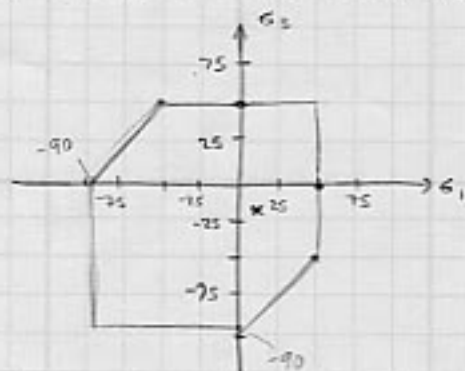


1. The stress (in kpsi) at a point is given by

$$\begin{aligned}\sigma_1 &= 10 \\ \sigma_2 &= 0 \\ \sigma_3 &= -20\end{aligned}$$

Calculate the factor of safety against failure if the material is:

a) Brittle w/ $S_{ut} = 50$, $S_{uc} = 90$ & using the modified-Mohr theory.



$|\sigma_3| > \sigma_1$ quadrant 4

$$N = \frac{S_{ut} S_{uc}}{S_{uc} \sigma_1 - S_{ut} (\sigma_1 + \sigma_3)}$$

$$N = \frac{50(90)}{90(10) - 50(-10)} = \boxed{3.2}$$

b) ductile with $S_y = 40$ using max. shear stress and von Mises

(i) Maximum Shear Stress Criterion

$$N = \frac{S_y}{\max(|\sigma_1|, |\sigma_2|, |\sigma_3 - \sigma_1|)} = \frac{40}{\max(10, 20, 30)} = \frac{40}{30} = \boxed{1.3}$$

(ii) Von Mises criterion

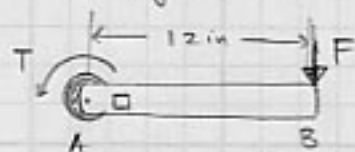
$$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1 \sigma_2 - \sigma_2 \sigma_3 - \sigma_3 \sigma_1} \quad \sigma_2 = 0$$

$$\sigma' = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_3 \sigma_1} = \sqrt{10^2 + (-20)^2 - (-20)(10)} = \sqrt{700}$$

$$N = \frac{S_y}{\sigma'} = \frac{40}{\sqrt{700}} = \frac{4}{\sqrt{7}} = \boxed{1.5}$$

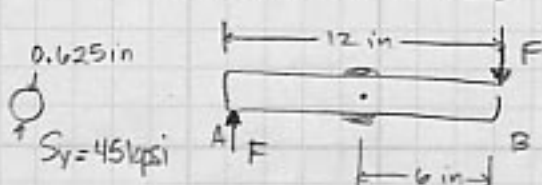
2. Consider two designs of a lug wrench for an automobile

a) single ended



this part of the member is in torsion. Shearing occurs

b) double ended



this part is in torsion shearing occurs

Given:
 $\phi = 0.625 \text{ in}$
 $S_y = 45 \text{ kpsi}$

a) The largest bending moment will occur at point A, where the handle turns into the knub.

$$M = F(12 \text{ in}) = 12F \text{ [lb}\cdot\text{in]}$$

(i) The stress due to the bending moment can be solved with

$$\sigma_x = \frac{Mc}{I} \quad \text{where } c = \frac{d}{2} \quad \text{and } I = \frac{\pi d^4}{64}$$

$$\sigma_x = \frac{(12F \text{ [lb}\cdot\text{in}])(.313 \text{ [in]})}{\frac{\pi (.625 \text{ [in]})^4}{64}} = F \cdot 501 \text{ [psi]}$$

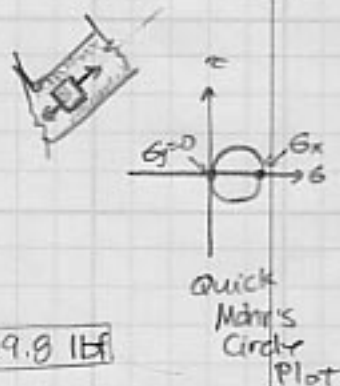
At the point... σ_x is the only stress component

$$\sigma_1 = \sigma_x \quad \sigma_2 = 0 \text{ psi} \quad \sigma_3 = 0 \text{ psi}$$

Von Mises: $\sigma' = \sigma_1 = F \cdot 501 \text{ [psi]}$

Distortion Energy Theory: $S_y = \sigma'$

$$45 \text{ kpsi} = F \cdot 501 \text{ [psi]} \quad \rightarrow \quad \boxed{F = 89.8 \text{ lbf}}$$



(ii) Find the shear stress due to torsion to see if force will cause the lug wrench to fail in shear.

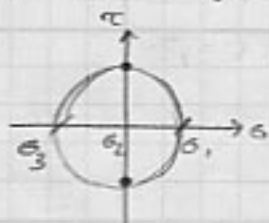
$$\tau = \frac{Tc}{J} \quad \text{where } c = \frac{d}{2} \quad \text{and } J = \frac{\pi d^4}{32}$$

2 cont...



$$\tau = \frac{(1077.6 \text{ lb-in})(.313 \text{ in})}{\frac{\pi (.625 \text{ in})^4}{32}} = 22.5 \text{ kpsi}$$

the only stress acting on the knob is τ



$$\sigma_1 = \tau \quad \sigma_2 = 0 \quad \sigma_3 = -\tau$$

$$\sigma' = \sqrt{\sigma_1^2 - \sigma_2 \sigma_3 + \sigma_3^2} = \sqrt{\tau^2 + \tau^2 + \tau^2} = \tau\sqrt{3}$$

$$\sigma' = 39.0 \text{ kpsi}$$

Quick Mohr's circle plot

The maximum von Mises stress case for case (a) is on the upper surface of the handle (arm) near the point where it transitions to the stub ... the max. force w/o yielding

$$F = 89.8 \text{ lbf}$$

(b) The largest bending moment will occur where the handle turns into the knob

$$M = F(6 \text{ in}) = 6F \text{ [lb-in]}$$

(i) the stress due to bending can be solved with $\sigma_x = \frac{Mc}{I}$

$$\sigma_x = \frac{(6F \text{ [lb-in]})(.313 \text{ [in]})}{\frac{\pi (.625 \text{ in})^4}{64}} = 251 * F \text{ [psi]}$$

At this point, the only stress component is σ_x .

$$\sigma_1 = 251 * F \text{ [psi]} \quad \sigma_2 = \sigma_3 = 0$$

Von Mises: $\sigma' = \sigma_1 = 251 * F \text{ [psi]}$

Distortion Energy Theory: $S_y = \sigma' \dots 45 \text{ kpsi} = 251 * F \text{ [psi]}$

$$F = 179 \text{ lbf}$$

(ii) Find the shear stress ...

$$\tau = \frac{Tc}{J} = \frac{2(179 \text{ lbf})(6 \text{ in})(.313 \text{ in})}{\frac{\pi (.625 \text{ in})^4}{32}} = 45 \text{ kpsi}$$

$$\sigma_1 = 45 \text{ kpsi} \quad \sigma_2 = 0 \quad \sigma_3 = -45 \text{ kpsi}$$

Von Mises: $\sigma' = 78 \text{ kpsi}$ & this is greater than S_y

... force in handle is limited by shear stress in the knob

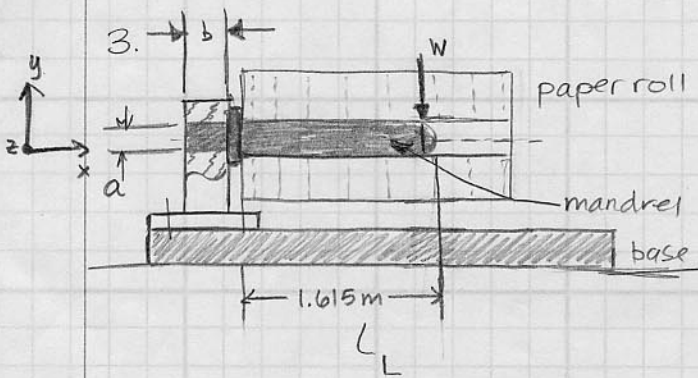
$$N_s = \frac{S_y}{\sigma'} = \frac{S_y}{\tau\sqrt{3}} = \frac{S_y J}{F(12 \text{ in})(.313 \text{ in})\sqrt{3}} = 1$$

$$F = 103.6 \text{ lbf}$$



$T = 2(179 \text{ lbf})(6 \text{ in})$
because you have 2 forces acting on the lug wrench





Weight of roll: $53.9 \text{ kN} = W$

Determine suitable dimensions for a and b to provide a factor of safety of 1.5 if:

- a) the beam is a ductile material with $S_y = 300 \text{ MPa}$
- b) the beam is a brittle material with $S_{ut} = 150 \text{ MPa}$, $S_{uc} = 570 \text{ MPa}$

- the maximum internal shear and moment occur at a section where the mandrel root leaves the stanchion

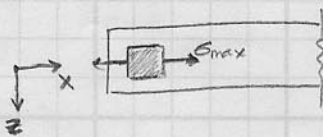
$$V_{\max} = \frac{2WL}{b} = \frac{2(53.9 \text{ kN})(1.615 \text{ m})}{b} = \frac{174 \text{ kN}}{b \text{ m}^2}$$

$$M_{\max} = WL = (53.9 \text{ kN})(1.615 \text{ m}) = 87 \text{ kN}\cdot\text{m}$$

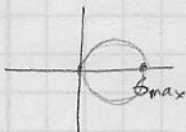
a) Bending stress maximum at the top or bottom of the mandrel at a section where the mandrel root leaves the stanchion.

$$\sigma_{\max} = \frac{M_{\max}(\frac{a}{2})}{I} \quad * I = \frac{\pi a^4}{64}$$

$$\sigma_{\max} = \frac{87 \text{ kN}\cdot\text{m} \cdot \frac{a}{2}}{\pi a^4 / 64} = \frac{887}{a^3} \text{ [kPa]}$$



$$\sigma_1 = \frac{88.7}{a^3} \text{ [kPa]} \quad \sigma_2 = \sigma_3 = 0$$



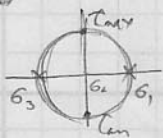
$$N_s = \frac{S_y}{\sigma_1} \rightarrow \sigma_1 = \frac{S_y}{N_s} \rightarrow \frac{88.7 \text{ kPa}}{a^3} = \frac{300 \text{ MPa}}{1.5}$$

$$\rightarrow a = 164 \text{ mm}$$

We know a... $\tau_{\max} = \frac{4V_{\max}}{3A} = \frac{4(\frac{174}{b}) \text{ kN/m}}{3(\frac{\pi a^2}{4})} = \frac{10994 \text{ kN}}{b \text{ m}^2}$



$$\sigma_1 = \tau_{\max} \quad \sigma_2 = 0 \quad \sigma_3 = -\tau_{\max}$$



Distortion Energy Theory

$$\sigma' = \tau_{\max} \sqrt{3} \quad \text{and} \quad N_s = \frac{S_y}{\sigma'} \rightarrow N_s \cdot \tau_{\max} = \frac{S_y}{\sqrt{3}} \approx 300 \text{ MPa}$$

$$\rightarrow b = 95 \text{ mm}$$

3. cont.

(b) All three brittle failure theories have the same fail/safe boundary for this condition (slope of load line is zero).

$$N_s = \frac{S_{ut}}{\sigma_1} = \frac{S_{ut}}{\sigma_{max}} \rightarrow \text{from part a}$$

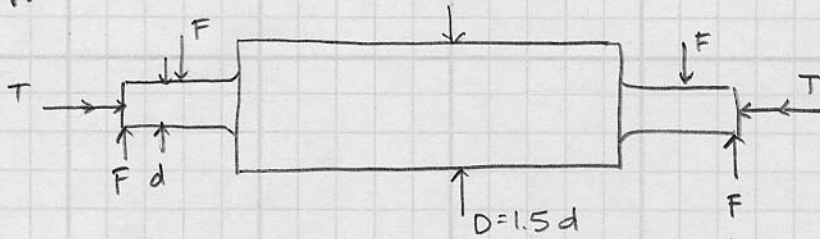
$$\sigma_{max} = \frac{S_{ut}}{N_s} \rightarrow \frac{88.7 \text{ MPa}}{1.5} = \frac{150 \text{ MPa}}{1.5} \rightarrow \boxed{a = 207 \text{ mm}}$$

now use τ_{max} equations from before

$$N_s = \frac{S_{ut}}{\sigma_1} = \frac{S_{ut}}{\tau_{max}}$$

$$\tau_{max} = \frac{S_{ut}}{N_s} \Rightarrow \frac{16(174) \frac{\text{KN}}{\text{m}}}{3(\pi a^2)b} = \frac{150 \text{ MPa}}{1.5} \rightarrow \boxed{b = 69 \text{ mm}}$$

4.



$$S_y = 150 \text{ MPa}$$

$$N = 3$$

$$150 \times 10^6 \frac{\text{N}}{\text{m}^2} \times \frac{1 \text{ m}^2}{(1000 \text{ mm})^2}$$

$$= 150 \frac{\text{N}}{\text{mm}^2}$$

$$\sigma_x = K_t \frac{M_c}{I} = \frac{(1.68)(3500)\left(\frac{d}{2}\right)}{\left(\frac{\pi d^4}{64}\right)} = \frac{59924}{d^3}$$

$$\tau_{xy} = K_{ts} \frac{T_c}{J} = \frac{(1.42)(8000)\left(\frac{d}{2}\right)}{\left(\frac{\pi d^4}{32}\right)} = \frac{57885}{d^3}$$

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \frac{29962}{d^3} + \sqrt{\left(\frac{29962}{d^3}\right)^2 + \left(\frac{57885}{d^3}\right)^2}$$

$$= \frac{29962 + 65180}{d^3} = \frac{95142}{d^3}$$

$$\sigma_1 = \frac{95142}{d^3}$$

$$\sigma_3 = \frac{-35218}{d^3}$$

$$\sigma' = \sqrt{\sigma_1^2 + \sigma_3^2} - \sigma_1, \sigma_3$$

$$\sigma' = \frac{1}{d^3} \sqrt{(95142)^2 + (-35218)^2} - (95142) - (-35218)$$

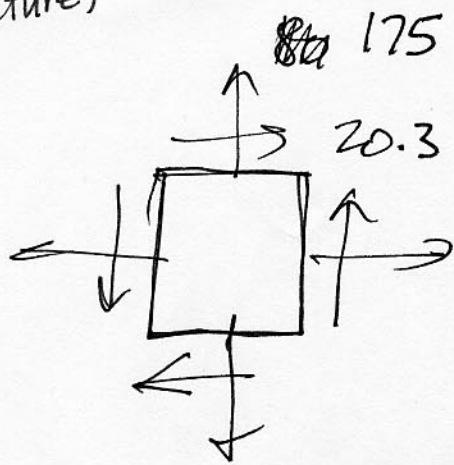
$$\sigma' = \frac{116803}{d^3}$$

$$\frac{S_y}{\sigma'} = N$$

$$\frac{150 \frac{\text{N}}{\text{mm}^2} (d^3)}{116803} = 3$$

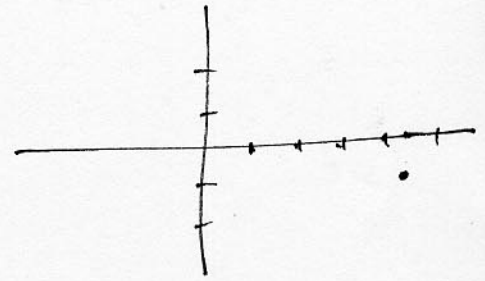
$$d = 13.3 \text{ mm}$$

5. (from lecture)



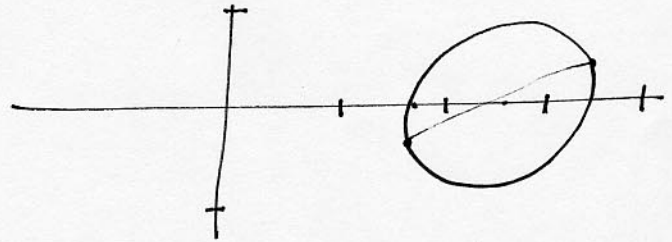
$$87.5 + \bar{\sigma}$$

$$\bar{\sigma} = \frac{P}{A}$$



$$\sigma_1 = \approx 180$$

$$\sigma_3 = \approx 80$$



$$\max(|\sigma_1|, |\sigma_3|, |\sigma_3 - \sigma_1|) = \frac{S_y}{N}$$

$$|\sigma_1| = S_y / N$$

$$\frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{S_y}{N} = 207$$

$$\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 = \left(207 + \frac{\sigma_x + \sigma_y}{2}\right)^2 = 0$$

$$g(\bar{\sigma}) = 0$$

$$\rightarrow \bar{\sigma} = P/A_x \rightarrow P$$

Now σ_1, σ_3

p	1.50E+07	J	7.09717E-07
ro	0.035	A	0.00063114
ri	0.032		
t	0.003		
T	4.50E+02		
Sy	2.90E+08		
N	1.4		

P	st	sr	tau	s1	s2	abs s1	abs s2	abs s1-s2	sprime	N
-1.00E+06	-1.50E+09	160000000	2.03E+07	1.60E+08	-1.50E+09	1.60E+08	1.50E+09	1.66E+09	1.66E+09	1.74E-01
-1.00E+05	-7.84E+07	160000000	2.03E+07	1.62E+08	-8.02E+07	1.62E+08	8.02E+07	2.42E+08	2.42E+08	1.20E+00
-1.00E+04	6.42E+07	160000000	2.03E+07	1.64E+08	6.00E+07	1.64E+08	6.00E+07	1.04E+08	1.64E+08	1.77E+00
-1.00E+03	7.84E+07	160000000	2.03E+07	1.65E+08	7.36E+07	1.65E+08	7.36E+07	9.11E+07	1.65E+08	1.76E+00
-1.00E+02	7.98E+07	160000000	2.03E+07	1.65E+08	7.50E+07	1.65E+08	7.50E+07	8.98E+07	1.65E+08	1.76E+00
-1.00E+01	8.00E+07	160000000	2.03E+07	1.65E+08	7.51E+07	1.65E+08	7.51E+07	8.97E+07	1.65E+08	1.76E+00
-1.00E+00	8.00E+07	160000000	2.03E+07	1.65E+08	7.51E+07	1.65E+08	7.51E+07	8.97E+07	1.65E+08	1.76E+00
0.00E+00	8.00E+07	160000000	2.03E+07	1.65E+08	7.51E+07	1.65E+08	7.51E+07	8.97E+07	1.65E+08	1.76E+00
1.00E+01	8.00E+07	160000000	2.03E+07	1.65E+08	7.52E+07	1.65E+08	7.52E+07	8.97E+07	1.65E+08	1.76E+00
1.00E+02	8.02E+07	160000000	2.03E+07	1.65E+08	7.53E+07	1.65E+08	7.53E+07	8.96E+07	1.65E+08	1.76E+00
1.00E+03	8.16E+07	160000000	2.03E+07	1.65E+08	7.66E+07	1.65E+08	7.66E+07	8.83E+07	1.65E+08	1.76E+00
1.00E+04	9.58E+07	160000000	2.03E+07	1.66E+08	9.00E+07	1.66E+08	9.00E+07	7.59E+07	1.66E+08	1.75E+00
1.00E+05	2.38E+08	160000000	2.03E+07	2.43E+08	1.55E+08	2.43E+08	1.55E+08	8.83E+07	2.43E+08	1.19E+00
1.00E+06	1.66E+09	160000000	2.03E+07	1.66E+09	1.60E+08	1.66E+09	1.60E+08	1.50E+09	1.66E+09	1.74E-01
6.00E+04	1.75E+08	160000000	2.03E+07	1.89E+08	1.46E+08	1.89E+08	1.46E+08	4.33E+07	1.89E+08	1.53E+00
7.00E+04	1.91E+08	160000000	2.03E+07	2.01E+08	1.50E+08	2.01E+08	1.50E+08	5.10E+07	2.01E+08	1.44E+00
8.00E+04	2.07E+08	160000000	2.03E+07	2.14E+08	1.52E+08	2.14E+08	1.52E+08	6.19E+07	2.14E+08	1.35E+00
-8.00E+04	-4.68E+07	160000000	2.03E+07	1.62E+08	-4.87E+07	1.62E+08	4.87E+07	2.11E+08	2.11E+08	1.38E+00
-7.00E+04	-3.09E+07	160000000	2.03E+07	1.62E+08	-3.30E+07	1.62E+08	3.30E+07	1.95E+08	1.95E+08	1.49E+00
-6.00E+04	-1.51E+07	160000000	2.03E+07	1.62E+08	-1.74E+07	1.62E+08	1.74E+07	1.80E+08	1.80E+08	1.61E+00
7.25E+04	1.95E+08	160000000	2.03E+07	2.04E+08	1.51E+08	2.04E+08	1.51E+08	5.35E+07	2.04E+08	1.42E+00
7.50E+04	1.99E+08	160000000	2.03E+07	2.07E+08	1.51E+08	2.07E+08	1.51E+08	5.62E+07	2.07E+08	1.40E+00
7.75E+04	2.03E+08	160000000	2.03E+07	2.11E+08	1.52E+08	2.11E+08	1.52E+08	5.90E+07	2.11E+08	1.38E+00
-7.50E+04	-4.28E+07	160000000	2.03E+07	1.62E+08	-4.48E+07	1.62E+08	4.48E+07	2.07E+08	2.07E+08	1.40E+00
-7.50E+04	-3.88E+07	160000000	2.03E+07	1.62E+08	-4.09E+07	1.62E+08	4.09E+07	2.03E+08	2.03E+08	1.43E+00
-7.25E+04	-3.49E+07	160000000	2.03E+07	1.62E+08	-3.70E+07	1.62E+08	3.70E+07	1.99E+08	1.99E+08	1.46E+00

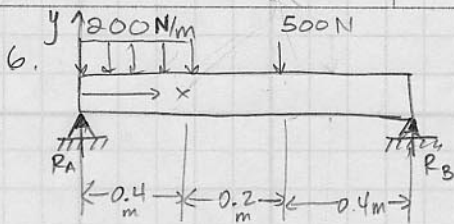
(Pa)

* *

p	2.00E+07		J	1.57E+20
P	1.00E+05			
t	0.005			
T	8.00E+02			
Sy	2.90E+08			
N	1.4			

ro	ri	A	st	sr	tau	s1	s2	abs s1	abs s2	abs	sprime	N
0.018	0.013	4.87E-04	2.31E+08	52000000	9.17E-20	2.31E+08	5.20E+07	2.31E+08	5.20E+07	1.79E+08	2.31E+08	1.25E+00
0.019	0.014	5.18E-04	2.21E+08	56000000	9.68E-20	2.21E+08	5.60E+07	2.21E+08	5.60E+07	1.65E+08	2.21E+08	1.31E+00
0.02	0.015	5.50E-04	2.12E+08	60000000	1.02E-19	2.12E+08	6.00E+07	2.12E+08	6.00E+07	1.52E+08	2.12E+08	1.37E+00
0.021	0.016	5.81E-04	2.04E+08	64000000	1.07E-19	2.04E+08	6.40E+07	2.04E+08	6.40E+07	1.40E+08	2.04E+08	1.42E+00
0.022	0.017	6.12E-04	1.97E+08	68000000	1.12E-19	1.97E+08	6.80E+07	1.97E+08	6.80E+07	1.29E+08	1.97E+08	1.47E+00
0.023	0.018	6.44E-04	1.91E+08	72000000	1.17E-19	1.91E+08	7.20E+07	1.91E+08	7.20E+07	1.19E+08	1.91E+08	1.52E+00
0.024	0.019	6.75E-04	1.86E+08	76000000	1.22E-19	1.86E+08	7.60E+07	1.86E+08	7.60E+07	1.10E+08	1.86E+08	1.56E+00
0.025	0.02	7.07E-04	1.82E+08	80000000	1.27E-19	1.82E+08	8.00E+07	1.82E+08	8.00E+07	1.02E+08	1.82E+08	1.60E+00
0.026	0.021	7.38E-04	1.78E+08	84000000	1.32E-19	1.78E+08	8.40E+07	1.78E+08	8.40E+07	9.35E+07	1.78E+08	1.63E+00
0.027	0.022	7.69E-04	1.74E+08	88000000	1.38E-19	1.74E+08	8.80E+07	1.74E+08	8.80E+07	8.60E+07	1.74E+08	1.67E+00
0.028	0.023	8.01E-04	1.71E+08	92000000	1.43E-19	1.71E+08	9.20E+07	1.71E+08	9.20E+07	7.89E+07	1.71E+08	1.70E+00
0.029	0.024	8.32E-04	1.68E+08	96000000	1.48E-19	1.68E+08	9.60E+07	1.68E+08	9.60E+07	7.22E+07	1.68E+08	1.72E+00
0.03	0.025	8.64E-04	1.66E+08	100000000	1.53E-19	1.66E+08	1.00E+08	1.66E+08	1.00E+08	6.58E+07	1.66E+08	1.75E+00
0.031	0.026	8.95E-04	1.64E+08	104000000	1.58E-19	1.64E+08	1.04E+08	1.64E+08	1.04E+08	5.97E+07	1.64E+08	1.77E+00
0.032	0.027	9.26E-04	1.62E+08	108000000	1.63E-19	1.62E+08	1.08E+08	1.62E+08	1.08E+08	5.40E+07	1.62E+08	1.79E+00
0.033	0.028	9.58E-04	1.60E+08	112000000	1.68E-19	1.60E+08	1.12E+08	1.60E+08	1.12E+08	4.84E+07	1.60E+08	1.81E+00
0.034	0.029	9.89E-04	1.59E+08	116000000	1.73E-19	1.59E+08	1.16E+08	1.59E+08	1.16E+08	4.31E+07	1.59E+08	1.82E+00
0.035	0.03	1.02E-03	1.58E+08	120000000	1.78E-19	1.58E+08	1.20E+08	1.58E+08	1.20E+08	3.80E+07	1.58E+08	1.84E+00
0.036	0.031	1.05E-03	1.57E+08	124000000	1.83E-19	1.57E+08	1.24E+08	1.57E+08	1.24E+08	3.31E+07	1.57E+08	1.85E+00
0.037	0.032	1.08E-03	1.56E+08	128000000	1.89E-19	1.56E+08	1.28E+08	1.56E+08	1.28E+08	2.83E+07	1.56E+08	1.86E+00
0.038	0.033	1.11E-03	1.56E+08	132000000	1.94E-19	1.56E+08	1.32E+08	1.56E+08	1.32E+08	2.37E+07	1.56E+08	1.86E+00
0.039	0.034	1.15E-03	1.55E+08	136000000	1.99E-19	1.55E+08	1.36E+08	1.55E+08	1.36E+08	1.93E+07	1.55E+08	1.87E+00
0.04	0.035	1.18E-03	1.55E+08	140000000	2.04E-19	1.55E+08	1.40E+08	1.55E+08	1.40E+08	1.49E+07	1.55E+08	1.87E+00
0.041	0.036	1.21E-03	1.55E+08	144000000	2.09E-19	1.55E+08	1.44E+08	1.55E+08	1.44E+08	1.07E+07	1.55E+08	1.87E+00
0.042	0.037	1.24E-03	1.55E+08	148000000	2.14E-19	1.55E+08	1.48E+08	1.55E+08	1.48E+08	6.63E+06	1.55E+08	1.88E+00
0.043	0.038	1.27E-03	1.55E+08	152000000	2.19E-19	1.55E+08	1.52E+08	1.55E+08	1.52E+08	2.63E+06	1.55E+08	1.88E+00
0.044	0.039	1.30E-03	1.55E+08	156000000	2.24E-19	1.56E+08	1.56E+08	1.56E+08	1.56E+08	1.26E+06	1.56E+08	1.86E+00
0.045	0.04	1.33E-03	1.55E+08	160000000	2.29E-19	1.60E+08	1.55E+08	1.60E+08	1.55E+08	5.07E+06	1.60E+08	1.81E+00
0.046	0.041	1.37E-03	1.55E+08	164000000	2.34E-19	1.64E+08	1.55E+08	1.64E+08	1.55E+08	8.79E+06	1.64E+08	1.77E+00
0.047	0.042	1.40E-03	1.56E+08	168000000	2.39E-19	1.68E+08	1.56E+08	1.68E+08	1.56E+08	1.24E+07	1.68E+08	1.73E+00
0.048	0.043	1.43E-03	1.56E+08	172000000	2.45E-19	1.72E+08	1.56E+08	1.72E+08	1.56E+08	1.60E+07	1.72E+08	1.69E+00
0.049	0.044	1.46E-03	1.56E+08	176000000	2.50E-19	1.76E+08	1.56E+08	1.76E+08	1.56E+08	1.95E+07	1.76E+08	1.65E+00
0.05	0.045	1.49E-03	1.57E+08	180000000	2.55E-19	1.80E+08	1.57E+08	1.80E+08	1.57E+08	2.30E+07	1.80E+08	1.61E+00
0.051	0.046	1.52E-03	1.58E+08	184000000	2.60E-19	1.84E+08	1.58E+08	1.84E+08	1.58E+08	2.63E+07	1.84E+08	1.58E+00
0.052	0.047	1.55E-03	1.58E+08	188000000	2.65E-19	1.88E+08	1.58E+08	1.88E+08	1.58E+08	2.97E+07	1.88E+08	1.54E+00
0.053	0.048	1.59E-03	1.59E+08	192000000	2.70E-19	1.92E+08	1.59E+08	1.92E+08	1.59E+08	3.29E+07	1.92E+08	1.51E+00
0.054	0.049	1.62E-03	1.60E+08	196000000	2.75E-19	1.96E+08	1.60E+08	1.96E+08	1.60E+08	3.62E+07	1.96E+08	1.48E+00
0.055	0.05	1.65E-03	1.61E+08	200000000	2.80E-19	2.00E+08	1.61E+08	2.00E+08	1.61E+08	3.93E+07	2.00E+08	1.45E+00
0.056	0.051	1.68E-03	1.62E+08	204000000	2.85E-19	2.04E+08	1.62E+08	2.04E+08	1.62E+08	4.25E+07	2.04E+08	1.42E+00
0.057	0.052	1.71E-03	1.62E+08	208000000	2.90E-19	2.08E+08	1.62E+08	2.08E+08	1.62E+08	4.56E+07	2.08E+08	1.39E+00
0.0205	0.0155	5.65E-04	2.08E+08	62000000	1.04E-19	2.08E+08	6.20E+07	2.08E+08	6.20E+07	1.46E+08	2.08E+08	1.39E+00
0.0206	0.0156	5.68E-04	2.07E+08	62400000	1.05E-19	2.07E+08	6.24E+07	2.07E+08	6.24E+07	1.45E+08	2.07E+08	1.40E+00
0.0207	0.0157	5.71E-04	2.06E+08	62800000	1.05E-19	2.06E+08	6.28E+07	2.06E+08	6.28E+07	1.44E+08	2.06E+08	1.41E+00
0.0208	0.0158	5.75E-04	2.06E+08	63200000	1.06E-19	2.06E+08	6.32E+07	2.06E+08	6.32E+07	1.42E+08	2.06E+08	1.41E+00
0.0209	0.0159	5.78E-04	2.05E+08	63600000	1.06E-19	2.05E+08	6.36E+07	2.05E+08	6.36E+07	1.41E+08	2.05E+08	1.42E+00
0.0565	0.0515	1.70E-03	1.62E+08	206000000	2.88E-19	2.06E+08	1.62E+08	2.06E+08	1.62E+08	4.40E+07	2.06E+08	1.41E+00
0.0566	0.0516	1.70E-03	1.62E+08	206400000	2.88E-19	2.06E+08	1.62E+08	2.06E+08	1.62E+08	4.43E+07	2.06E+08	1.41E+00
0.0567	0.0517	1.70E-03	1.62E+08	206800000	2.89E-19	2.07E+08	1.62E+08	2.07E+08	1.62E+08	4.46E+07	2.07E+08	1.40E+00
0.0568	0.0518	1.71E-03	1.62E+08	207200000	2.89E-19	2.07E+08	1.62E+08	2.07E+08	1.62E+08	4.49E+07	2.07E+08	1.40E+00
0.0569	0.0519	1.71E-03	1.62E+08	207600000	2.90E-19	2.08E+08	1.62E+08	2.08E+08	1.62E+08	4.53E+07	2.08E+08	1.40E+00
0.057	0.052	1.71E-03	1.62E+08	208000000	2.90E-19	2.08E+08	1.62E+08	2.08E+08	1.62E+08	4.56E+07	2.08E+08	1.39E+00

(m)



$$I = 2.85 \times 10^{-8} \text{ m}^4$$

$$C = 0.02 \text{ m}$$

$$E = \text{Steel}$$

FROM BEAM THEORY

$$B.C. \begin{cases} M(0) = 0 & V(0) = 0 \\ M(1) = 0 \end{cases}$$

$$0 \leq x \leq 0.4$$

$$q = -200 \text{ N/m}$$

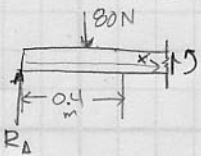
$$EI \frac{d^4 v}{dx^4} = q \quad (EI) \frac{d^2 v}{dx^2} = M$$

$$EI \frac{d^3 v}{dx^3} = qx + C_1 \quad 0 = 0 + C_1$$

$$EI \frac{d^2 v}{dx^2} = M = \frac{qx^2}{2} + C_1 x + C_2$$

$$M = \frac{qx^2}{2} \quad \text{for } 0 \leq x \leq 0.4$$

$$0.4 \leq x \leq 0.6$$

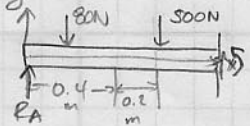


$$\sum M = 0: -M - (80 \text{ N})(x - 0.2 \text{ m}) + (264 \text{ N})(x) = 0$$

$$M = 16 \text{ N}\cdot\text{m} + (264 \text{ N})(x \cdot \text{m})$$

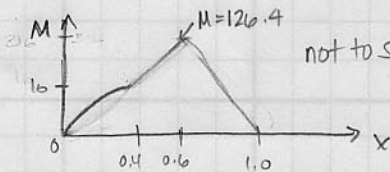
$$M = 16 + 264x \quad \text{for } 0.4 \leq x \leq 0.6$$

$$0.6 \leq x \leq 1.0$$



$$\sum M = 0: -M + (264 \text{ N})(x) - (80 \text{ N})(x - 0.2) - (500 \text{ N})(x - 0.6) = 0$$

$$M = 316 - 316x \quad \text{for } 0.6 \leq x \leq 1.0$$



$$\sigma_{\max} = \frac{M_{\max} C}{I} = \frac{(126 \text{ N})(0.02 \text{ m})}{2.85 \times 10^{-8} \text{ m}^4}$$

$$\sigma_{\max} = 88.7 \text{ MPa}$$

ON TOP OF BEAM $\leftarrow \sigma_{\max} \rightarrow \sigma_1 = 88.7 \text{ MPa}$

$$\sigma_2 = 0$$

$$\sigma_3 = 0$$

(i) Von Mises: $N = \frac{\sigma_1}{\sigma'} = \frac{300 \text{ MPa}}{88.7 \text{ MPa}} = \boxed{3.4}$

(ii) Modified Mohr: Quad I and IV where $N = \frac{S_{ut}}{\sigma_1}$ $1 \leq N \leq 1.7$ ✓

$$N = \frac{150 \text{ MPa}}{88.7 \text{ MPa}} = \boxed{1.7}$$