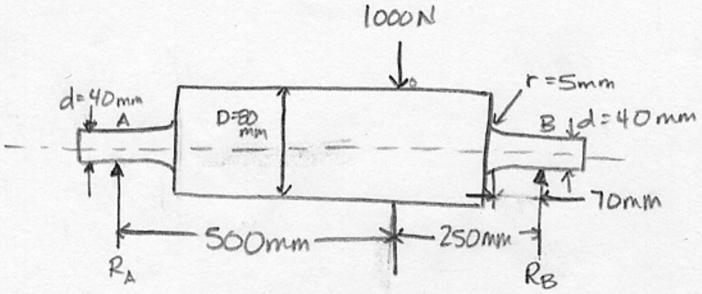
ME111 - HW#5 SOLUTIONS

4.54



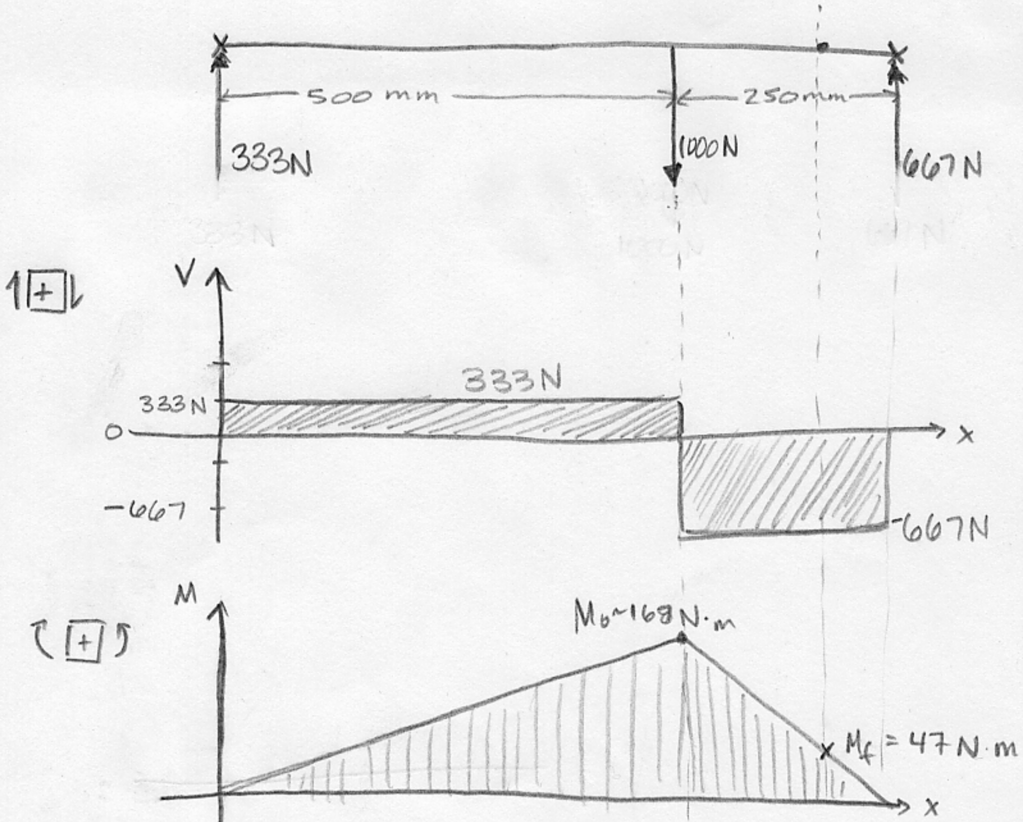
FIND MAX. STRESS AT SHAFT FILLET.
CRITICAL SHAFT FILLET IS 70 MM FROM B

Find the external loads with summation of forces and moments

$+\circlearrowleft \Sigma M_B = 0: 0 = 1000N(.250m) - R_A(.750m) \quad \therefore R_A = 333N$

$+\uparrow \Sigma F_y = 0: R_A - 1000N + R_B = 0 \quad \therefore R_B = 667N$

Now prepare to look at internal loads w/ shear & moment diagrams



at 500mm

$+\circlearrowleft \Sigma M_0 = 0: 0 = M_0 - 333N(500mm)$

$\therefore M_0 = 168$

167 N·m

$\circlearrowleft + \Sigma M_f = 0: M_f + 667N(70mm)$

$M_f = 47 N\cdot m$

(fig. 4.35a)

$$\sigma_{nom} = \frac{32M}{\pi d^3} = \frac{32(47N\cdot m)}{\pi(.04m)^3} = 7.5 MPa$$

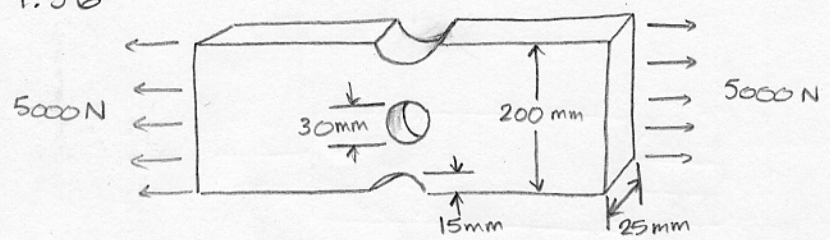
$\frac{r}{d}(\text{critical}) = \frac{5}{40} = 0.125$

$\frac{D}{d} = \frac{80}{40} = 2$

FROM FIGURE (4.35a), $K_t = 1.65$

$\sigma_{max} = \sigma_{nom} K_t = 7.5(1.65) = \boxed{12.4 MPa}$

4.56



WHAT IS VALUE OF THE MAXIMUM STRESS AT BOTH THE HOLE AND THE NOTCH?

nominal stress equal to bar - hole and notch

$$\sigma_{nom(\text{hole and notch})} = \frac{5000\text{ N}}{(.2\text{m}-.06\text{m})(.025\text{m})} = 1.43\text{ MPa}$$

NOTCH
(figure 4.39)

$$\frac{H}{h} = \frac{200\text{mm}}{170\text{mm}} = 1.2$$

$$\frac{r}{h} = \frac{15\text{mm}}{170\text{mm}} = 0.09$$

(from figure 4.39b)

$K_t = 2.45$

$\therefore \sigma_{N\,max} = 2.45(1.43\text{ MPa}) = \boxed{3.5\text{ MPa}}$

HOLE
(figure 4.40)

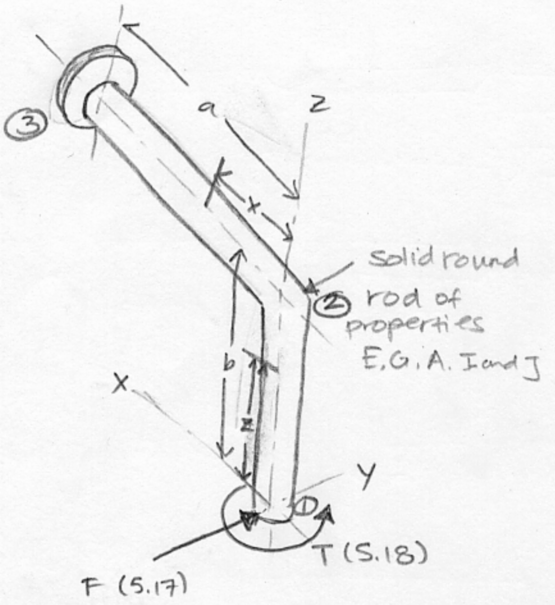
$$\frac{d}{b} = \frac{30\text{mm}}{200\text{mm}} = 0.15$$

(from figure 4.40b)

$K_t = 2.5$

$\therefore \sigma_{H\,max} = 2.5(1.43\text{ MPa}) = \boxed{3.6\text{ MPa}}$

5.17



BRACKET IS LOADED WITH FORCE IN THE Y-DIRECTON DERIVE AN EXPRESSION FOR THE DEFLECTION OF THE FREE END IN THE Y-DIRECTION.

*shear force is negligible

Look at external forces...

$\circlearrowleft + \Sigma M_{1-2} = 0 : \ 0 = M_{1-2} - Fz = 0 \qquad \therefore M_{1-2} = Fz$

$\circlearrowleft + \Sigma M_{2-3} = 0 : \ 0 = M_{2-3} - Fx = 0 \qquad \therefore M_{2-3} = Fx \ ; T_{2-3} = Fb$

Castigliano's Method (Table 5.3)

general energy equation for $\overbrace{M_{1-2} \text{ and } M_{2-3}}^{\text{bending}}$ + energy eq. for torsion T_{2-3}

$$U = \int_0^b \frac{(Fz)^2}{2EI} dz + \int_0^a \frac{(Fx)^2}{2EI} dx + \int_0^a \frac{(Fb)^2}{2GJ} dx$$

$$U = \frac{F^2}{2EI} \int_0^b z^2 dz + \frac{F^2}{2EI} \int_0^a x^2 dx + \frac{F^2 b^2}{2GJ} \int_0^a dx$$

$$U = \frac{F^2}{2EI}\left(\frac{b^3}{3} + \frac{a^3}{3} \right) + \frac{F^2 b^2 a}{2GJ}$$

$$\delta = \frac{\partial U}{\partial F} = \frac{F}{3EI}(b^3 + a^3) + \frac{Fb^2 a}{GJ} = \boxed{\frac{Fb^3}{3EI} + \frac{Fa^3}{3EI} + \frac{Fb^2 a}{GJ}}$$

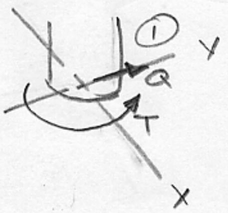
5.18 (same as diagram for 5.17... now look at T instead of F)

BRACKET IS LOADED WITH A TORQUE ABOUT THE Z-AXIS. DERIVE EXPRESSION FOR RESULTING DEFLECTION OF THE FREE END IN THE Y-DIRECTION.

*shear force is negligible

place dummy load at ① along y-axis

Q is a dummy load



look at external forces

$\circlearrowleft + \Sigma M_{1-2} = 0 : 0 = M_{1-2} - Qz \qquad \therefore M_{1-2} = Qz$

$T_{1-2} = T$

$\circlearrowleft + \Sigma M_{2-3} = 0 : 0 = M_{2-3} - Qx - T \quad \therefore M_{2-3} = Qx + T$

$T_{2-3} = Qb$

Castigliano's Method (5.3) Use gen. deflection eq.

bending for M_{1-2} and M_{2-3} ... torsion for T_{1-2} ; T_{2-3} 4 eqns

$$\delta = \int_0^b \frac{M_{1-2}(\partial M_{1-2}/\partial Q)}{EI} dz + \int_0^b \frac{T_{1-2}(\partial T_{1-2}/\partial Q)}{GJ} dz + \int_0^a \frac{M_{2-3}(\partial M_{2-3}/\partial Q)}{EI} dx + \int_0^a \frac{T_{2-3}(\partial T_{2-3}/\partial Q)}{GJ} dx$$

V=Q is neglected

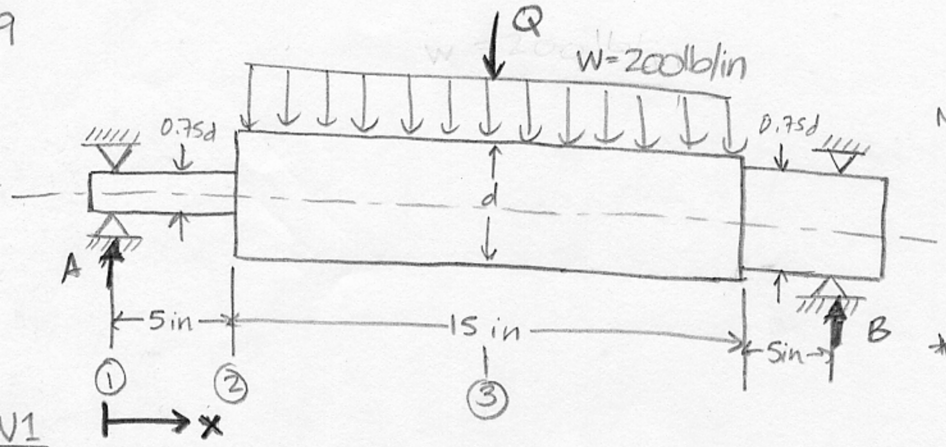
$$\delta = \int_0^b \frac{Qz(z)}{EI} dz + \int_0^b \frac{T_{1-2}(0)}{GJ} dz + \int_0^a \frac{Qx+T(x)}{EI} dx + \int_0^a \frac{Qb(b)}{GJ} dx$$

set Q=0

$$\delta = \int_0^a \frac{Tx}{EI} dx$$

$$\boxed{\delta = \frac{Ta^2}{2EI}}$$

5.19



USING CASTIGLIANO'S METHOD, DETERMINE REQUIRED DIAMETER d TO LIMIT THE DEFLECTION TO 0.2 mm.

* transverse shear is negligible

SOLUTON1

Add a dummy load Q at middle of bar

First look at external forces

$$+\uparrow \Sigma F=0: 0 = A - 3000 - Q + B = 0$$

$$\circlearrowleft + \Sigma M_A = 0: \; Q(12.5in) + 3000(12.5in) - B(25in) = 0$$

$$B = \frac{Q+3000}{2} = \frac{Q}{2} + 1500$$

$$\therefore A = \frac{Q}{2} + 1500$$

Look at moments w.r.t. Q

$$\Sigma + \circlearrowleft M_{1-2} = 0 : 0 = M_{1-2} - \left(1500 + \frac{Q}{2}\right)(x) \rightarrow M_{1-2} = \frac{Q}{2}x + 1500x$$

$$\Sigma + \circlearrowleft M_{2-3} = 0 : 0 = M_{2-3} - \left(1500 + \frac{Q}{2}\right)(x) + 200(x-5)\left(\frac{x-5}{2}\right) \qquad 100(x^2 - 10x + 25)$$

$$M_{2-3} = 1500x + \frac{Q}{2}x - 100x^2 + 1000x + 2500$$

$$M_{2-3} = -100x^2 + \frac{Qx}{2} + 2500x - 2500$$

"general deflection equation" (table 5.3)

* Due to symmetry... integrate x between 0 and 12.5 and then double

Let I_L and I_s represent large (d) and small (0.75d) respectively

$$\delta = 2\int_0^5 \frac{M_{1-2}(\partial M_{1-2}/\partial Q)}{EI_s}dx + 2\int_5^{12.5} \frac{M_{2-3}(\delta M_{2-3}/\partial Q)}{EI_L}dx$$

$$\delta = 2\int_0^5 \frac{\left(\frac{Q}{2}x + 1500x\right)\left(\frac{x}{2}\right)}{EI_s}dx + 2\int_5^{12.5} \frac{\left(-100x^2 + \frac{Qx}{2} + 2500x - 2500\right)\left(\frac{x}{2}\right)}{EI_L}dx$$

5.19 continued

Let $Q=0$

$$\delta = \cancel{2}\int_0^5 \frac{1500x^2}{\cancel{2}EI_s}dx + \cancel{2}\int_5^{12.5} \frac{(-100x^3+2500x^2-2500x)}{\cancel{2}EI_L}dx$$

$$\delta = \frac{1500}{EI_s}\left.\frac{x^3}{3}\right|_{x=0}^{x=5} + \frac{100}{EI_L}\left(-\frac{x^4}{4}+25\frac{x^3}{3}-25\frac{x^2}{2}\right|_{x=5}^{x=12.5}\Bigg)$$

$$\delta = \frac{62{,}500}{EI_s} + \frac{764{,}648}{EI_L}$$

from appendix C-1 ... $E = 30\times10^6$ psi

$$I_L = \frac{\pi d^4}{64} = 0.04909\,d^4$$

$$I_s = \frac{\pi(0.75d)^4}{64} = 0.01553\,d^4$$

plug in $\delta = 0.2\text{ mm} = 0.00787\text{ in}$

$$0.00787 = \frac{62{,}500}{(30\times10^6)(0.01553d^4)} + \frac{764{,}648}{(30\times10^6)(0.04909d^4)} = \frac{0.13415}{d^4} + \frac{0.51924}{d^4}$$

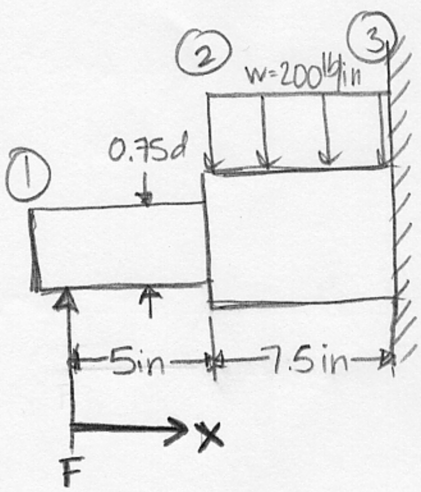
$$d^4 = 83.023$$

$$\boxed{d = 3.02\text{ in}}$$

SOLUTION 2

Because of symmetry consider half the shaft as a cantilever with the deflection caused by the 1500 lb bearing force being 0.2 mm (.00787 in)

* neglect transverse shear
* consider only bending
* even though F is known to be 1500 lb... keep it as a variable until $\partial M/\partial F$ is taken
* Let I_s and I_L pertain to the small and large portions of the shaft, respectfully.



5.19 continued

$\circlearrowleft^{+}\Sigma M_{1-2} = 0: 0 = M_{1-2} - Fx \quad \therefore M_{1-2} = Fx$

$\circlearrowleft^{+}\Sigma M_{2-3} = 0: 0 = M_{2-3} - F_x + 200(x-5)\left(\frac{x-5}{2}\right) \quad \therefore M_{2-3} = Fx - 100x^2 + 1000x - 2500$

using gen. def. eq. (Table 5.3)

$$\delta = \int_0^5 \frac{M_{1-2}(\partial M_{1-2}/\partial F)}{EI_s}dx + \int_5^{12.5} \frac{M_{2-3}(\partial M_{2-3}/\partial F)}{EI_L}dx$$

$$= \frac{1}{EI_s}\int_0^5 Fx(x)\,dx + \frac{1}{EI_L}\int_5^{12.5} (Fx - 100x^2 + 1000x - 2500)(x)\,dx$$

$$= \frac{F}{EI_s}\left(\frac{x^3}{3}\Big|_{x=0}^{x=5}\right) + \frac{1}{EI_L}\left(F\frac{x^3}{3} - 100\frac{x^4}{4} + 1000\frac{x^3}{3} - 2500\frac{x^2}{2}\Big|_{x=5}^{x=12.5}\right)$$

$$= \frac{1}{EI_s}(41.667F) + \frac{1}{EI_L}(609.37F - 594,727 + 609,375 - 164,062)$$

substituting

$F = 1500\,lb$

$\delta = 0.00787\,in$

$E = 30\times10^6\,psi$

$I_L = 0.04909d^4$

$I_s = 0.01553d^4$

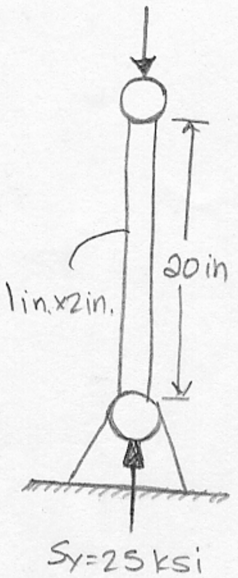
$$0.00787 = \frac{41.667(1500)}{30\times10^6(0.01553d^4)} + \frac{(609.37)(1500) - 594,727 + 609,375 - 164,062}{30\times10^6(0.04909d^4)}$$

$$0.00787 = \frac{0.1342}{d^4} + \frac{0.5192}{d^4}$$

$d^4 = 83.019$ and $\boxed{d = 3.02\,in}$

5.25

(a)



WHAT AXIAL COMPRESSIVE LOAD CAN BE APPLIED?

Safety factor of 4

Appendix B-1 : least radius of gyration

$$\rho = 0.289h = 0.289(1) = 0.289 \text{ in.}$$

$$\frac{L_e}{\rho} = \frac{20}{0.289} = 69.2$$

Eq (5.13)

$$\frac{L_e}{\rho} = \left(\frac{2\pi^2 E}{S_y}\right)^{1/2} \quad \text{where } E = 10.4 \times 10^6 \text{ psi (App. C-1)}$$

$$\frac{L_e}{\rho} = \left[\frac{2\pi^2 (10.4\times10^6)}{25\times10^3}\right]^{1/2} = 90.6$$

← Euler and Johnson are tangent here

∴ Johnson equation applies (69.2 < 90.6)

Eq (5.12)

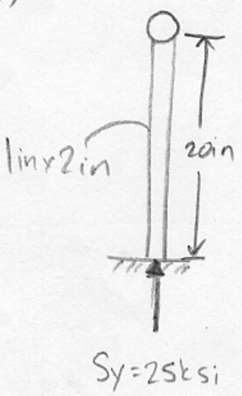
$$S_{cr} = S_y - \frac{S_y^2}{4\pi^2 E}\left(\frac{L_e}{\rho}\right)^2$$

$$S_{cr} = 25{,}000 - \frac{25{,}000^2}{4\pi^2(10.4\times10^6)}(69.2)^2 = 17{,}703 \text{ psi}$$

$$P_{cr} = S_{cr}A = 17{,}703\,(1)(2) = 35{,}406 \text{ lb}$$

safety factor of 4 ... $\boxed{P = 8{,}851.5 \text{ lb}}$

(b)



WHAT AXIAL COMPRESSIVE LOAD CAN BE APPLIED?

from figure 5.27(e) assume $L_e = 2.1L = 42\text{in}$

$$\frac{L_e}{\rho} = \frac{42}{0.289} = 145.33$$

← Euler equation applies (145.33 > 90.6)

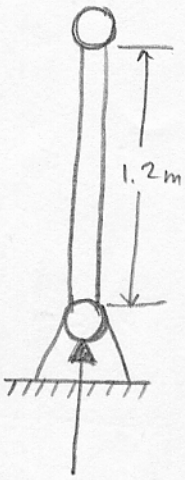
Eq (5.11)

$$S_{cr} = \frac{\pi^2 E}{(L_e/\rho)^2} = \frac{(3.14)^2(10.4\times10^6 \text{ psi})}{(145.33)^2} = 4{,}854 \text{ lb}$$

$$P_{cr} = S_{cr}A = 4{,}854\,(1)(2) = 9{,}708 \text{ lb}$$

safety factor of 4 ... $\boxed{P = 2{,}427 \text{ lb}}$

5.26



WHAT COMPRESSIVE LOAD CAN BE CARRIED WITH A SAFETY FACTOR OF 3?

$S_y = 350$ MPa
$\rho = 8$mm
$\rho_{min} = 5$mm

Use ρ_{min}

$$\frac{L_e}{\rho_{min}} = \frac{1200}{5} = 240$$

(eq 5.13)

$$\frac{L_e}{\rho_{min}} = \left(\frac{2\pi^2 E}{S_y}\right)^{1/2} = \left[\frac{2\pi^2(207)}{0.350}\right]^{1/2} = 108.05$$

Euler and Johnson equations tangent
∴ Euler applies
(240 > 108.05)

(eq 5.11)

$$S_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L_e/\rho)^2}$$

where $E = 207\times10^9$ Pa (Appendix C-1)

$$S_{cr} = \frac{\pi^2(207\times10^3\text{ MPa})}{(240)^2} = 35.47\text{ MPa}$$

* because cross-sectional area is not given, the load capacity with SF=3 can only be given as

$$P = \frac{35.5\,A}{3}$$

$$P = 11.8\,A$$

where "P" is in Newtons and "A" in mm²