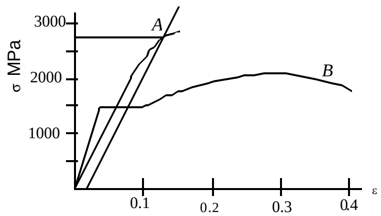
ME111
Mid-Term Exam Solution
October 25, 1999

Problem 1 (25 Points)



Problems (a) - (f), refer to figure above.

(a) Characterize material A and B as ductile or brittle. A brittle, B ductile
(b) Which is the stiffest material? B (highest E value)
(c) Which has the highest ultimate strength? A
(d) If material A has a specified 2% offset strain, determine its approximate yield strength using the above figure. $S_y = 3{,}750\ MPa$
(e) Which material has the largest modulus of resilience? A
(f) Which material has the largest modulus of toughness? B

(g) If a metal has the following experimentally determined properties:

$$E = 72\,GPa, \quad S_u = 200\,MPa, \quad U_R = 0.24\,MPa, \quad U_T = 23.1\,MPa$$

Estimate the yield strength, Brinell hardness, and strain at fracture.
Is the material brittle or ductile? Why?

$$U_R = \frac{S_y^2}{2E} \Rightarrow S_y = 186\ MPa \qquad U_T = \left(\frac{S_y + S_u}{2}\right)\varepsilon_f \Rightarrow \varepsilon_f = 0.12$$

$$S_u = 3.45 H_B \Rightarrow H_B = 58\ \text{(soft material)}$$

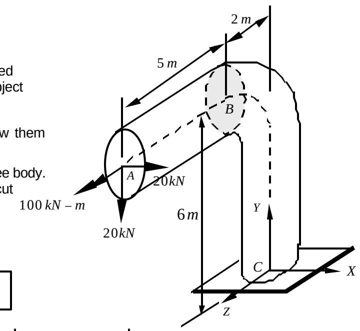
Since the fracture strain is greater than 5%, this material is ductile.

Problem 2 (25 Points)

A circular bar is bent into a bracket and supported by the plate at C as shown. The bracket is subject to forces and a torque at A.

(a) Determine all support reactions at C and show them acting in a drawing

(b) Cut the bracket at B and consider AB as a free body. Determine the internal forces acting on the cut at B and show them acting in a drawing.



(a) $\sum \mathbf{F} = \mathbf{0}$

$$F_x = -20; \quad F_y = 20; \quad F_z = 0$$

$$\sum \mathbf{M} + \sum \mathbf{r} \times \mathbf{F} = \mathbf{0}$$

$$[M_{CX}\mathbf{i} + M_{CY}\mathbf{j} + M_{CZ}\mathbf{k}] + [100\mathbf{k}] + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 7 \\ 20 & -20 & 0 \end{vmatrix} = \mathbf{0}$$

$$M_{CX}\mathbf{i} + M_{CY}\mathbf{j} + M_{CZ}\mathbf{k} + 100\mathbf{k} + (140\mathbf{i} + 140\mathbf{j} - 120\mathbf{k}) = \mathbf{0}$$

$$M_{CX} = -140; \quad M_{CY} = -140; \quad M_{CZ} = 20$$

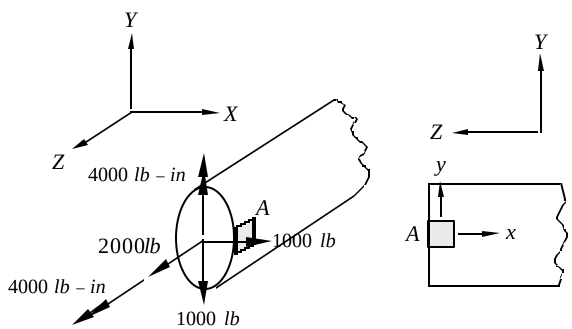
(b) $\sum \mathbf{F} = \mathbf{0}$ $\quad F_x = -20; \quad F_y = 20; \quad F_z = 0$

$$\sum \mathbf{M} + \sum \mathbf{r} \times \mathbf{F} = \mathbf{0}$$

$$M_{BX}\mathbf{i} + M_{BY}\mathbf{j} + M_{BZ}\mathbf{k} + 100\mathbf{k} + (100\mathbf{i} + 100\mathbf{j}) = \mathbf{0}$$

$$M_{BX} = -100; \quad M_{BY} = -100; \quad M_{CZ} = -100$$

Problem 3 (25 Points)



A circular bar of diameter $D = 4\ in$ is subject to the internal resultants shown. Determine the “x-y” state of plane stress at point A and show the stress components in a drawing of the “stress element.”

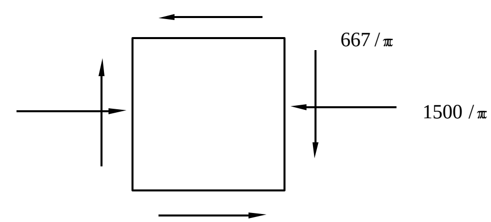
For your convenience, note that:

$$A = \frac{\pi D^2}{4} = 4\pi$$

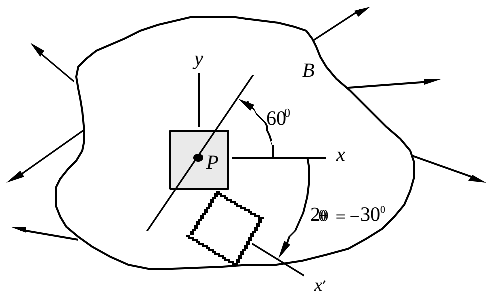
$$I_x = I_y = \frac{\pi D^4}{64} = 4\pi$$

$$J_z = \frac{\pi D^4}{32} = 8\pi$$

	Sign
$P_X :\ ----$	
$P_Y :\ \tau_{xy} = \frac{4}{3}\cdot\frac{V_Y}{A} = \frac{4}{3}\cdot\frac{1000}{4\pi} = 333/\pi$	positive
$P_Z :\ \sigma_x = \frac{P_Z}{A} = \frac{2000}{4\pi} = 500/\pi$	positive
$M_Y :\ \sigma_x = \frac{Mc}{I} = \frac{4000 * 2}{4\pi} = 2000/\pi$	negative
$M_Z :\ \tau_{xy} = \frac{Tc}{J_Z} = \frac{4000 * 2}{8\pi} = 1000/\pi$	negative



Problem 4 (25 Points)

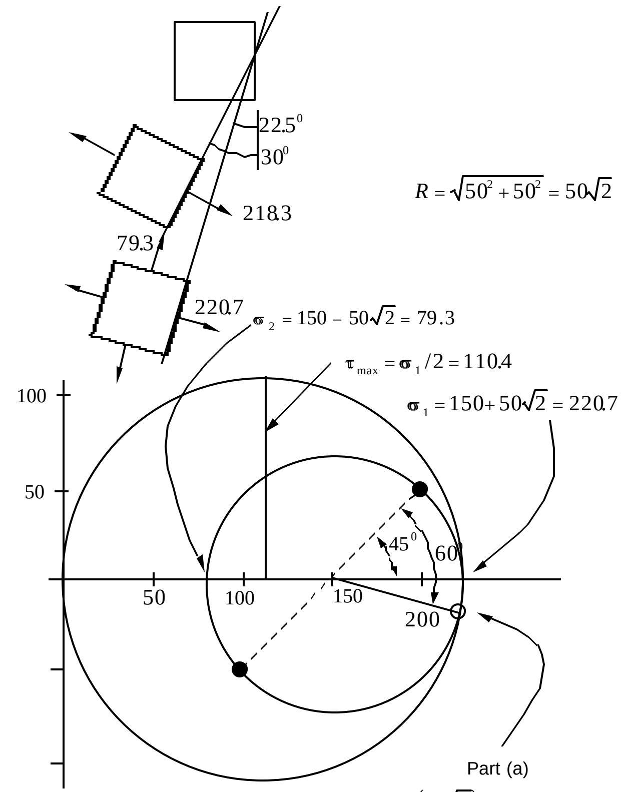


The state of plane stress in a body B at point P is given as:

$$\sigma_x = 200, \quad \sigma_y = 100, \quad \tau_{xy} = -50$$

(a) Determine the normal stress across the line AB at the point P (use any approach you wish).

(b) **Using Mohr's circle**, determine the principal stresses and maximum shear stresses and show them acting on a properly oriented element.



Part (a)

$$\sigma_{x'} = 150 + (50\sqrt{2})\cos(-15^0) = 218.3$$