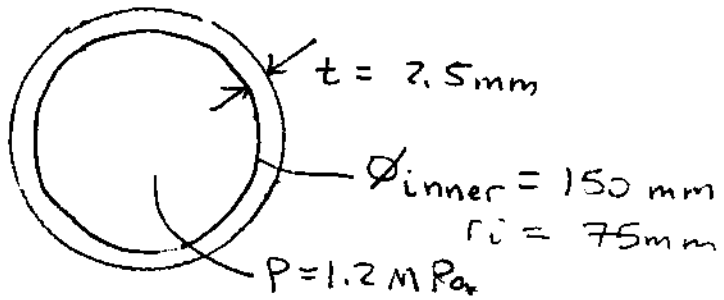
1.) DOWLING 6.6

SPHERICAL PRESSURE VESSEL



FIND: MAXIMUM NORMAL STRESS, SHEAR STRESS, DESCRIBE THE PLANES WHERE THESE ACT.

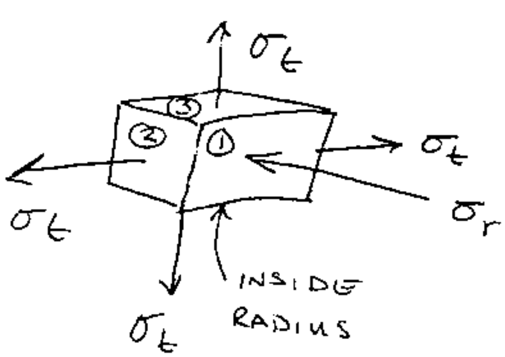
$$\frac{r_i}{t} = \frac{75mm}{2.5mm} = 30$$

USE THIN WALL APPROXIMATIONS FOR STRESSES,

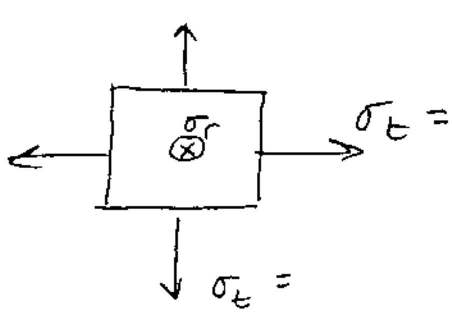
$\sigma_t = \frac{pr_i}{2t}$ $\quad$ $\sigma_r = -p$ at inside surface.

$\sigma_t =$ $\quad$ $\sigma_r = -1.2MPa$

WE EXAMINE AN ELEMENT ON THE INSIDE SURFACE.

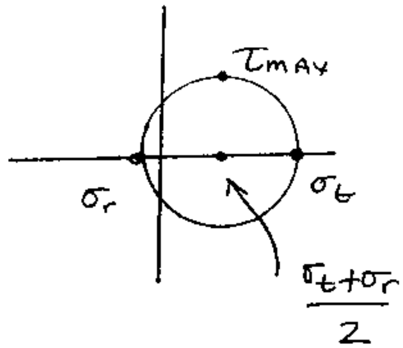


LOOK AT FACE ①



NOTICE WE HAVE THIS ELEMENT IN PRINCIPAL STRESS ORIENTATION, SO WE CAN DRAW MOHR'S CIRCLE.

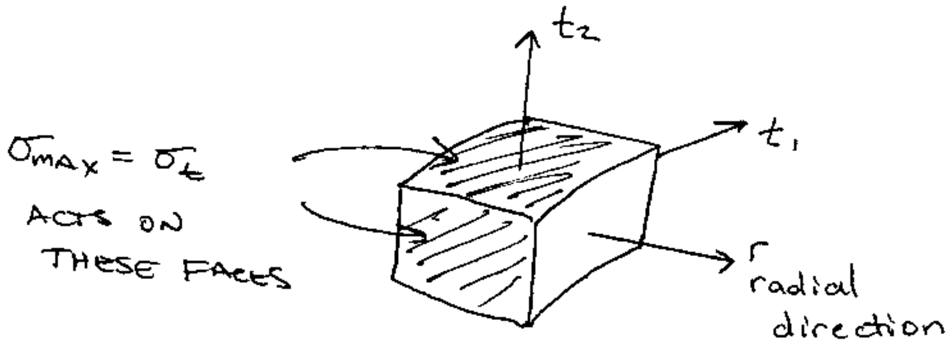
..) CONT)

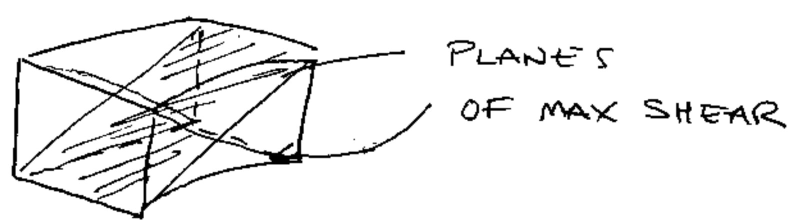


NOTE $\tau_{MAX} = \left|\frac{\sigma_t - \sigma_r}{2}\right|$

$$\tau_{MAX} = \frac{pr}{4t} + \frac{p}{2}$$

$=$



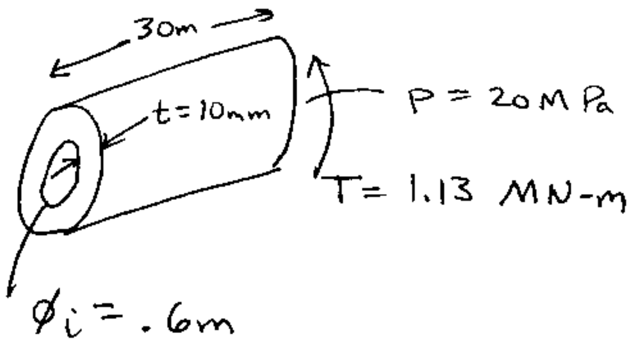


2. DOWLING 6.7

FIND

1.) $\sigma_1, \sigma_2, \sigma_3$

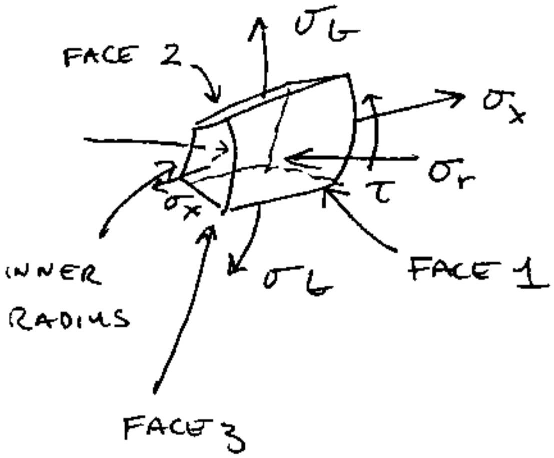
2.) τ_1, τ_2, τ_3



STRATEGY

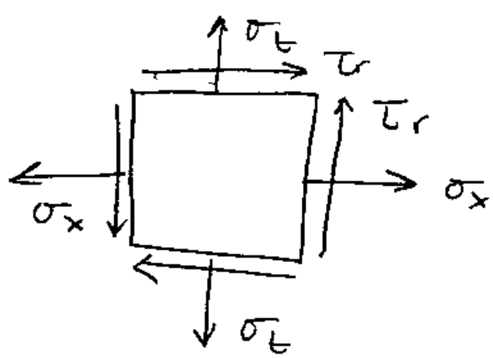
1.) DRAW INNER SURFACE ELEMENT.

2.) DRAW 3-D MOHRS CIRCLE



⟹

FACE 1



NOTE

1.) $\frac{r}{t} = \frac{300mm}{10mm} = 30$ USE THIN WALL EQNS

2.) DIRECTION OF TORQUE IS NOT IMPORTANT WE'LL STILL GET SAME $\sigma_1, \sigma_2, \sigma_3$.

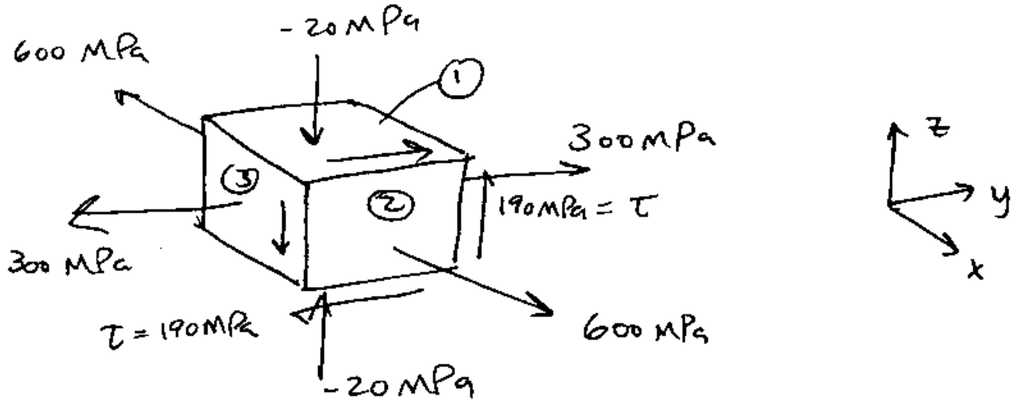
$$\sigma_x = \frac{pr}{2t} = 300 \text{ MPa} \qquad J = \frac{\pi((r+t)^4 - r^4)}{2}$$

$$\sigma_t = \frac{pr}{t} = 600 \text{ MPa} \qquad J = .00178 \text{ m}^4$$

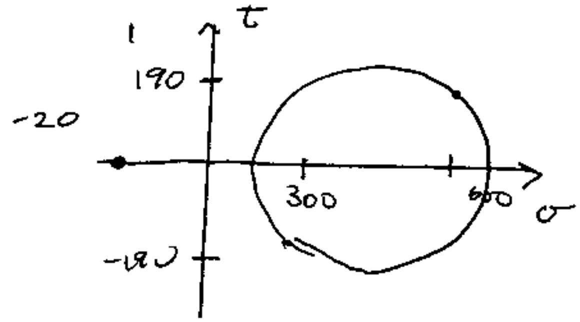
$$\tau_r = \frac{Tr}{J} = \frac{(1.13 \times 10^6 \text{ Nm})(.3m)}{.00178 m^4} = 190 \text{ MPa}$$

$$\sigma_r = -20 \text{ MPa}$$

OUR STRESS CUBE LOOKS LIKE



MOHR'S CIRCLE ON FACE 1



$$\sigma_1, \sigma_2 = \frac{300+600}{2} \pm \sqrt{\left(\frac{300-600}{2}\right)^2 + 190^2}$$

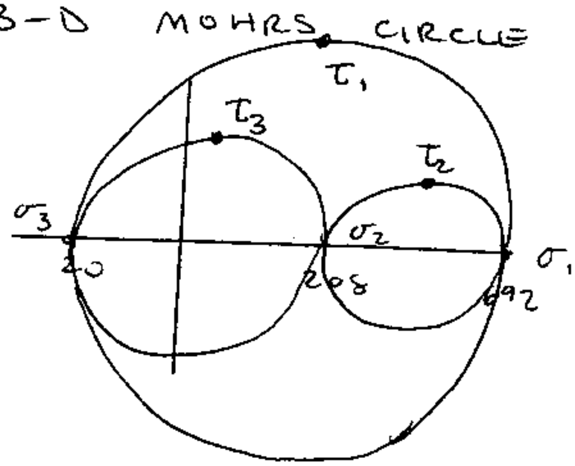
$$\sigma_1, \sigma_2 = 450 \pm 242$$

$$\sigma_1 = 692 \text{ MPa}$$

$$\sigma_2 = 208 \text{ MPa}$$

WE KNOW $\sigma_3 = -20$ MPa

3-D MOHRS CIRCLE



$\sigma_1 = 692$ MPa

$\sigma_2 = 208$ MPa

$\sigma_3 = -20$ MPa

$$\tau_1 = \left|\frac{\sigma_1-\sigma_3}{2}\right| = 356 \text{ MPa}$$

$$\tau_2 = \left|\frac{\sigma_1-\sigma_2}{2}\right| = 242 \text{ MPa}$$

$$\tau_3 = \left|\frac{\sigma_2-\sigma_3}{2}\right| = 114 \text{ MPa}$$

5.) DOWLING 6.16

(5)

HAVE PRESSURED CYLINDER
USING EQNS A1, A2 DEVELOP EQN
FOR MAX SHEAR AS FUNCTION OF
RADIAL POSITION R IN TERMS OF
r_1, r_2, p. SHOW MAX SHEAR OCCURS
AT INNER WALL $(R = r_1)$

EQN A1.)

$$\sigma_t = \frac{p r_1^2}{r_2^2 - r_1^2}\left(\frac{r_2^2}{R^2} + 1\right)$$

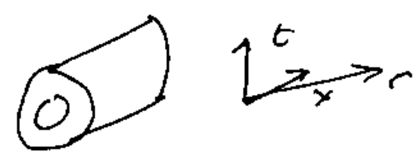
$$\sigma_r = \frac{-p r_1^2}{r_2^2 - r_1^2}\left(\frac{r_2^2}{R^2} - 1\right)$$

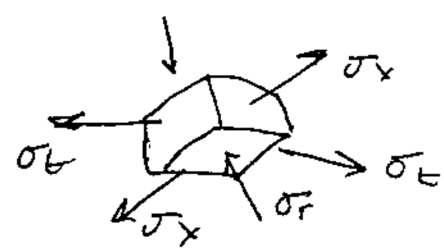
EQN A.2)

$$\sigma_x = \frac{p r_1^2}{r_2^2 - r_1^2}$$

ASSUME

1.) IGNORE END EFFECTS





NOTE $\frac{r_2^2}{R^2} + 1 > 1$

SO $\sigma_t > \sigma_x$

AND $\sigma_t > \sigma_r$

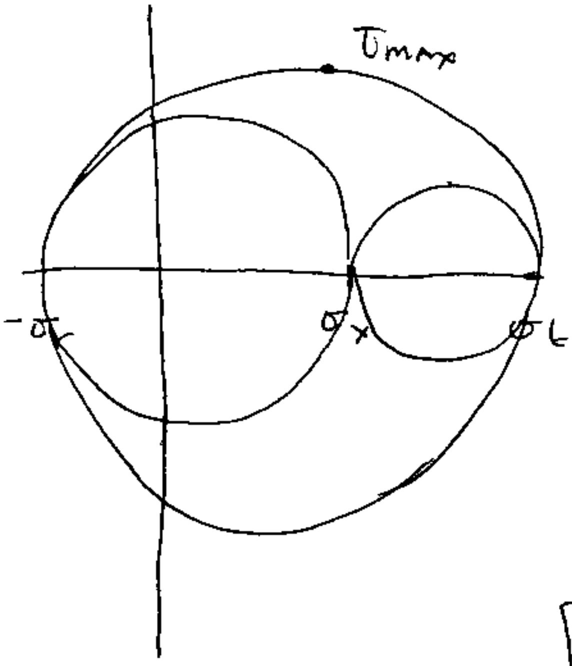
$\sigma_x > \sigma_r$

$\sigma_1 = \sigma_t$

$\sigma_2 = \sigma_x$

$\sigma_3 = \sigma_r$

(CONT)



(6)

$$\tau_{max} = \left| \frac{\sigma_t - \sigma_r}{2} \right|$$

$$\tau_{max} = \frac{p r_1^2}{r_2^2 - r_1^2} \left(\frac{r_2^2}{R^2} + 1 + \frac{r_2^2}{R^2} - 1 \right)$$

$$\boxed{\tau_{max} = \frac{2 p r_1^2 r_2^2}{R^2 (r_2^2 - r_1^2)}}$$

$$r_1 \le R \le R_2$$

$\tau_{max} \propto \frac{1}{R^2}$ SO @ $R_{MINIMUM}$, τ IS MAXIMUM, SO MAX STRESS OCCURS AT THE MIDDLE.

b.) $d_i = 80mm$ $\quad r_1 = 40mm$

$d_o = 100mm$ $\quad r_2 = 50mm$

$p = 100 MPa$

@ $R = r_1$ FIND PRINCIPAL NORMAL STRESSES AND SHEAR STRESSES.

$$\sigma_t = \frac{100MPa\,(0.04m)^2}{(.05)^2 - (.04m)^2} \left(\frac{.05^2}{.04^2} + 1 \right) = 455\ MPa$$

$$\sigma_r = \frac{-100MPa\,(.04m)^2}{(.05m)^2 - (.04m)^2} \left(\frac{.05^2}{.04^2} - 1 \right) = -100\ MPa$$

: CONT)

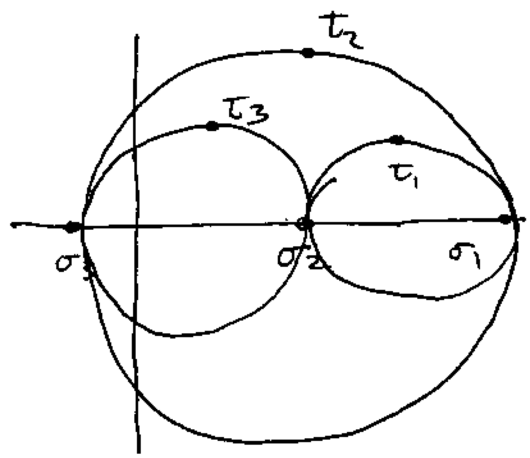
$$\sigma_x = \frac{100\,MPa\,(.04m)^2}{(.05m)^2 - (.04m)^2} = 178\ MPa$$

SINCE NO SHEAR STRESS WE HAVE

$\sigma_1 = \sigma_t = 455\ MPa$

$\sigma_2 = \sigma_x = 178\ MPa$

$\sigma_3 = \sigma_r = -100\ MPa$



$$\tau_1 = \left|\frac{\sigma_1 - \sigma_2}{2}\right| = 138.5\ MPa$$

$$\tau_2 = \left|\frac{\sigma_1 - \sigma_3}{2}\right| = 277.5\ MPa$$

$$\tau_3 = \left|\frac{\sigma_2 - \sigma_3}{2}\right| = 139\ MPa$$