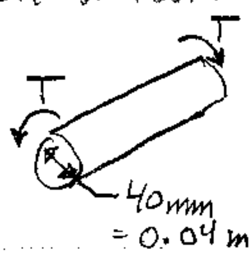
# Solutions to Problem Set #3

4.12) What torque is required to produce a maximum shear stress of 400 MPa:

a) In a round shaft of 40mm diameter?




Assumptions:
1) Bar is straight
2) Torque is along longitudinal axis
3) No stress raisers
4) The material is homogeneous & perfectly elastic

From Eq. 4.4
$$\tau = \frac{16T}{\pi d^3} \Rightarrow T = \frac{\pi d^3 \tau}{16} = \frac{\pi(0.04m)^3(400 \times 10^6 Pa)}{16}$$

$$\boxed{T = 5030\ N \cdot m}$$

b) From Eq. 4.5
$$\tau = \frac{4.8T}{a^3} \Longrightarrow \boxed{T = \frac{\tau a^3}{4.8} = \frac{(400 \times 10^6 Pa)(0.04m)^3}{4.8} = 5330\ N \cdot m}$$

note $a = b$... in Eq. 4.5

4.17) What bending moment is required to produce a maximum normal stress of 400 MPa:

a) In a straight round rod of 40-mm diameter?

Assumptions & Picture are same as 4.12 (a)

From Eq. 4.8:
$$M = \frac{\sigma_{max} \pi d^3}{32} = \frac{(400 \times 10^6 Pa)\pi(.04m)^3}{32} = 2510\ N \cdot m$$

b) In a straight square rod, 40mm on a side (with bending about the X-axis as shown for a rectangular section in Appendix B-2)?

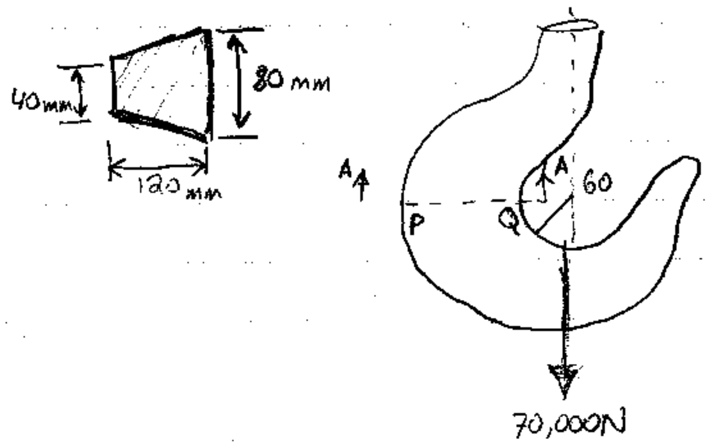
Note: Appendix B-2a is actually more useful!

14.17 b) con't:

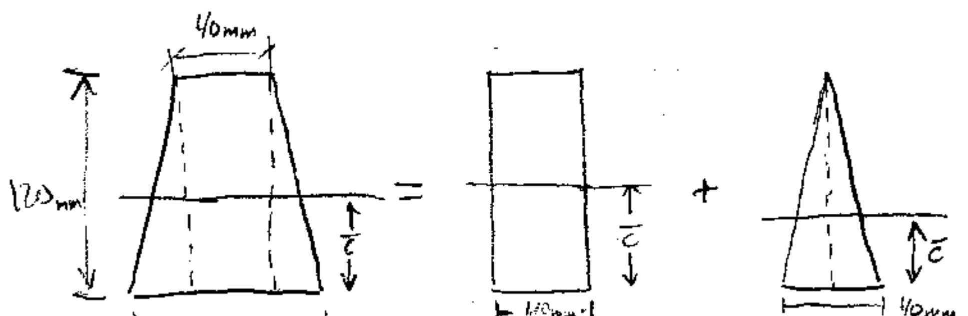
From Eq. 4.7: $\sigma = \frac{M}{Z} \Rightarrow M = \frac{\sigma_{max}}{Z}$, $Z = \frac{bh^2}{6} = \frac{b^3}{6}$

$$M = \frac{b^3 \sigma}{6} = \frac{(0.04\,m)^3 (400 \times 10^6\,Pa)}{6} = 4270\ N \cdot m$$

4.21) Critical section AA of a crane hook (below) is considered, for purposes of analysis, to be trapezoidal with dimensions as shown. Determine the resultant stress (bending plus direct tension) at points P & Q.




1) Step 1: Find $\bar{c}$ and $\bar{r}$, 'cuz we know we'll need 'em in Fig. 4.11




A trapezoid is a rectangle plus two triangles!

$\bar{c}_{rect} = 60\,mm = 0.06\,m$ $\qquad$ $\bar{c}_{tri} = 40\,mm = 0.04\,m$

Now, we look @ weighted average of the $\bar{c}$'s...

$$\bar{c}_{trap} = \frac{A\bar{c}\,(rect) + A\bar{c}\,(triangle)}{A\,(trap)} = \frac{(.0048\,m^2)(.06\,m) + (.0024\,m^2)(.04\,m)}{(.0048 + .0024\,m^2)}$$

$$\bar{c}_{trap} = 0.053\,m$$

4.21 con't)

2.) Find I, cuz we'll need that too!

$I_{trap}$ (about $\bar{c}_{trap}$) = $I_{rect} + I_{tri}$ (each about $\bar{c}_{trap}$) ← Parallel axis theorem!

Assume that the material is homogeneous, with a unit weight of 1. (i.e. in the "$mr^2$" term of the parallel axis theorem, we can use the area as the mass).

$$I_{trap} = I_{rect} + m(\bar{c}_{rect} - \bar{c}_{trap})^2 + I_{triangle} + m(\bar{c}_{tri} - \bar{c}_{trap})^2$$

$$= \frac{bh^3}{12} + A_{rect}(\bar{c}_{rect} - \bar{c}_{trap})^2 + \frac{bh^3}{36} + A_{tri}(\bar{c}_{tri} - \bar{c}_{trap})^2$$

$$= \frac{(.04m)(.12m)^3}{12} + (.0048m^2)(.06 - .053m)^2 + \frac{(.04m)(.12m)^3}{36} + (.0024m^2)(.04 - .053m)^2$$

$$= 5.76\times10^{-6}m^4 + 2.35\times10^{-7}m^4 + 1.92\times10^{-6}m^4 + 4.056\times10^{-7}m^4$$

or

$$= 5.76\times10^{-6}m^4 + 2.13\times10^{-7}m^4 + 1.92\times10^{-6}m^4 + 4.26\times10^{-7}m^4$$

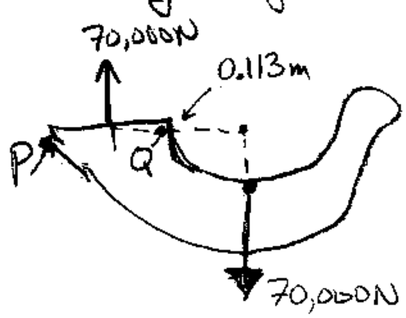
↙ depending on round off

$$I = 8.32\times10^{-6}\,m^4$$

3. $\frac{\bar{r}}{\bar{c}} = \frac{.06\,m\text{ (the radius of the crane)} + .053\,m\,(\bar{c})}{.053\,m} = 2.13 \xrightarrow{\text{Fig 4.11}} K_i = 1.52,\ K_o = 0.73$

BLEACH!

4. Free Body Diagram Time (NOT "Force Body Diagram"!)




So...

$\sigma = \frac{P}{A}$, $\sigma = K\frac{Mc}{I}$

↗ tension

So at P, the moment from the load is opposite to the tensile stress, & at Q the moment wants to deform Q in tension as well!

at P

c = dist. from neutral axis to most extreme fiber!

$$\sigma = \frac{P}{A} - K_o \frac{Mc}{I} = \frac{70\,000\ N}{.0072\ m^2} - .73\frac{\left(70000(.06+.053)\ N\cdot m\right)(.066\ m)}{8.32\times10^{-6}\ m^4}$$

$$= 9.72\ MPa - 46.41\ MPa = \boxed{-36.7\ MPa}$$

at Q

$$\sigma = \frac{P}{A} + K_i \frac{Mc}{I}$$

$$= 9.72\ MPa + 1.52\frac{[70{,}000(.113\ )N\cdot m](.053\ m)}{8.32\times10^{-6}\ m} = 9.72 + 77.29\ MPa$$

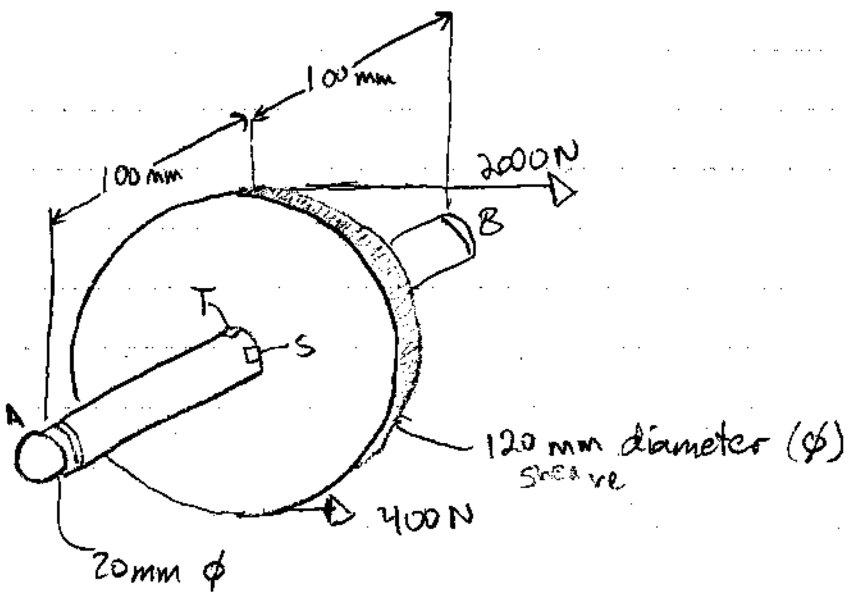
$$\boxed{\sigma = 87.01\ MPa}$$

4.31) The shaft shown below is 200 mm long between self-aligning bearings A and B. Belt forces are applied to a sheave in the center, as shown. The left end of the shaft is connected to a clutch by means of a flexible coupling. Nothing is attached to the right end.

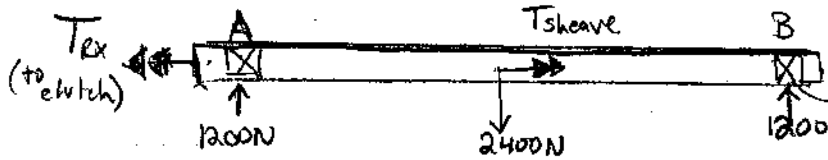
a) Determine and make a sketch showing the stresses acting on the top and side elements, T & S, located adjacent to the sheave (neglect stress concentrations).

b) Draw 3-D Mohr's circles for T & S.

c) Draw the stress state at S of a principle element & a maximum shear element.

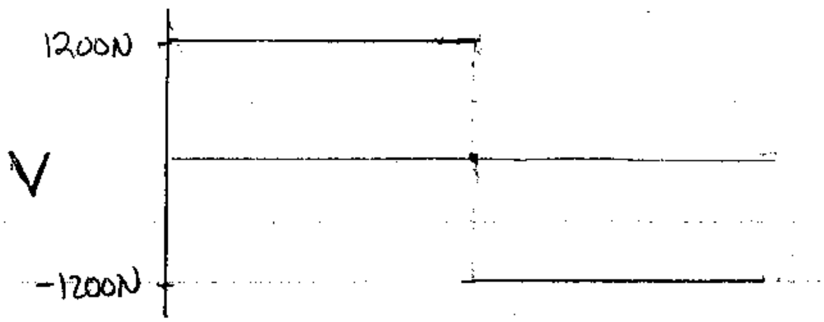


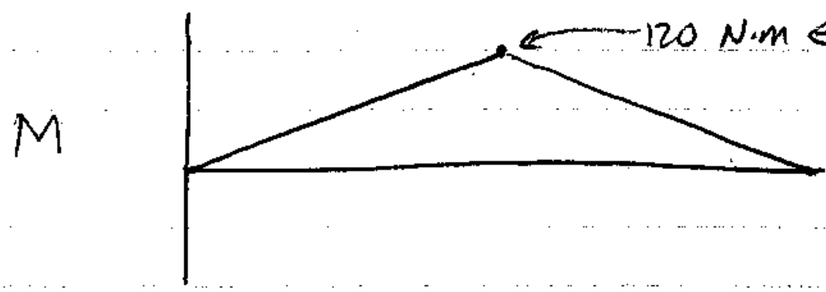
4.31 con't)




symbol for bearing

1200 N ← From $\Sigma F_y = 0$/ Symmetry!



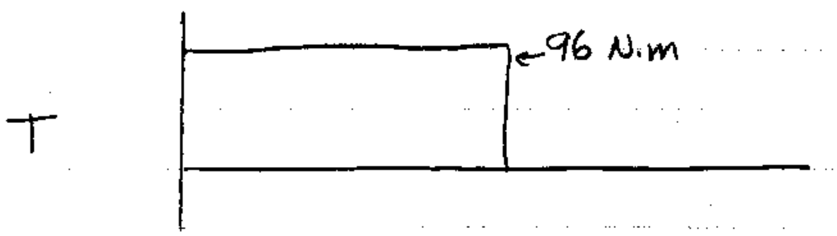




120 N·m ← (1200N)(.1 m) ← from value of the graph above

$\Sigma M = 0$ for the sheave

$$0 = T_{sheave} + (2000N)(.06m) - (400N)(.06m)$$

$$T_{sheave} = 96\ N.m$$




Now, for torsion (eq. 4.4)

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(96N.m)}{\pi(.02m)^3} = 61.12\ MPa$$

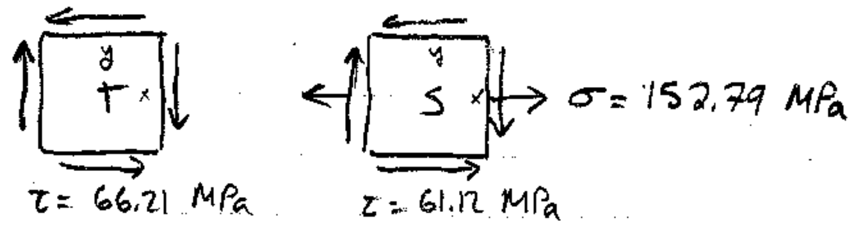
For bending (Eq. 4.8)

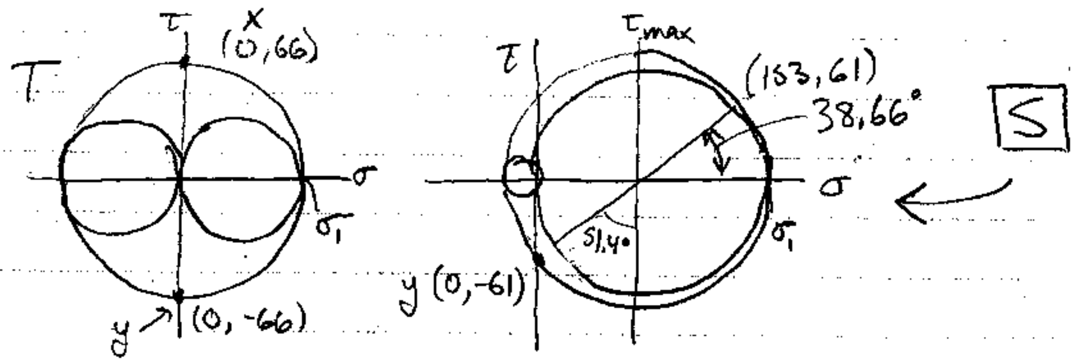
$$\sigma = \frac{My}{I} = \frac{32M}{\pi d^3} = \frac{32(120\ N.m)}{\pi(.02\ m)^3} = 152.79\ MPa$$

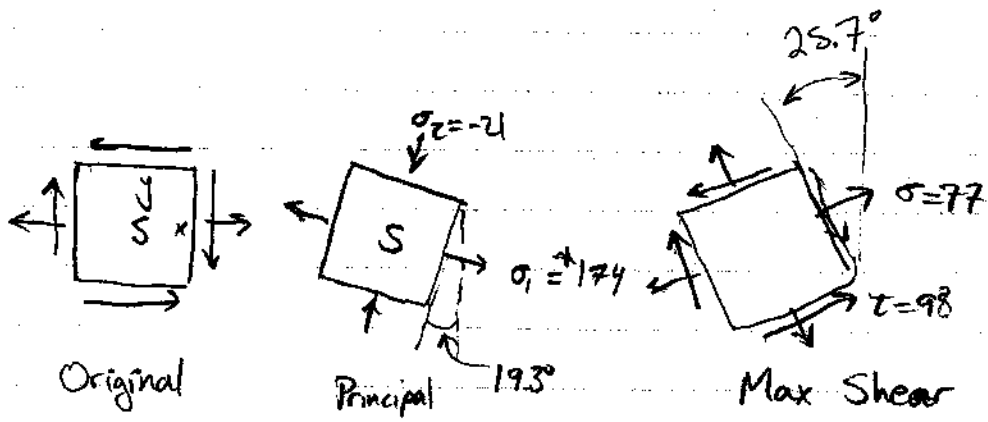
For transverse Shear (Eq. 4.13)

$$\tau = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{(1200N)}{(.01)^2\pi} = 5.09\ MPa$$

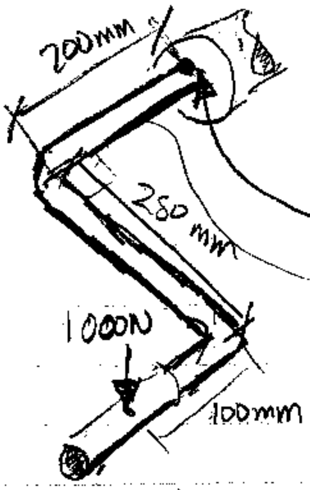
4.31 con't)


y
T
x
y
S
x
σ = 152.79 MPa
τ = 66.21 MPa
τ = 61.12 MPa


T
τ
x
(0,66)
σ
σ1
y
(0,-66)
τ
τmax
(153,61)
38.66°
S
σ
51.4°
σ1
y (0,-61)


y
S
x
Original
σz = -21
S
σ1 = 174
19.3°
Principal
25.7°
σ = 77
τ = 98
Max Shear

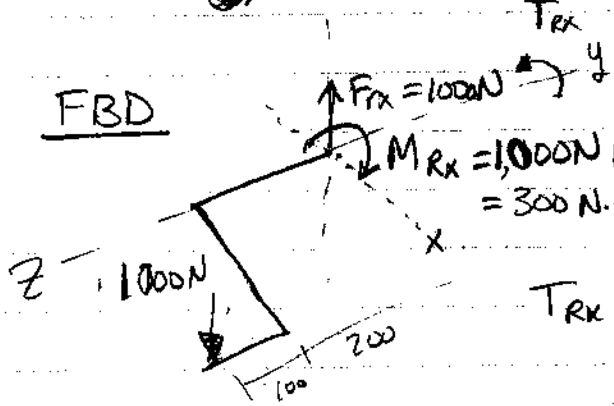
4.34)




a) Find the location at highest bending stress. Make a Mohr's circle at that point.

Highest stress point (furthest from the load — it's a giant lever!)

FBD




$F_{Rx} = 1000N$

$M_{Rx} = 1{,}000N\,(300\,mm)$
$= 300\ N{\cdot}m$ about the x-axis

$$T_{Rx} = (1000\,N)(.25\,m) = 250\ N{\cdot}m$$

Bending Moment (Eq. 4.8)

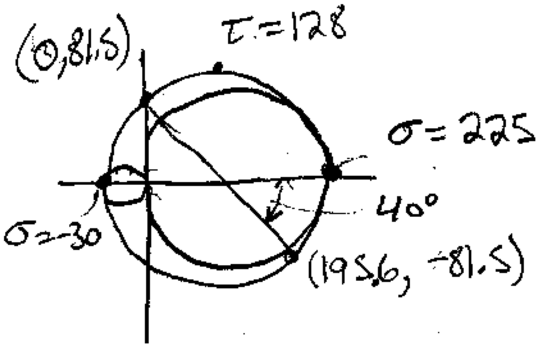
$$\sigma = \frac{32M}{\pi d^3} = \frac{32\,(300\,N{\cdot}m)}{\pi\,(.025\,m)^3} = 195.6\ MPa$$

Torsion (Eq. 4.4)

$$\tau = \frac{16T}{\pi d^3} = \frac{16\,(250\,N)}{\pi\,(.025\,m)^3} = 81.5\ MPa$$

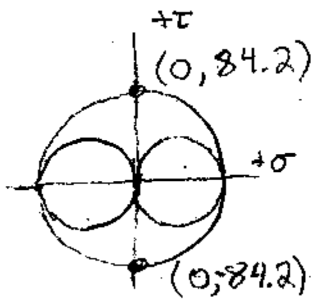
Transverse Shear (Eq. 4.13)

$$\tau = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\,\frac{1000\,N}{\pi(.0125\,m)^2} = 2.7\ MPa$$




$\tau_{max} = 128$ MPa, $\sigma_{max} = 225$ MPa

Point a




Point b