

ME111
 Instructor: Peter Pinsky
 Class #5
 October 6, 2000

Today's Topics

- Stresses caused by axial, bending, shear and torsion loadings.
- Stresses caused by combinations of loads – combined stresses

Reading Assignment

Juvinall Sections 4.1-4.8

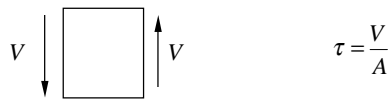
5.0 The Stress Tensor

Recall that stress is defined as load per unit area:

Normal stress



Shear stress



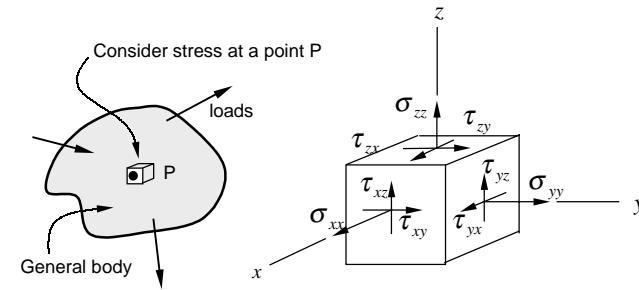
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5.1 Stress in Three Dimensions

- Stress tensor at point is expressed in terms of *components* with respect to a set of fixed coordinate axes.
- Consider a general state of stress in 3-d:



- There are 3 components of **normal stress**

$$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$$

- There are 6 components of **shear stress**, but only 3 are *independent*

$$\tau_{xy}, \tau_{yz}, \tau_{zx}$$

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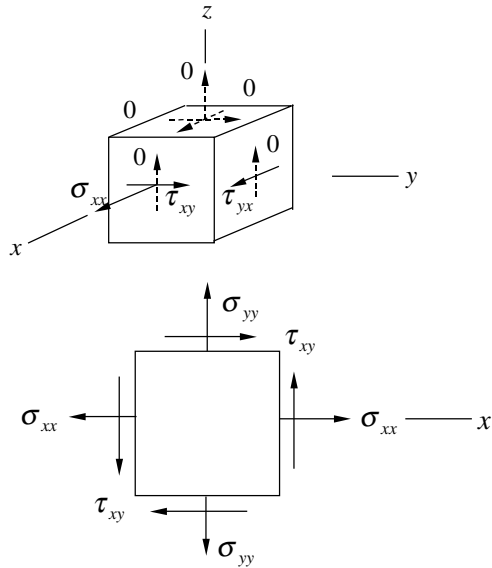
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5.2 Plane Stress

- We'll often work with the simpler, but also important, case of 2-d stresses

- Plane stress assumption:

$$\sigma_{zz} = 0, \tau_{zx} = 0, \sigma_{yz} = 0$$



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5.3 Axial Loading

Normal stress due to axial force $\sigma = \frac{P}{A}$

5.4 Bending

Normal stress due to bending moment $\sigma = \frac{My}{I}$ $\sigma_{\max} = \frac{Mc}{I}$

Shear stress due to bending moment $\tau = \frac{VQ}{It}$ $Q = \bar{y}'A'$ $\tau_{\max} = \frac{4V}{3A}$

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5.5 Direct Shear Loading

$$\tau = \frac{V}{A}$$

5.6 Torsional Loading

Shear stress due to torque

$$\tau = \frac{Tr}{J} \quad J = \frac{\pi D^4}{32} \quad \tau_{\max} = \frac{Tc}{J}$$

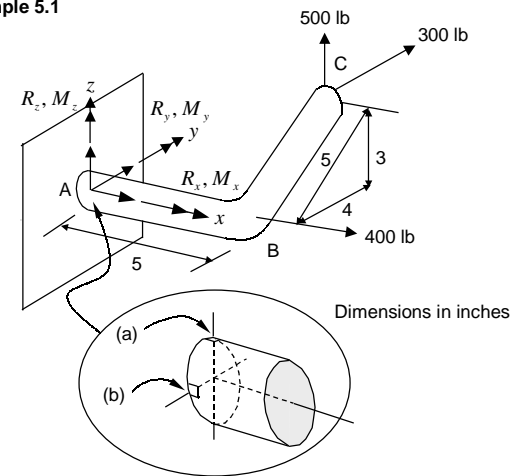
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5.7 Combined Stresses

Example 5.1



The solid rod shown has a radius of 0.75 in. Determine the stresses at points (a) and (b).

Solution approach:

1. Compute internal forces at A.
2. Determine normal and shear stresses due to each of the individual internal forces.
3. Combine the normal and shear stresses.

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1. Compute Reactions at A.

Equilibrium:

$$\sum \mathbf{F} = \mathbf{0}: \quad \begin{Bmatrix} R_x \\ R_y \\ R_z \end{Bmatrix} + \begin{Bmatrix} 400 \\ 300 \\ 500 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \begin{Bmatrix} R_x \\ R_y \\ R_z \end{Bmatrix} = \begin{Bmatrix} -400 \\ -300 \\ -500 \end{Bmatrix}$$

$$\sum \mathbf{M}_A = \mathbf{0} \quad \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} + \mathbf{r} \times \mathbf{F} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

where $\mathbf{r} = \begin{Bmatrix} 5 \\ 4 \\ 3 \end{Bmatrix}$ $\mathbf{F} = \begin{Bmatrix} 0 \\ 300 \\ 500 \end{Bmatrix}$

Recall $\mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 4 & 3 \\ 0 & 300 & 500 \end{vmatrix}$

$$= 1100\mathbf{i} - 2500\mathbf{j} + 1500\mathbf{k}$$

So that

$$\begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} + \begin{Bmatrix} 1100 \\ -2500 \\ 1500 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{Bmatrix} -1100 \\ 2500 \\ -1500 \end{Bmatrix}$$

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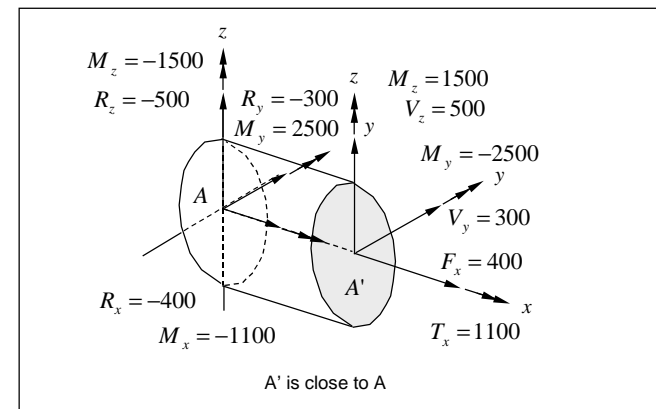
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2. Compute Internal Forces Near Support.

Reaction Forces at A:

$$R_x = -400, \quad R_y = -300, \quad R_z = -500$$

$$M_x = -1100, \quad M_y = 2500, \quad M_z = -1500,$$



Reaction Forces at A':

$$F_x = 400, \quad V_y = 300, \quad V_z = 500$$

$$T_x = 1100, \quad M_y = -2500, \quad M_z = 1500,$$

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Bending Moments, Torques and Axial Forces at A'

Stresses Due to Internal Force Resultants

Normal stress due to axial force $\sigma = \frac{P}{A}$

Normal stress due to bending moment $\sigma = \frac{My}{I} \quad I = \frac{\pi D^4}{64} \quad \sigma_{\max} = \frac{Mc}{I}$

Shear stress due to bending moment $\tau = \frac{VQ}{It} \quad Q = \bar{y}'A' \quad \tau_{\max} = \frac{4V}{3A}$

Shear stress due to torque $\tau = \frac{Tr}{J} \quad J = \frac{\pi D^4}{32} \quad \tau_{\max} = \frac{Tc}{J}$

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Mathcad Worksheets

$T_x := 1100$

$M_y := 2500$

$M_z := 1500$

$P_x := 400$

$V_y := 300$

$V_z := 500$

$D := 1.5$

$c := \frac{D}{2}$

$I := \pi \frac{D^4}{64}$

$J := \pi \frac{D^4}{32}$

$A := \pi \frac{D^2}{4}$

Stress due to P_x

$\sigma_1 := \frac{P_x}{A}$

Stress due to V_y

$\tau_2 := \frac{4}{3} \frac{V_y}{A}$

Stress due to V_z

$\tau_3 := \frac{4}{3} \frac{V_z}{A}$

Stress due to T_x

$\tau_4 := \frac{T_x c}{J}$

Stress due to M_y

$\sigma_5 := \frac{M_y c}{I}$

Stress due to M_z

$\sigma_6 := \frac{M_z c}{I}$

$\sigma_1 = 226.354 \quad \tau_2 = 226.354 \quad \tau_3 = 377.256$

$\tau_4 = 1.66 \times 10^3 \quad \sigma_5 = 7.545 \times 10^3 \quad \sigma_6 = 4.527 \times 10^3$

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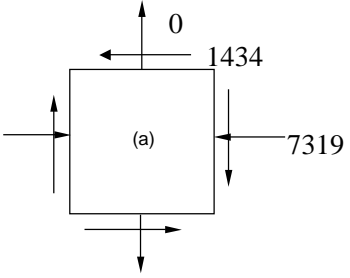
Mathcad Worksheets

Stress at point (a)

$$\sigma_x := \sigma_1 - \sigma_5$$

$$\sigma_y := 0$$

$$\tau_{xy} := -\tau_4 + \tau_2$$

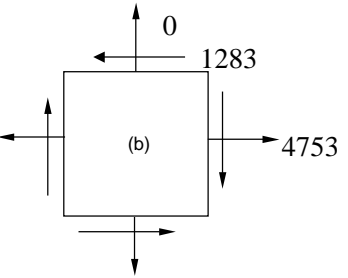
$$\sigma_x = -7.319 \times 10^3 \quad \sigma_y = 0 \quad \tau_{xy} = -1.434 \times 10^3$$


Stress at point (b)

$$\sigma_x := \sigma_1 + \sigma_6$$

$$\sigma_y := 0$$

$$\tau_{xy} := -\tau_4 + \tau_3$$

$$\sigma_x = 4.753 \times 10^3 \quad \sigma_y = 0 \quad \tau_{xy} = -1.283 \times 10^3$$


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