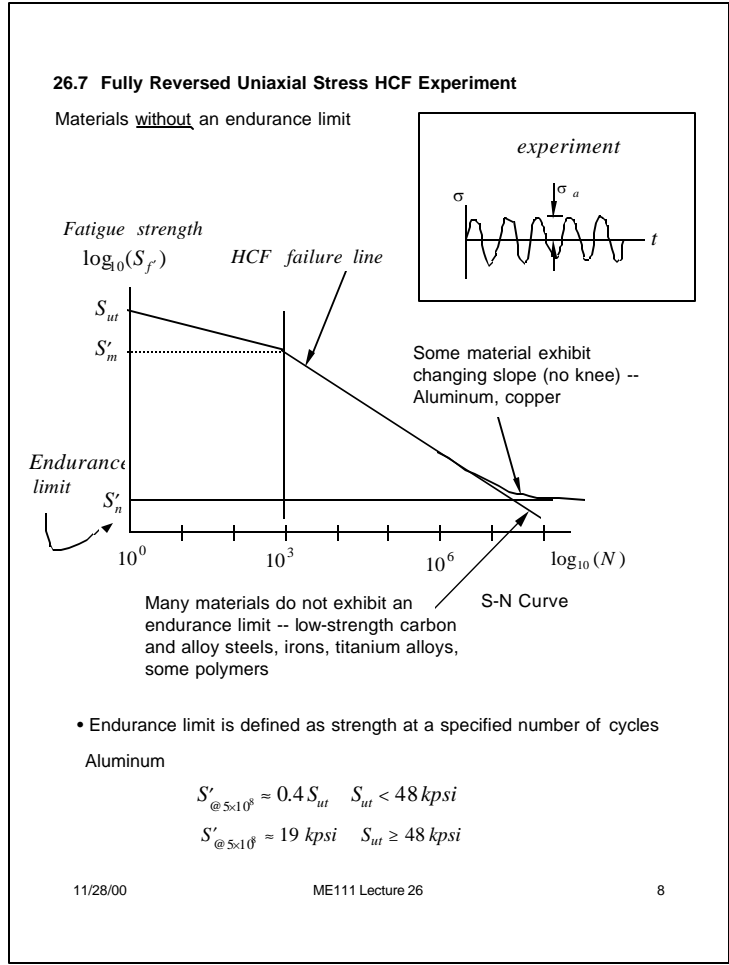
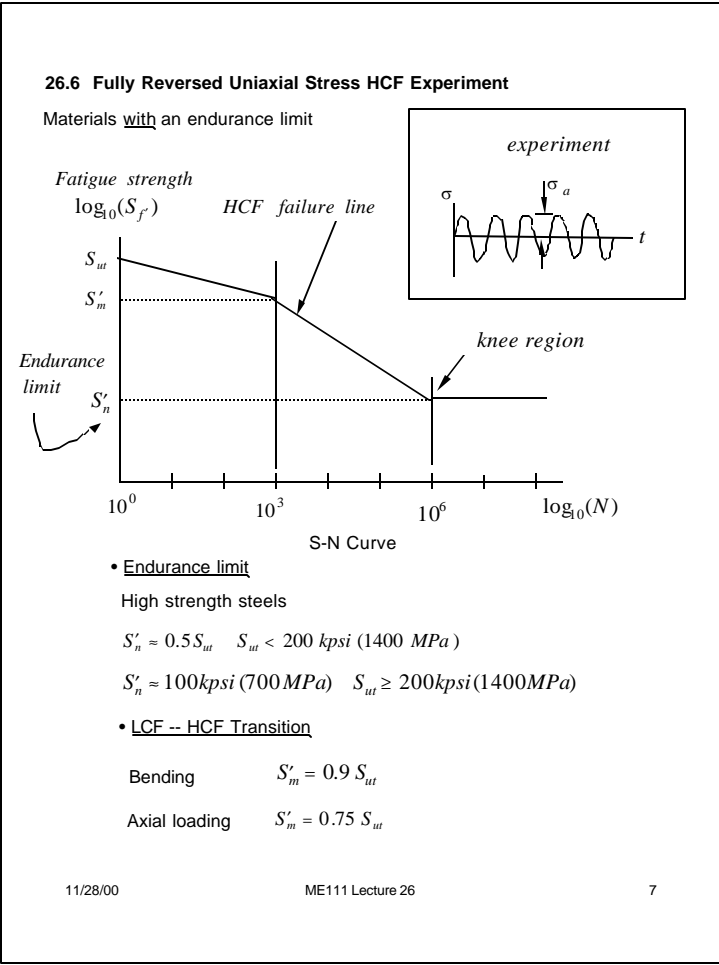


Fully reversed



Fluctuating

| | | |
|--------------------|--|--------------------------|
| Stress range | $\Delta \sigma = \sigma_{\max} - \sigma_{\min}$ | Fatigue life: |
| Alternating stress | $\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$ | $\sigma_m > 0$ reduced |
| Mean stress | $\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$ | $\sigma_m < 0$ increased |



26.8 Basic relationship between S'_n , S' and S_{ut}

Steels

$$S'_n = 0.5 S_{ut} \quad S_{ut} < 200 \text{ kpsi (1400 MPa)}$$

$$S'_n = 100 \text{ kpsi (700 MPa)} \quad S_{ut} \geq 200 \text{ kpsi (1400 MPa)}$$

Irons

$$S'_n = 0.4 S_{ut} \quad S_{ut} < 60 \text{ kpsi (400 MPa)}$$

$$S'_n = 24 \text{ kpsi (160 MPa)} \quad S_{ut} \geq 60 \text{ kpsi (400 MPa)}$$

Aluminums (No endurance limit)

$$S'_{@5 \times 10^8} = 0.4 S_{ut} \quad S_{ut} < 48 \text{ kpsi (330 MPa)}$$

$$S'_{@5 \times 10^8} = 19 \text{ kpsi} \quad S_{ut} \geq 48 \text{ kpsi (330 MPa)}$$

Copper Alloys (No endurance limit)

$$S'_{@5 \times 10^8} = 0.4 S_{ut} \quad S_{ut} < 40 \text{ kpsi (280 MPa)}$$

$$S'_{@5 \times 10^8} = 14 \text{ kpsi} \quad S_{ut} \geq 40 \text{ kpsi (280 MPa)}$$

26.9 Correcting experimental data for design

Corrected endurance limit:

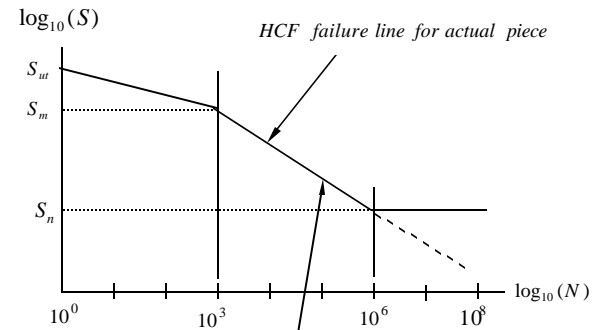
$$S_n = C_L C_G C_S S'_n$$

Corrected fatigue strength:

$$S = C_L C_G C_S S'$$

See Juvinall Sect 8.8 for coefficients

True fatigue strength



Fatigue strength at $N > 10^3$ cycles:

$$S_n = aN^b$$

$$\log_{10}(a) = \log_{10}(S_m) - 3b$$

$$b = \frac{1}{-3} \log\left(\frac{S_m}{S_n}\right)$$

26.10 Factor of Safety

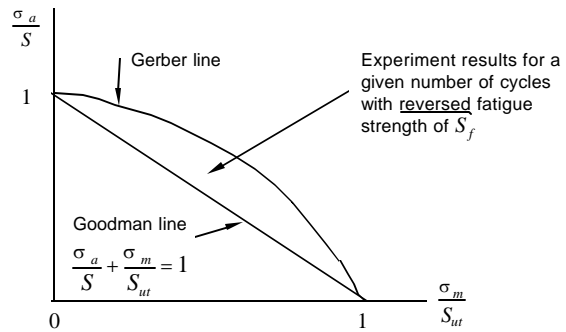
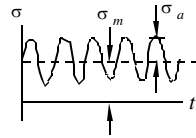
• Design for HCF:

$$N_f = \frac{S_f}{\sigma_a}$$

• Design for Infinite Life Fatigue:

$$N_f = \frac{S_e}{\sigma_a}$$

26.11 Fluctuating Uniaxial Stress HCF

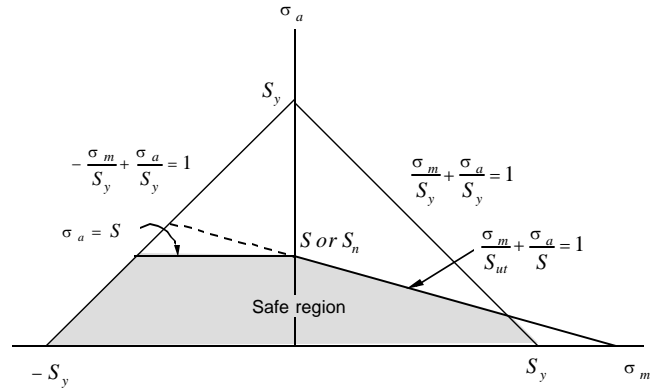


11/28/00

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11

Safe/Failure Regions



Important note:

If the material exhibits an endurance limit, the above figure can be used for infinite life fatigue by replacing

S with S_n

11/28/00

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12

Factor of Safety -- 1

Example -- 1

The alternating stress is constant over the life of the part but the mean stress can increase:

$$\frac{\sigma_m}{S_y} + \frac{\sigma_a}{S_y} = 1 \Rightarrow \sigma_{m, fail} = \left(1 - \frac{\sigma_a}{S_y}\right) S_y$$

$$N_f = \frac{\sigma_{m, fail}}{\sigma_m} = \frac{\left(1 - \frac{\sigma_a}{S_y}\right) S_y}{\sigma_m}$$

11/28/00 ME111 Lecture 26 13

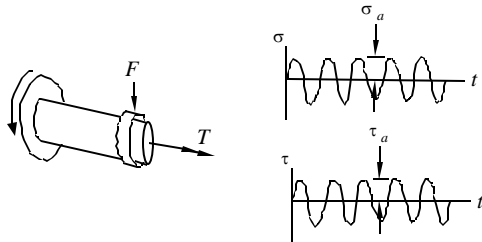
Factor of Safety -- 2

Example 2

The alternating stress and mean stress can increase in a fixed ratio, OB_2 :

11/28/00 ME111 Lecture 26 14

26.12 Fully Reversed Multiaxial Stresses HCF



- Assume stress components are synchronous (in phase) (called simple multiaxial stress)

- von Mises (effective) alternating stress 3-d:

$$\sigma'_a = \sqrt{\frac{(\sigma_{ax} - \sigma_{ay})^2 + (\sigma_{ay} - \sigma_{az})^2 + (\sigma_{az} - \sigma_{ax})^2 + 6(\tau_{axy}^2 + \tau_{ayz}^2 + \tau_{azx}^2)}{2}}$$

- von Mises (effective) alternating stress in plane stress:

$$\sigma'_a = \sqrt{\sigma_{ax}^2 + \sigma_{ay}^2 - \sigma_{ax}\sigma_{ay} + 3\tau_{axy}^2}$$

Factor of Safety

- Design for HCF:

$$N_f = \frac{S_f}{\sigma'_a}$$

- Design for Infinite Life Fatigue:

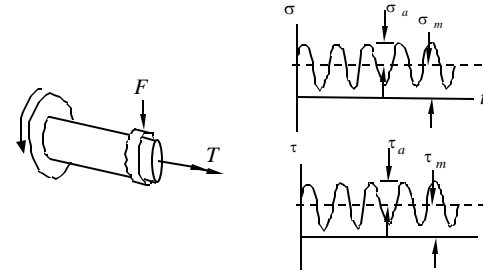
$$N_f = \frac{S_e}{\sigma'_a}$$

11/28/00

ME111 Lecture 26

15

26.13 Fluctuating Multiaxial Stresses HCF



- Assume stress components are synchronous (in phase) (called simple multiaxial stress)

- von Mises (effective) alternating stress in plane stress:

$$\sigma'_a = \sqrt{\sigma_{ax}^2 + \sigma_{ay}^2 - \sigma_{ax}\sigma_{ay} + 3\tau_{axy}^2}$$

- Effective mean stress in plane stress:

$$\sigma'_m = \sigma_{mx} + \sigma_{my} + \sigma_{mz} \quad (\text{Sines method})$$

$$\sigma'_m = \sqrt{\sigma_{mx}^2 + \sigma_{my}^2 - \sigma_{mx}\sigma_{my} + 3\tau_{mxy}^2} \quad (\text{von Mises method})$$

Factor of Safety

- Design for HCF:

Same as Fluctuating Uniaxial Stress HCF

- Design for Infinite Life Fatigue:

Same as Fluctuating Uniaxial Stress HCF

11/28/00

ME111 Lecture 26

16