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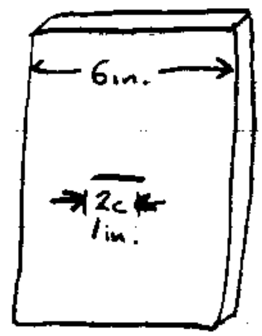
PROBLEM SET #6

ME111
FALL 2000

6.2

FIND: P_{max}

GIVEN: $2w = 6$ in.
$t = .035$ in.
7075-T651 Al: $S_u = 78$ ksi
$S_y = 70$ ksi
central crack: $2c = 1$ in.
$K_{IC} = 60 \text{ ksi}\sqrt{\text{in.}}$
Thin Plate



SOLUTION:

$$K_{Ic} = (1.8\sqrt{c})\,\sigma_g \quad (6.2)$$

$$\sigma_g = \frac{K_{IC}}{1.8\sqrt{c}} = \frac{60 \text{ ksi}\sqrt{\text{in.}}}{1.8(\sqrt{.5 \text{ in.}})}$$

$$\sigma_g = 47.14 \text{ ksi}$$

$$P = \sigma_g(2wt) = (47.14 \text{ ksi})(6 \text{ in.})(.035 \text{ in.})$$

$$P = 9.90 \text{ kips}$$

$$\boxed{P_{max} = 9{,}899 \text{ lb}}$$

6.7

FIND: P_{MAX}

GIVEN: thin plate

$2w = 8$ in.

$t = .05$ in.

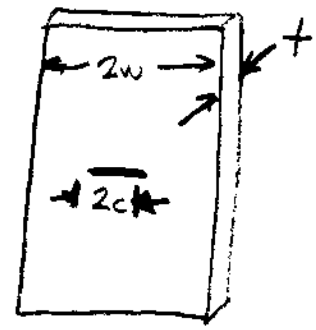
Material: Leaded beryllium copper

$S_u = 98$ ksi

$S_y = 117$ ksi

$K_{Ic} = 70$ ksi$\sqrt{in}$

$2c = 1.5$ in.



SOLUTION: $K_{Ic} = (1.8\sqrt{c})\,\sigma_g \quad (6.2)$

$$\sigma_g = \frac{K_{Ic}}{1.8\sqrt{c}} = \frac{70 \text{ ksi}\sqrt{\text{in.}}}{1.8\sqrt{.75 \text{ in.}}} = 44.91 \text{ ksi}$$

$$P_{max} = \sigma_g(2wt) = (44.91 \text{ ksi})(8 \text{ in.})(.05 \text{ in.}) = 17.96 \text{ kips}$$

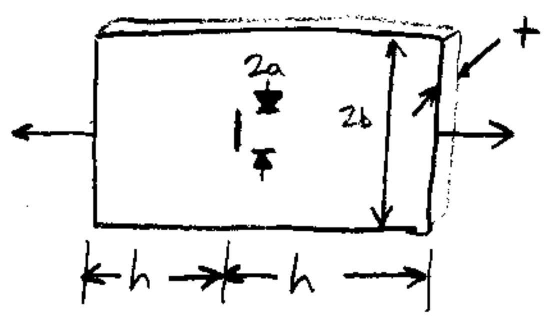
$$\boxed{P_{max} = 17{,}962 \text{ lb}}$$

PROBLEM 3

FIND: P_{max} if crack half-length is (a) 10 mm and (b) 24 mm

GIVEN: Material: AISI 1144 Steel
$K_c = 115 \text{ MPa}\sqrt{m}$

$b = 40$ mm
$t = 15$ mm
$h = 20$ mm
$N = 3$ ← factor of safety



SOLUTION: $N = \dfrac{K_c}{K}$ $\quad K = \beta \sigma \sqrt{\pi a}$ (from Lecture 16)

① FIND β: PART A: $a/b = .25$ $\quad h/b = .5$
$\beta \approx 1.25$
PART B: $a/b = .6$ $\quad h/b = .5$
$\beta \approx 2.5$

FROM FIG 5-19 in Lecture 16

② $\sigma = \dfrac{K}{\beta\sqrt{\pi a}} = \dfrac{K_c}{N\beta\sqrt{\pi a}}$

$$\frac{P}{2bt} = \frac{K_c}{N\beta\sqrt{\pi a}}$$

$$P = \frac{2bt\,K_c}{N\beta\sqrt{\pi a}}$$

(a) $P = \dfrac{(2)(.04m)(.015m)(115 \text{ MPa}\sqrt{m})}{(3)(1.25)(\pi \cdot .01m)^{\frac{1}{2}}}$

$P_{max} = 207.6$ kPa

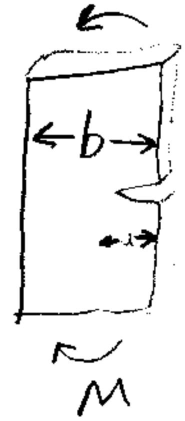
(b) $P = \dfrac{(2)(.04m)(.015m)(115 \text{ MPa}\sqrt{m})}{(3)(2.5)(\pi \cdot .024m)^{\frac{1}{2}}} = 67.0$ kPa

(a) $P_{max} = 207.6$ kPa
(b) $P_{max} = 67.0$ kPa

PROBLEM 4

FIND: LARGEST EDGE CRACK PERMITTED

GIVEN: $b = 20\,mm$
$t = 10\,mm$
$M = 10\,N \cdot m$
$N = 2.5$
$K_c = 3\,MPa\sqrt{m}$



SOLUTION: ① FIND σ_g

$$\sigma_g = \frac{My}{I} = \frac{M\left(\frac{b}{2}\right)}{\frac{tb^3}{12}} = \frac{6M}{tb^2}$$

$$= \frac{(6)(10\,N \cdot m)}{(.01\,m)(.02\,m)^2} = 15\,MPa$$

② $$K = \beta \sigma_g \sqrt{\pi a}$$

$$\frac{K_c}{N} = \beta \sigma_g \sqrt{\pi a}$$

$$\beta\sqrt{a} = \frac{K_c}{N \sigma_g \sqrt{\pi}} = \frac{(3\,MPa\sqrt{m})}{(2.5)(15\,MPa)(\sqrt{\pi})} = .0451$$

③ Make a guess for a, find corresponding β and check against $\beta\sqrt{a} = .0451$

a	β from chart		$\beta\sqrt{a}$	
2 mm	$\frac{a}{b} = \frac{a}{h} = \frac{1}{10}$,	$\beta = 1.05$	.0469	→ choose smaller a
1 mm	$\frac{a}{h} = \frac{1}{20}$	$\beta = 1.03$	.0325	→ choose larger a
1.5 mm	$\frac{a}{h} = .075$	$\beta = 1.06$	.0411	→ choose larger a
1.8 mm	$\frac{a}{h} = .09$	$\beta = 1.05$	.0445	→ choose larger a
1.85 mm	$\frac{a}{h} = .0925$	$\beta = 1.04$	.045	close enough

$$\boxed{a \approx 1.85\,mm}$$

answers might vary slightly due to interpretation of chart.

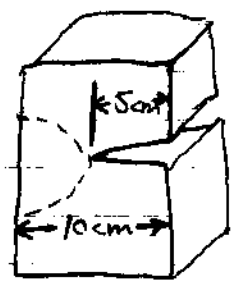
PROBLEM #6

FIND: Strength of joint as a percentage of strength if the joint were solid as limited by (i) fracture and (ii) fully plastic yielding

Comment on suitability of steel

GIVEN: (a) $S_y = 550$ MPa $K_C = 55$ MPa$\sqrt{m}$

(b) $S_y = 400$ MPa $K_c = 200$ MPa$\sqrt{m}$



SOLUTION: (a) ① FIND a_{eff} ← from lecture 16 notes

(i) $a_{eff} = a + \frac{1}{\pi}\left(\frac{K}{S_y}\right)^2 = .05\,m + \frac{1}{\pi}\left(\frac{55\text{ MPa}\sqrt{m}}{550\text{ MPa}\sqrt{m}}\right)^2$

$a_{eff} = .05318$

$\sigma = \frac{K}{\beta\sqrt{\pi a_{eff}}}$ $\beta = 1.5$ ← $(a/b = .5 \quad h/b = \infty)$ from fig 5-22

$= \frac{55\text{ MPa}\sqrt{m}}{1.5\,(\pi \cdot .05318)^{.5}} = 89.7$ MPa

Strength $= \frac{\sigma}{S_y} = \frac{89.7\text{ MPa}}{550\text{ MPa}} = 16.3\%$

Strength = 16.3%

(ii) $\sigma_{nom} = \frac{P_{nom}}{A}$ $\sigma_{notch} = \frac{P_{notch}}{\frac{1}{2}A}$

$\sigma_{nom} = \sigma_{notch}$

$\frac{P_{nom}}{A} = \frac{P_{notch}}{\frac{1}{2}A}$

$\frac{P_{notch}}{P_{nom}} = \frac{1}{2}$

Strength = 50%

#6 (continued)

(b)
$$a_{eff} = a + \frac{1}{\pi}\left(\frac{K}{S_y}\right)^2$$
$$= .05\,m + \frac{1}{\pi}\left(\frac{200\ MPa\sqrt{m}}{400\ MPa\sqrt{m}}\right)^2$$
$$= .1296$$

Effective crack length is longer than width of plate. The material will yield before reaching 200°C.

(c) This steel is not suitable for these temperatures.