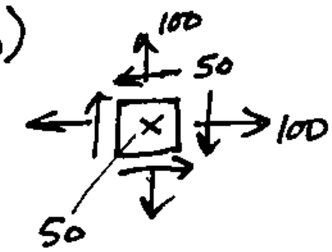
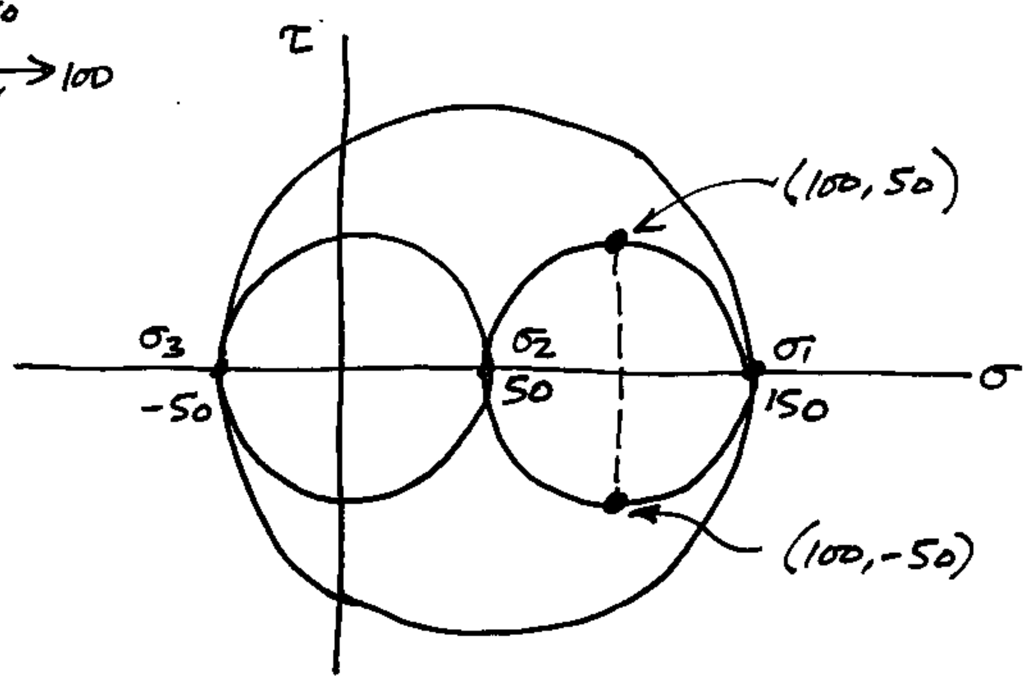
ME111 Mid-Term Solution

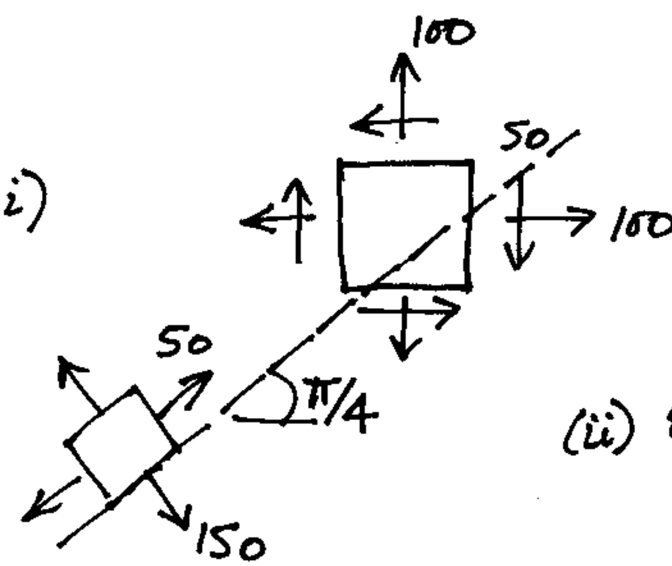
Autumn 2000

Problem (1)

(a) Principal stresses are the largest normal stresses for a given stress state

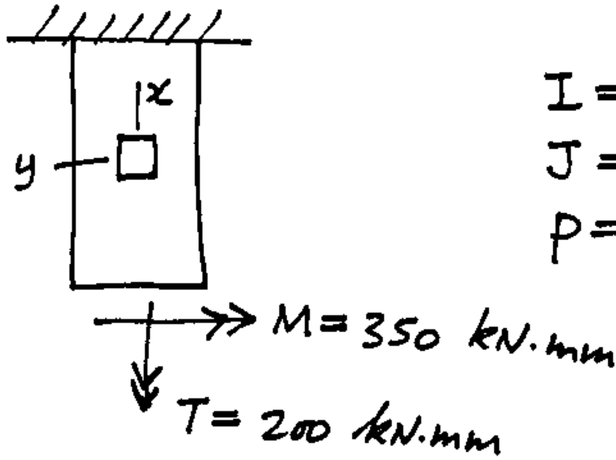
(b) 



(i) 

(ii) $\tau_{max} = 100$ MPa

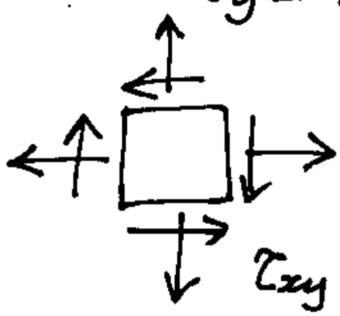
(c)



$I = 2.7 \times 10^4 \ mm^4$

$J = 2I$

$p = 20$ MPa



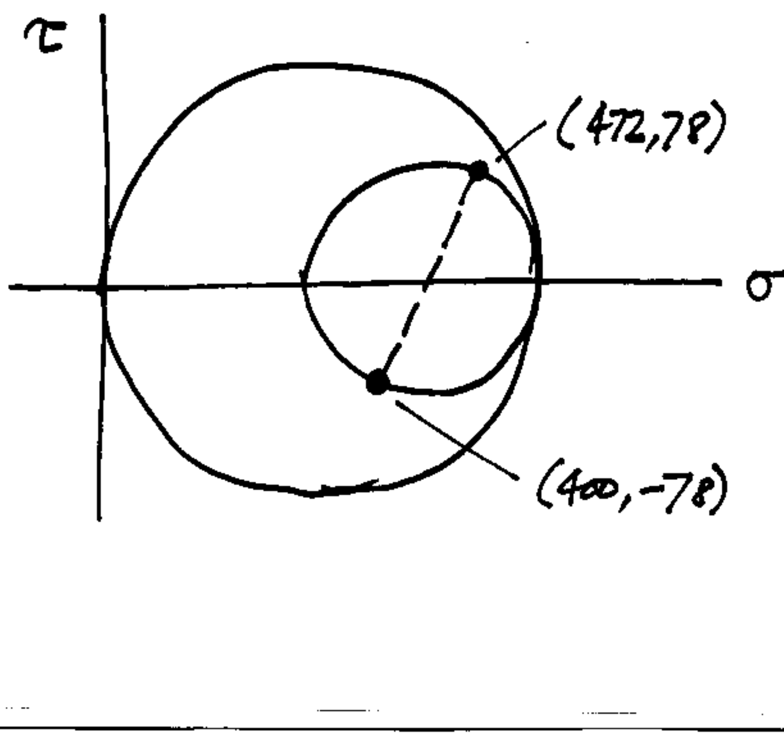
$\sigma_y = \frac{pr}{t}$

$\sigma_x = \frac{pr}{2t} + \frac{Mc}{I}$

$\tau_{xy} = \frac{Tc}{J}$

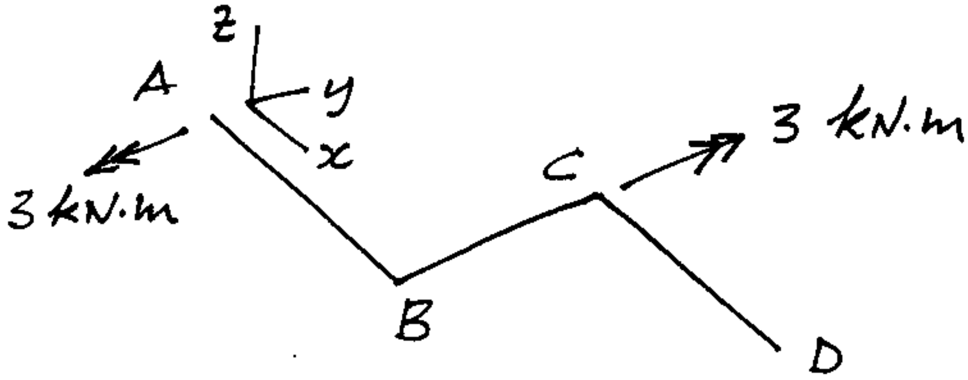
$$\sigma_x = \frac{(20)(20)}{(2)(1)} + \frac{(35 \times 10^4)(21)}{2.7 \times 10^4} = 472 \text{ MPa}$$

$$\sigma_y = \frac{(20)(20)}{1} = 400 \text{ MPa} \ ; \ \tau_{xy} = \frac{(2 \times 10^5)(21)}{5.4 \times 10^4} = 78 \text{ MPa}$$



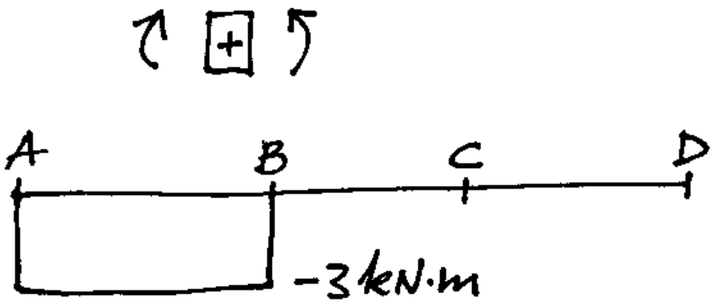
Problem (2)

(a)

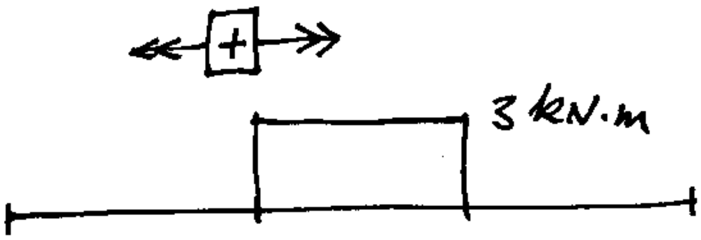


$M_y = -3$ kN·m

(b)

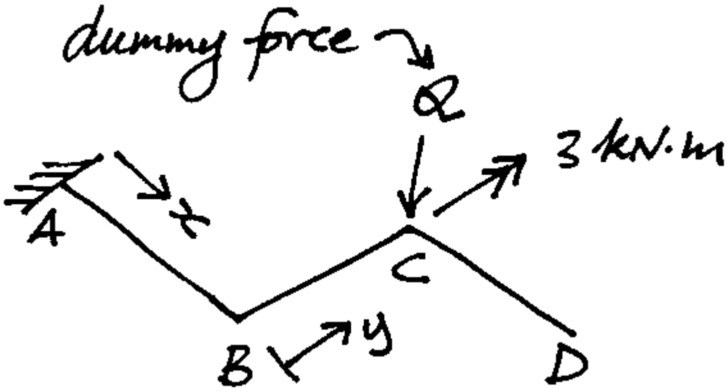


Bending Moment



Torque

(c)



Use N, mm
MPa

Member A-B

$$M_{AB} = -3\times10^6 - Q(2000-x) \qquad T_{AB} = -2000Q$$

$$M_{AB}|_{Q=0} = -3\times10^6 \qquad T_{AB}|_{Q=0} = 0$$

$$\frac{\partial M_{AB}}{\partial Q} = -(2000-x)$$

Member B-C

$$M_{BC} = -Q(2000-y) \qquad T_{BC} = 3\times10^6$$

$$M_{BC}|_{Q=0} = 0 \qquad T_{BC}|_{Q=0} = 3\times10^6$$

$$\frac{\partial T_{BC}}{\partial Q} = 0$$

$$\delta = \sum\int \frac{M(Q=0)\,\partial M/\partial Q}{EI}\,dx + \sum\int \frac{T(Q=0)\,\partial T/\partial Q}{GJ}$$

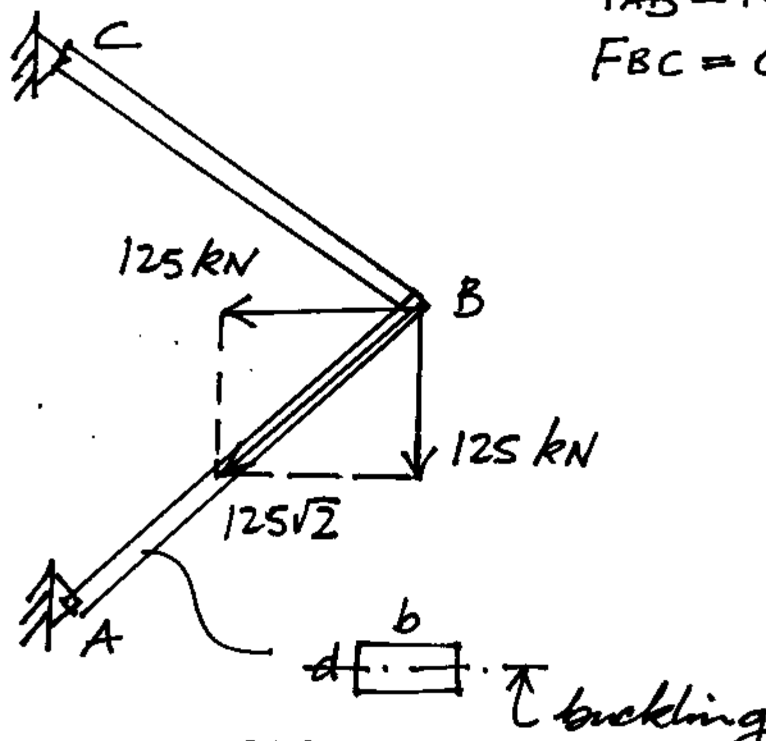
$$= \int_0^{2000} \frac{-(-3\times10^6)(2000-x)}{EI}\,dx$$

$$E = 60{,}000 \text{ MPa} \qquad I = \frac{\pi d^4}{64} = \frac{\pi(50)^4}{64} = 307\times10^3 \text{ mm}^4$$

$$\delta = \frac{3\times10^6}{(6\times10^4)(307\times10^3)}\left[2000x - \frac{x^2}{2}\right]_0^{2000}$$

$$= 325 \text{ mm} \quad \text{Ans.}$$

Problem (3)



$F_{AB} = 125\sqrt{2}$ compr.
$F_{BC} = 0$

Compare S_r to $\pi\sqrt{\frac{2E}{S_y}}$:

$$S_r = \frac{L_{eff}}{\rho} = \frac{600}{\sqrt{I/A}}$$

$$= \frac{600}{\sqrt{\left(\frac{bd^3}{12}\right)\left(\frac{1}{bd}\right)}} = \frac{600}{d/\sqrt{12}} = \frac{600}{(25)/\sqrt{12}} = 83.1$$

$$(S_r)_{y/2} = \pi\sqrt{\frac{2E}{S_y}} = \pi\sqrt{\frac{(2)(2\times10^5)}{400}} = \pi\sqrt{1000} = 99.3$$

$S_r < (S_r)_{y/2} \Rightarrow$ Johnson Theory

$$P_{cr} = (\sigma_{cr})(A) = \left(S_y - \frac{1}{E}\left(\frac{S_y S_r}{2\pi}\right)\right)(bd)$$
$$= 260 \text{ kN}$$

$$FS = \frac{P_{cr}}{P} = \frac{260}{125\sqrt{2}} = \underline{1.47} \text{ Ans.}$$