1. The stress (in kpsi) at a point is given by:

\[ \sigma_1 = 10 \]
\[ \sigma_2 = 0 \]
\[ \sigma_3 = -20 \]

Calculate the factor of safety against failure if the material is:

a) Brittle with \( S_{ut} = 50, S_{uc} = 90 \) using the modified Mohr theory.

\[ N = \frac{S_{ut} \cdot S_{uc}}{S_{uc} \cdot \sigma_1 - S_{ut} \cdot (\sigma_1 + \sigma_3)} \]
\[ N = \frac{50 \cdot 90}{90(10) - 50(-10)} = 3.2 \]

b) Ductile with \( S_y = 40 \) using max. shear stress and von Mises.

(i) Maximum shear stress criterion

\[ N = \frac{S_y}{\max(10, 20, 30)} = \frac{40}{\max(10, 20, 30)} = \frac{40}{30} = 1.3 \]

(ii) Von Mises criterion

\[ \sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\sigma_1\sigma_2 - 2\sigma_2\sigma_3 - 2\sigma_3\sigma_1} \]
\[ \sigma_2 = 0 \]
\[ \sigma' = \sqrt{10^2 + 20^2 - 2 \cdot 20 \cdot 0 - 2 \cdot 10 \cdot 0 - 2 \cdot 0 \cdot 10} = \sqrt{700} \]
\[ N = \frac{S_y}{\sigma'} = \frac{40}{\sqrt{700}} = \frac{4}{\sqrt{700}} = 1.5 \]
2. Consider two designs of a lug wrench for an automobile.

a) Single ended

This part of the member is in torsion. Shearing occurs.

b) Double ended

This part is in torsion. Shearing occurs.

Given:
\[ \phi = 0.625 \text{ in} \]
\[ S_y = 45 \text{ kpsi} \]

a) The largest bending moment will occur at point A, where the handle turns into the knob.

\[ M = F(12 \text{ in}) = 12F \text{ [lb-in]} \]

i) The stress due to the bending moment can be solved with

\[ \sigma_x = \frac{Mc}{I} \quad \text{where} \quad c = \frac{d}{2} \quad \text{and} \quad I = \frac{\pi d^4}{64} \]

\[ \sigma_x = \frac{(12F \text{ [lb-in]})(0.313 \text{ [in]})}{\pi (0.625 \text{ [in]})^4} = F \times 501 \text{ [psi]} \]

At the point... \( \sigma_x \) is the only stress component.

\( \sigma_1 = \sigma_x \quad \sigma_2 = 0 \text{ psi} \quad \sigma_3 = 0 \text{ psi} \)

Von Mises: \( \sigma' = \sigma_x = F \times 501 \text{ [psi]} \)

Distortion Energy Theory: \( S_y = \sigma' \)

\[ 45 \text{ kpsi} = F \times 501 \text{ [psi]} \quad \rightarrow \quad F = 89.8 \text{ lb} \]

ii) Find the shear stress due to torsion to see if force will cause the lug wrench to fail in shear.

\[ \tau = \frac{Tc}{J} \quad \text{where} \quad c = \frac{d}{2} \quad \text{and} \quad J = \frac{\pi d^4}{32} \]
\[ \sigma = \frac{(1077.6 \text{ lb-in})(.313\text{ in})}{\pi (.625\text{ in})^4} = 22.5 \text{ kpsi} \]

the only stress acting on the knub is \( \sigma \)

\[ \sigma_1 = \sigma, \quad \sigma_2 = 0, \quad \sigma_3 = 0 \]

\[ \sigma' = \sqrt{\sigma_1^2 - \sigma_2\sigma_3 + \sigma_3^2} = \sqrt{0^2 + 0^2 + 0^2} = 0 \]

\[ \sigma' = 39.0 \text{ kpsi} \]

The maximum von Mises stress case for case (a) is on the upper surface of the handle (arm) near the point where \( \sigma \) transitions to the stub ... the max. force \%Y of yielding

\[ F = 89.8 \text{ lbf} \]

(b) The largest bending moment will occur where the handle turns into the knub.

\[ M = F(l) = r F \left( \frac{\text{lb-in}}{l} \right) \]

(i) the stress due to bending can be solved with \( \sigma_x = \frac{M_x}{I} \)

\[ \sigma_x = \frac{(6F)(\text{lb-in})(.313\text{ in})}{\pi (.625\text{ in})^4} = 251 + F \text{ [psi]} \]

At this point, the only stress component is \( \sigma_x \).

\[ \sigma_1 = \sigma_x, \quad \sigma_2 = \sigma_3 = 0 \]

Von Mises: \( \sigma' = \sigma_x = 251 + F \text{ [psi]} \)

Distortion Energy Theory: \( S_y = \sigma' \quad ... \quad 45 \text{ kpsi} = 251 + F \text{ [psi]} \)

\[ F = 179 \text{ lbf} \]

(ii) Find the shear stress...

\[ \tau = \frac{179\text{ lbf}\times(60\text{ in})}{\pi (.625\text{ in})^4} = 45 \text{ kpsi} \]

\[ \sigma_1 = 45 \text{ kpsi}, \quad \sigma_2 = 0, \quad \sigma_3 = -45 \text{ kpsi} \]

Von Mises: \( \sigma' = 78 \text{ kpsi} \quad \text{this is greater than} \quad S_y \quad \text{... force in handle is limited by shear stress in the knub.} \]

\[ N_s = \frac{S_y}{\sigma_x} \frac{\sigma_x}{\tau} = \frac{S_y \tau}{F(l) \times (12\text{ in})(.313\text{ in})^{1/2}} = 1 \]

\[ F = 103.6 \text{ lbf} \]
- The maximum internal shear and moment occur at a section where the mandrel root leaves the stanchion.

\[ V_{\text{max}} = \frac{2WL}{b} = \frac{2(53.9 \text{ kN})(1.615 \text{ m})}{b} = \frac{174 \text{ kN}}{b} \text{ m}^2 \]

\[ M_{\text{max}} = WL = (53.9 \text{ kN})(1.615 \text{ m}) = 87 \text{ kN} \cdot \text{m} \]

a) Bending stress maximum at the top or bottom of the mandrel at a section where the mandrel root leaves the stanchion.

\[ \sigma_{\text{max}} = \frac{M_{\text{max}}(\frac{a}{2})}{I} \quad \text{where} \quad I = \frac{\pi a^4}{64} \]

\[ \sigma_{\text{max}} = \frac{87 \text{ kN} \cdot \text{m} \cdot a \cdot 32}{\pi a^3} = \frac{887}{a^3} \text{ [kPa]} \]

\[ \sigma_1 = \frac{88.7}{a^3} \text{ [kPa]} \quad \sigma_2 = \sigma_3 = 0 \]

\[ N_s = \frac{S_y}{\sigma_1} \quad \text{so} \quad N_s = \frac{S_y}{N_s} \rightarrow \frac{887 \text{ kPa}}{a^3} = \frac{300 \text{ MPa}}{1.5} \]

\[ a = 164 \text{ mm} \]

We know a...

\[ \tau_{\text{max}} = \frac{4V_{\text{max}}}{3A} = \frac{4(174 \text{ kN} \cdot \text{m})}{3(\pi a^4)} \frac{1094 \text{ kN} \cdot \text{m}}{b^2} \]

\[ \sigma_1 = \tau_{\text{max}} \quad \sigma_2 = 0 \quad \sigma_3 = -\tau_{\text{max}} \]

Distortion Energy Theory

\[ \sigma' = \tau_{\text{max}} \frac{V_3}{b} \quad \text{and} \quad N_s = \frac{S_y}{\sigma'} \rightarrow N_s = \frac{S_y}{\sigma'} = \frac{S_y}{\sqrt{3}} \]

\[ b = 95 \text{ mm} \]
(b) All three brittle failure theories have the same fail-safe boundary for this condition (slope of loadline is zero).

\[ N_s = \frac{S_{ut}}{S_{ti}} = \frac{S_{ut}}{S_{max}} \rightarrow \text{for part 2} \]

\[ S_{max} = \frac{S_{ut}}{N_s} + \frac{88.7 \text{ MPa}}{A^2} \leq 150 \text{ MPa} \]

\[ \Rightarrow A = 0.077 \text{ mm} \]

Now use \( T_{max} \) equations from before

\[ N_s = \frac{S_{ut}}{S_{ti}} = \frac{S_{ut}}{T_{max}} \]

\[ T_{max} = \frac{S_{ut}}{N_s} \Rightarrow \frac{16(1+4\frac{b}{A})}{3(\pi A^2)b} \leq \frac{150 \text{ MPa}}{1.5} \]

\[ b = 69 \text{ mm} \]
\[ \sigma_x = K_t \frac{M_c}{I_1} = \frac{(1.08)(3500)(\frac{d}{2})}{(\pi d^4/64)} = \frac{59924}{d^3} \]

\[ c_{xy} = K_t \frac{J_c}{J} = \frac{(1.42)(8000)(\frac{d}{2})}{(\pi d^4/32)} = \frac{57085}{d^3} \]

\[ \sigma_1 = \sigma_x + \sqrt{(\sigma_x)^2 + (c_{xy})^2} = \frac{29962}{d^3} + \sqrt{\left(\frac{29962}{d^3}\right)^2 + \left(\frac{57085}{d^3}\right)^2} \]

\[ \sigma_1 = \frac{29962 + 65180}{d^3} = \frac{95142}{d^3} \]

\[ \sigma_3 = -\frac{35218}{d^3} \]

\[ \sigma' = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1 \sigma_3} \]

\[ \sigma' = \frac{1}{d^3} \sqrt{(95142)^2 + (-35218)^2 - (95142)(-35218)} \]

\[ \sigma' = \frac{116803}{d^3} \]

\[ \frac{S_Y}{\sigma'} = N \]

\[ \frac{150 \text{ MPa}}{116803 \text{ (d}^3\text{)}} = 3 \]

\[ d = 13.3 \text{ mm} \]
\[
\begin{align*}
\sigma_1 &= \approx 180 \\
\sigma_3 &= \approx 80 \\
\mu_{\max}(101, 152, 153 - \sigma, 1) &= \frac{S_y}{N} \\
|\sigma| &= \frac{S_y}{N} \\
\frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \rho_{xy}^2} &= \frac{S_y}{N} = 207 \\
\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \rho_{xy}^2 &= (207 + \frac{\sigma_x + \sigma_y}{2})^2 = 0 \\
g(\sigma) &= 0 \\
\rightarrow \bar{\sigma} &= \frac{P}{A_x} \rightarrow P
\end{align*}
\]

Note: \( \sigma_1, \sigma_3 \)
FROM BEAM THEORY

\[ q_x = -200 \text{ N/m} \]

\[
\frac{d^4v}{dx^4} = q_x + C_1x + C_2 \]
\[
\frac{d^2v}{dx^2} = M = \frac{q_x x^2}{2} + C_1x + C_2
\]

From 0 ≤ x ≤ 0.4

\[ M = \frac{q_x x^2}{2} \]

From 0.4 ≤ x ≤ 0.2

\[ M = -M_0(x-0.2)+M_1(x) \]

\[ M = 160N\cdot m + 964N(x) \]

For 0.4 ≤ x ≤ 0.2

\[ M = 16 + 184x \]

\[ R_A + R_B = 580 \text{ N} \]

\[ \tau M = 0: 600\text{ N/m} \cdot 0.2\text{ m} + 500\text{ N/m} \cdot 0.1\text{ m} - 316 \text{ N} = 0 \]

\[ R_B = 316 \text{ N} \]

\[ R_A = 264 \text{ N} \]

\[ \sigma_{\text{max}} = \frac{M_{\text{max}}c}{I} = \frac{126 \text{ N}(0.02\text{ m})}{2.85 \times 10^{-8} \text{ m}^4} = 88.7 \text{ MPa} \]

\[ \sigma_x = 88.7 \text{ MPa} \]

\[ \sigma_2 = 0 \]

\[ \sigma_3 = 0 \]

(i) Von Mises:

\[ N = \frac{5\sigma_x}{6} = \frac{300 \text{ MPa}}{88.7 \text{ MPa}} = 3.4 \]

(ii) Modified Mohr: Quad I and IV where \( N = \frac{5\sigma_x}{6} \)

\[ N = \frac{150 \text{ MPa}}{88.7 \text{ MPa}} = 1.7 \]