8.5)

Assumptions:

1. For steel, the tensile strength in psi is 500 times the Brinell hardness.
2. The curve in Fig. 8.5 is an accurate representation of the S-N data for steel.
3. For steel, the endurance limit in psi is 250 times the Brinell hardness.
4. For steel, the endurance limit for 10^3 cycle is 90% of the ultimate strength.

Analysis:

1. $S_n' = 0.5S_u$ in ksi.
2. S for 10^3 cycle $= 0.9S_u$
3.

S_u (ksi)	S_n (ksi)	S for 10^3 cycle (ksi)
95	47.5	85.5
185	92.5	166.5
240	100~125	216

Comments:

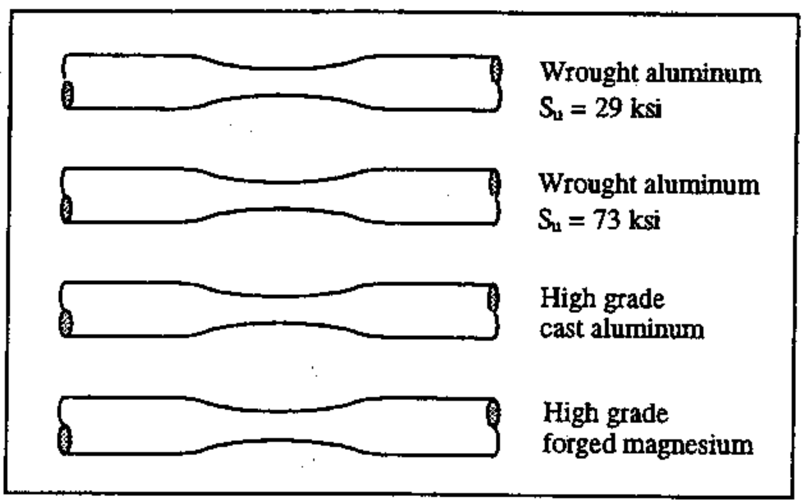
1. The relationship $S_n' = 0.5S_u$ is accurate only to ultimate tensile strength values of 200 ksi. The endurance limit may or may not continue to increase for greater tensile strength values depending on the composition of the steel.
2. For 10^3-cycle fatigue strength, actual stress is not as high as calçulated values because of significant yielding.

SOLUTION (8.6)

Known: Four known standard R.R. Moore specimens are given.

Find: Estimate the long-life rotating bending fatigue strength (state whether it is for 10^8 or 5×10^8 cycles).

Schematic and Given Data:



Assumptions:

1. The specimen is subjected to pure bending (i.e., zero transverse shear).
2. Figs. 8.8, 8.9, and 8.10 can be used to estimate the fatigue strength of aluminum and magnesium.

Analysis:

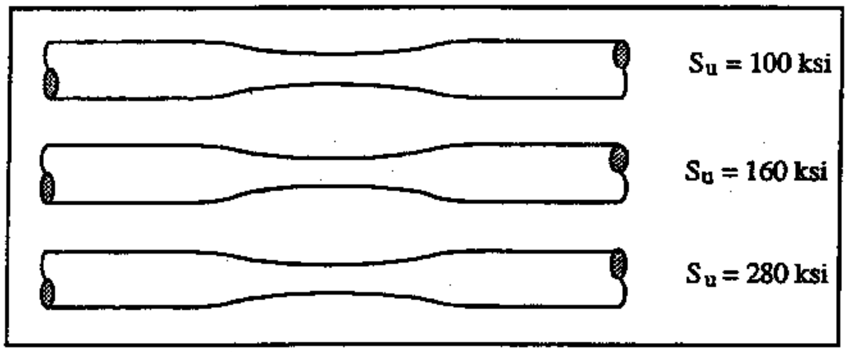
1. From Fig. 8.9, for the wrought aluminum having $S_u = 29$ ksi, the fatigue strength at 5×10^8 cycles is 12 ksi. ■
2. From Fig. 8.9, for the wrought aluminum having $S_u = 73$ ksi, the fatigue strength at 5×10^8 cycles is 19 ksi. ■
3. From Fig. 8.8, for the high grade cast aluminum, the fatigue strength at 5×10^8 cycles is 11 ksi for sand cast and 15 ksi for permanent mold cast. ■
4. From Fig. 8.10, for high grade forged magnesium, the fatigue strength at 10^8 cycles is 22 ksi. ■

SOLUTION (8.7)

Known: Standard R.R. Moore test specimens are made of steels having known ultimate tensile strengths.

Find: Estimate the rotating bending endurance limit and also the 10^3 cycle fatigue strength.

Schematic and Given Data:



Assumptions:

1. For steel, the tensile strength in psi is 500 times the Brinell hardness.
2. The curve in Fig. 8.5 is an accurate representation of the S-N data for steel.
3. For steel, the endurance limit in psi is 250 times the Brinell hardness.
4. For steel, the endurance limit for 10^3 cycle is 90% of the ultimate strength.

Analysis:

1. $S_n' = 0.5S_u$ in ksi.
2. S for 10^3 cycle $= 0.9S_u$
3.

S_u (ksi)	S_n (ksi)	S for 10^3 cycle (ksi)
100	50	90
160	80	144
280	100~125	252

Comments:

1. The relationship $S_n' = 0.5S_u$ is accurate only to ultimate tensile strength values of 200 ksi. The endurance limit may or may not continue to increase for greater tensile strength values depending on the composition of the steel.

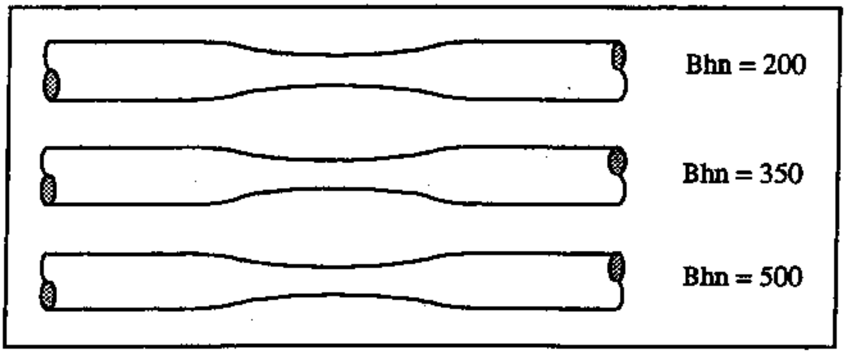
2. For the 10^3-cycle fatigue strength, the actual stress is not as high as calculated values because of significant yielding.

SOLUTION (8.8)

Known: Standard R.R. Moore test specimens are made of steels having known Brinell hardness.

Find: Estimate the rotating bending endurance limit and also the 10^3 cycle fatigue strength.

Schematic and Given Data:



Assumptions:
1. For steel, the tensile strength in psi is 500 times the Brinell hardness.
2. The curve in Fig. 8.5 is an accurate representation of the S-N data for steel.
3. For steel, the endurance limit in psi is 250 times the Brinell hardness.
4. For steel, the endurance limit for 10^3 cycle is 90% of the ultimate strength.

Analysis:
1. $S_u = 500$ Bhn in psi.
2. $S_n' = 0.25$ Bhn in ksi.
3. S for 10^3 cycle $= 0.9S_u$
4.

Bhn	S_u (ksi)	S_n' (ksi)	S for 10^3 cycle (ksi)
200	100	50	90
350	175	87.5	157.5
500	250	100~125	225

Comments:
1. The relationship $S_n' = 0.25$ Bhn is accurate only to Brinell hardness values of about 400.
2. For the 10^3-cycle fatigue strength, the actual stress is not as high as calculated values because of significant yielding.

SOLUTION (8.9)

Known: Standard R.R. Moore specimens are subjected to loading.

17) Consider a 3.5" diameter steel bar $S_u = 97$ ksi $S_y = 68$ ksi & machined surfaces. Estimate the fatigue strength for ① 10^6 or more cycles ② 5×10^4 cycles for a) bending b) axial, & c) torsional loading.

Assumptions: use table 8.1, Use Fig. 8.13 to estimate C_s, $C_G = .9$ for axial & torsion loading

1) 10^6 cycles (i.e. endurance limit)

$S_n = S_n' C_L C_G C_s$ ← see footnote c

a) bending − $C_L = 1.0$, $C_G = .8$, $C_s = .76$

$S_n' = .5 S_u = (.5)(97) = 48.5$ ksi

$S_n = (48.5\text{ ksi})(1)(.8)(.76) =$ **29.4 ksi**

Note, I did this problem graphically, so my answers are approximate!

b) axial − $C_L = 1$, $C_G = .8$ (between .7 & .8), $C_s = .76$

$S_n' = 48.5$ ksi

$S_n = (48.5\text{ ksi})(1)(.8)(.76) =$ **29.5 ksi**

c) torsion − $C_L = .58$, $C_G = .8$, $C_s = .76$

$S_n = (48.5\text{ ksi})(.58)(.8)(.76) =$ **17.1 ksi**

2. 10^3 cycle strength − plot S-N graph, & read off 5×10^4 cycles

Bending ← all these values from table 8.1

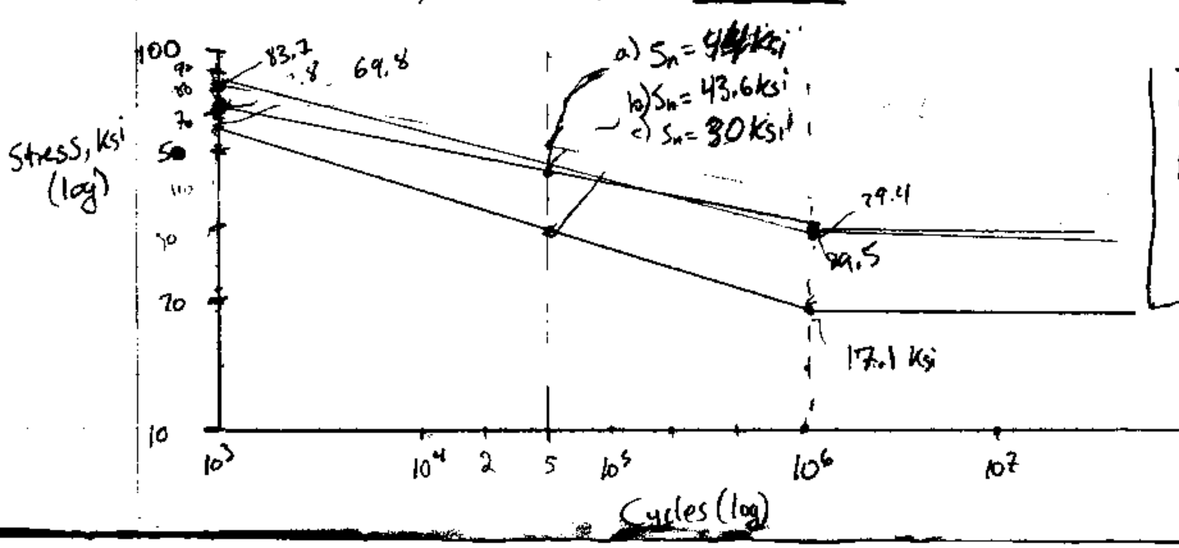
$.9 S_u = .9(97) = 83.7$ ksi

Axial

$.75 S_u = .75(97) = 72.8$ ksi

Torsion (see footnote e for table 8.1)

$.9 S_{us} = .9(.8) S_u = .9(.8)(97) = 69.8$ ksi



5×10^4 cycle Strength

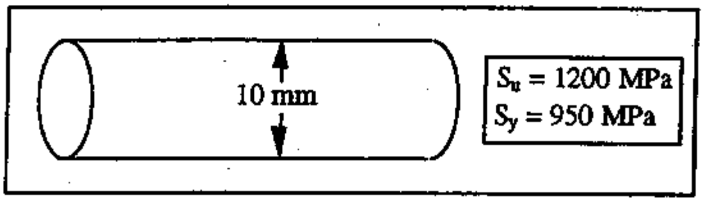
a) $S_n = 44$ ksi

b) $S_n = 43.6$ ksi

c) $S_n = 30$ ksi

8.19

Schematic and Given Data:



FINE GROUND SURFACE

FIND: FATIGUE STRENGTH CORRESPONDING to
a) 10^6 or more cycles
b) 2×10^5 cycles

Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_s.
4. The gradient factor, $C_G = 0.9$.

Analysis:

1. Endurance limits: (10^6 cycle strength)

   $S_n = S_n' C_L C_G C_s$

   For bending,

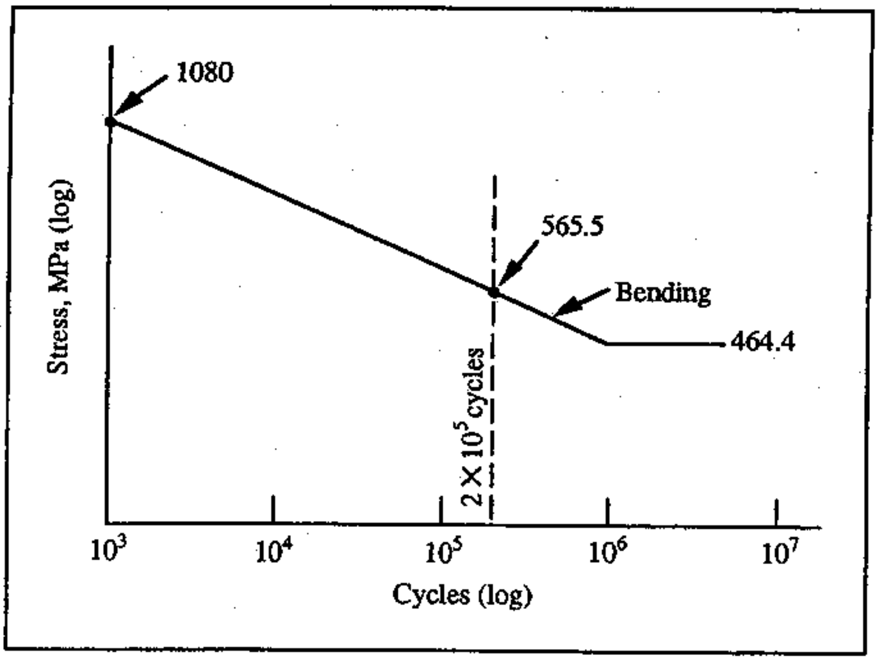
   $S_n' = 0.5\, S_u = 0.5(1200) = 600$ MPa (Fig. 8.5)
   $C_L = 1$ (Table 8.1)
   $C_G = 0.9$ (Table 8.1)
   $C_s = 0.86$ (Fig. 8.13)
   $S_n = (600)(1)(0.9)(0.86) = 464.4$ MPa

2. 10^3 cycle strength

   For bending,

   $0.9 S_u = 0.9(1200) = 1080$ MPa (Table 8.1)

3. S-N curves



4. 2×10^5 cycle strength

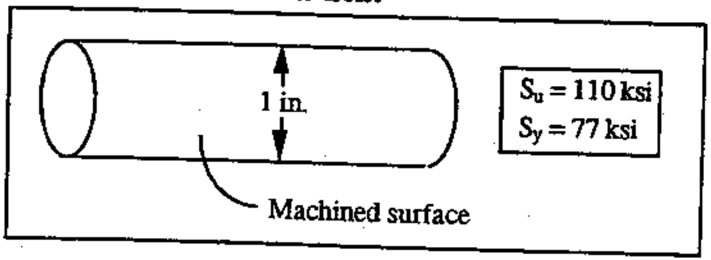
   Bending: 565.5 MPa

SOLUTION (8.21)

Known: A steel bar having known S_u and S_y has average machined surfaces.

Find: Plot on log-log coordinates estimated S-N curves for (a) bending, (b) axial, and (c) torsional loading. For each of the three types of loading, determine the fatigue strength corresponding to (1) 10^6 or more cycles and (2) 6×10^4 cycles.

Schematic and Given Data:



Assumptions:
1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_s.

Analysis:

1. <u>Endurance limits</u>: (10^6 cycle strength)

   $S_n = S_n' C_L C_G C_s$

   For bending,

   $S_n' = 0.5\ S_u = 0.5(110) = 55$ ksi (Fig. 8.5)

   $C_L = 1$ (Table 8.1)

   $C_G = 0.9$ (Table 8.1)

   $C_s = 0.74$ (Fig. 8.13)

   $S_n = (55)(1)(0.9)(0.74) = 36.6$ ksi ■

   For axial,

   $S_n' = 55$ ksi

   $C_L = 1$

   $C_G = 0.8$ (between 0.7 and 0.9)

   $C_s = 0.74$

   $S_n = 55(1)(0.8)(0.74) = 32.6$ ksi ■

   For torsion,

   $S_n' = 55$ ksi

   $C_L = 0.58$

   $C_G = 0.9$

   $C_s = 0.74$

   $S_n = 55(0.58)(0.9)(0.74) = 21.2$ ksi ■

2. <u>10^3 cycle strength</u>

   For bending,

   $0.9S_u = 0.9(110) = 99.0$ ksi (Table 8.1)

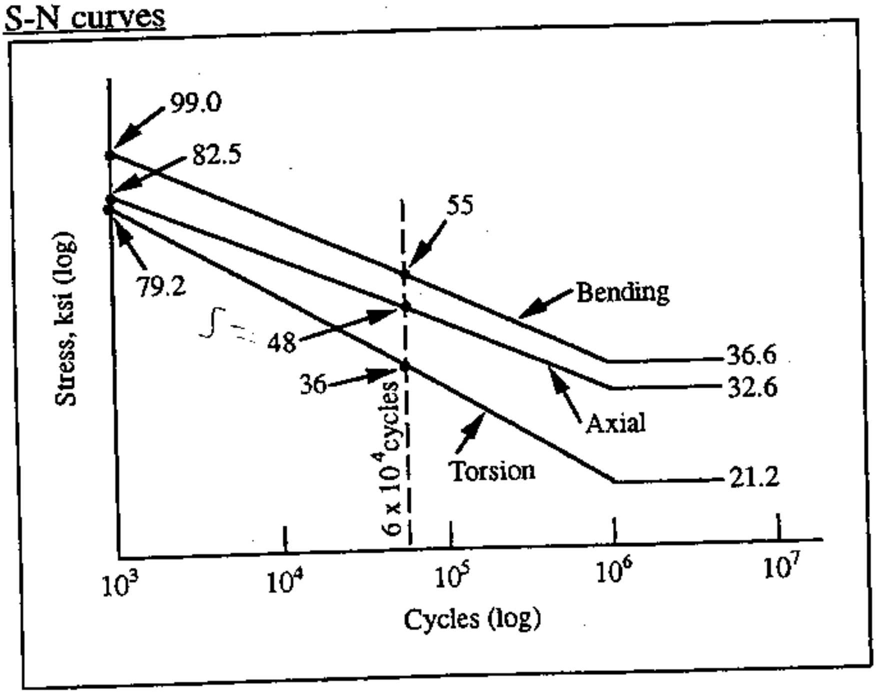
   For axial,

   $0.75S_u = 0.75(110) = 82.5$ ksi

   For torsion,

   $0.9S_{us} = 0.9(0.8)(110) = 79.2$ ksi

3. <u>S-N curves</u>



4. 6×10^4 cycle strength

Bending: 55 ksi
Axial: 48 ksi
Torsion: 36 ksi

Comments:

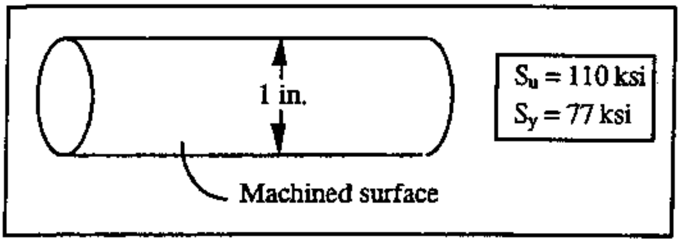
1. C_s is not used for correcting 10^3-cycle strength because for ductile parts this is close to static strength, which is unaffected by surface finish.
2. For critical designs pertinent test data should be used rather than the preceding rough approximation.

SOLUTION (8.23)

Known: A steel bar having known S_u and S_y has average machined surfaces.

Find: Determine the fatigue strength corresponding to (1) 10^6 or more cycles and (2) 6×10^4 cycles for the case of zero-to-maximum (rather than completely reversed) load fluctuations for bending, axial, and torsional loading.

Schematic and Given Data:

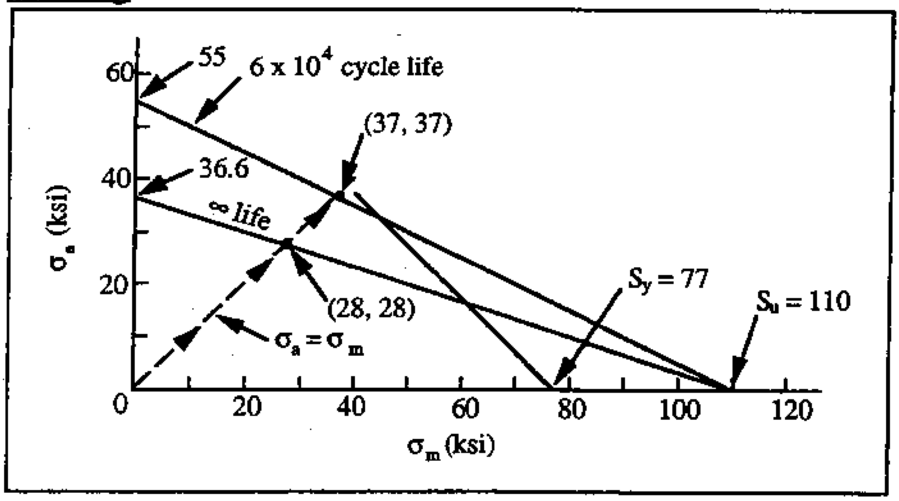


Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_s.

Analysis:

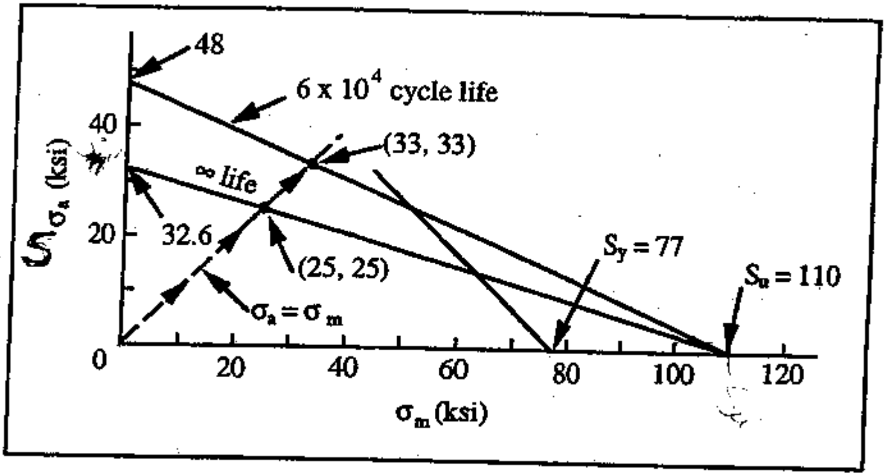
1. Bending



For ∞ life, $\sigma_{max} = 56$ ksi ■

For 6×10^4 cycles, $\sigma_{max} = 74$ ksi ■

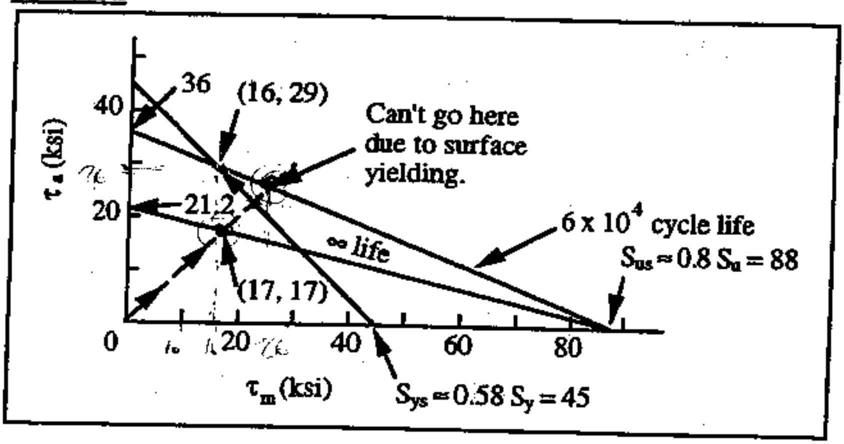
2. Axial



For ∞ life, $\sigma_{max} = 50$ ksi

For 6×10^4 cycles, $\sigma_{max} = 66$ ksi

3. Torsion



For ∞ life, $\tau_{max} = 34$ ksi

For 6×10^4 cycles, $\tau_{max} =$ ~~58~~ ksi*

*Only if the yielding indicated is acceptable, but if so, $\tau_a = 29$, and the load stress can be 0-52 ksi.

SOLUTION (8.26)

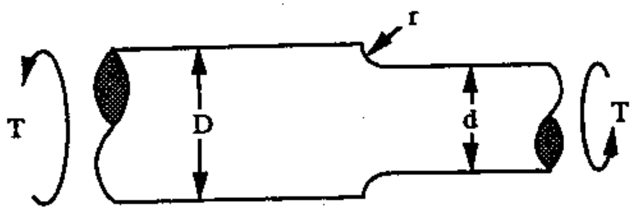
Known: A stepped shaft having known dimensions was machined from steel having known tensile properties.

Find:

(a) Estimate the torque T required to produce static yielding.

(b) Estimate the value of reversed torque, ±T required to produce eventual fatigue failure.

Schematic and Given Data:



D = 2 in.
d = 1 in.
r = 0.05 in.
S_u = 90 ksi
S_y = 75 ksi
Machined surfaces.

Assumption: The shaft is manufactured as specified with regard to the critical fillet.

Analysis:

1. From Eqs. 4.3 and 4.4, for static yielding,

$$\tau = \frac{Tc}{J} = \frac{16T}{\pi d^3} = \frac{16T}{\pi}$$

Equate this to shear yield,

$$S_{ys} \approx 0.58 S_y = 0.58(75) = 43.5 \text{ ksi}$$

$$\tau = \frac{16T}{\pi} = 43{,}500$$

$$T = \frac{\pi(43{,}500)}{16} = 8540 \text{ lb in.}$$ ■

2. For fatigue failure, the appropriate endurance limit is:

$$S_n = S_n' C_L C_G C_s$$

where $S_n' = 0.5S_u = 0.5(90)$

$C_L = 0.58$ (Table 8.1)

$C_G = 0.9$ (Table 8.1)

$C_s = 0.77$ (Fig. 8.13)

$$S_n = 0.5(90)(0.58)(0.9)(0.77) = 18.1 \text{ ksi}$$ ■

3. From Fig. 4.35, $K_t = 1.72$
From Fig. 8.24, $q = 0.78$
Thus, $K_f = 1 + (K_t - 1)q$ [Eq. (8.2)]
$= 1 + (0.72)(0.78) = 1.56$

4. For fatigue failure,

$$K_f \frac{16T}{\pi} = 18{,}100 \; ; \; T = \frac{18{,}100\pi}{1.56(16)} = 2{,}280 \text{ lb in.}$$ ■

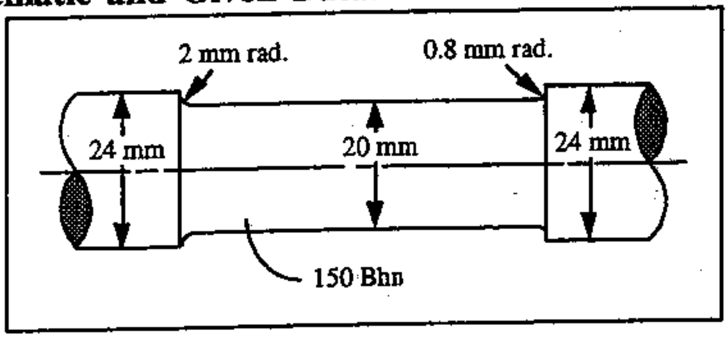
Comment: For static loading of a ductile material, the very first yielding at the notch-root is not significant; hence, ignore stress concentration.

SOLUTION (8.28)

Known: A machined shaft having a known hardness experiences completely reversed torsion.

Find: With a safety factor of 2, estimate the value of reversed torque that can be applied without causing eventual fatigue failure.

Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to the critical shaft geometry.

Analysis:

1. For steel,

$$S_u = 0.5 \text{ Bhn} = 0.5(150) = 75 \text{ ksi}$$

or, $S_u = 75 \text{ ksi} \left(\frac{6.890 \text{ MPa}}{\text{ksi}}\right) = 517 \text{ MPa}$

2. $S_n = S_n' C_L C_G C_s$

$S_n' = 0.5 S_u = 0.5(517)$ (Fig. 8.5)
$C_L = 0.58$ (Table 8.1)
$C_G = 0.9$ (Table 8.1)
$C_s = 0.78$ (Fig. 8.13)

$S_n = 0.5(517)(0.58)(0.9)(0.78) = 105.3 \text{ MPa}$ ■

3. At the critical point (0.8 mm radius), $r/d = 0.04$ and $D/d = 1.2$

From Fig. 4.35(c), $K_t = 1.65$
From Fig. 8.23, $q = 0.74$
Hence, $K_f = 1 + (K_t - 1)q$ [Eq.(8.2)]
$= 1 + (0.65)(0.74) = 1.48$

4. Therefore, the nominal value of reversed torsional stress can be $\tau = 105.3/1.48 = 71.1$ MPa.

But, $\tau = \frac{16T}{\pi d^3}$ or $T = \frac{\tau \pi d^3}{16}$

$$T = \frac{(71.1 \text{ MPa})\pi(20 \text{ mm})^3}{16} = 111{,}700 \text{ N·mm}$$

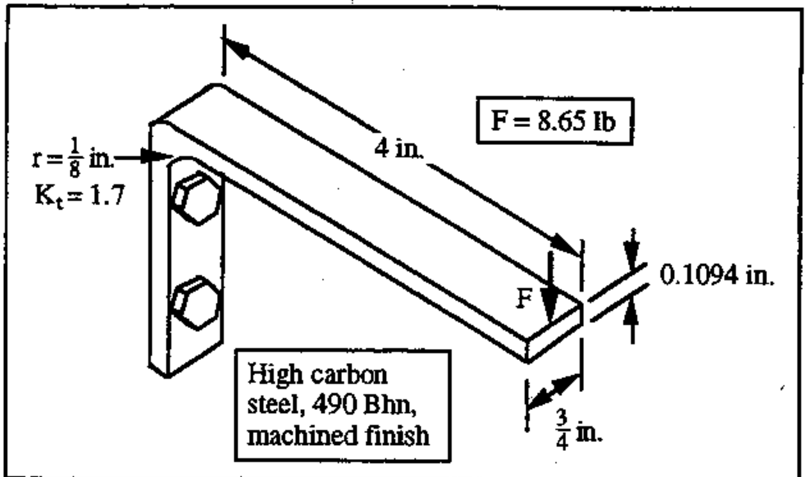
with SF = 2, $T = \frac{111.7 \text{ N·m}}{2} = 55.8 \text{ N·m}$ ■

SOLUTION (8.30)

Known: A cantilever beam is serving as a spring for a latching mechanism. When assembled, the free end is deflected 0.075 in., which correspond to a force F. When the latch operates, the end deflects as additional 0.15 in.

Find: Determine if eventual fatigue failure is expected.

Schematic and Given Data:



Assumption: The cantilever beam is linearly elastic.

Analysis:

1. Assuming linear elastic response, if $\delta_{min} = 0.075$ in. corresponds to 8.65 lb, then $\delta_{max} = 0.225$ in. corresponds to 25.95 lb. Therefore, $F_{min} = 8.65$ lb and $F_{max} = 25.95$ lb.
2. $S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]

   $S_n' \approx 100$ ksi* (Fig. 8.6)
   $C_L = 1$ (Table 8.1)
   $C_G = 1$ (Table 8.1)
   $C_s = 0.54$ (Fig. 8.13)

   $S_n = (100)(1)(1)(0.54) = 54$ ksi

   *The $S_n' = 0.25$ Bhn ksi estimation is accurate only to Brinell hardness values of about 400.
3. Operating stresses:

   $\sigma_m = \frac{M_m}{Z} K_f \quad \sigma_a = \frac{M_a}{Z} K_f$

   where

$$M_m = 4\,F_m = 4\left(\frac{25.95 + 8.65}{2}\right) = 69.2 \text{ in•lb}$$

$$M_a = 4\,F_a = 4\left(\frac{25.95 - 8.65}{2}\right) = 34.6 \text{ in•lb}$$

$$Z = \frac{I}{c} = \frac{bh^2}{6} = \frac{(0.75)(0.1094)^2}{6} \approx 0.0015 \text{ in.}^2$$

   From Fig. 8.24, $q \approx 1$

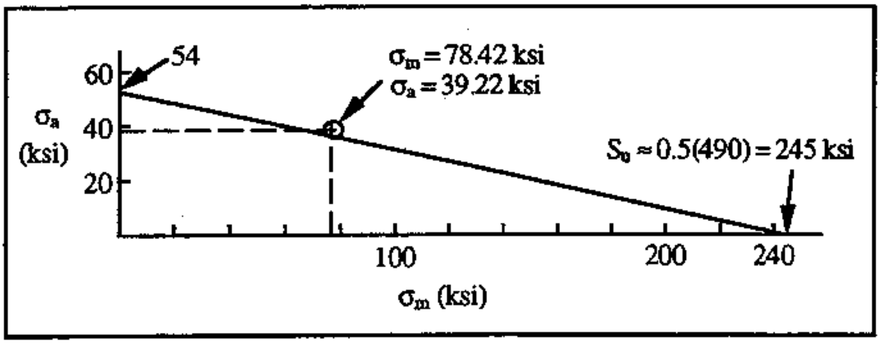
   $K_f = 1 + (1.7 - 1)(1) = 1.7$ [Eq.(8.2)]

   Therefore,

$$\sigma_m = \frac{69.2}{0.0015}(1.7) = 78.42 \text{ ksi}$$

$$\sigma_a = \frac{34.6}{0.0015}(1.7) = 39.22 \text{ ksi}$$

4.



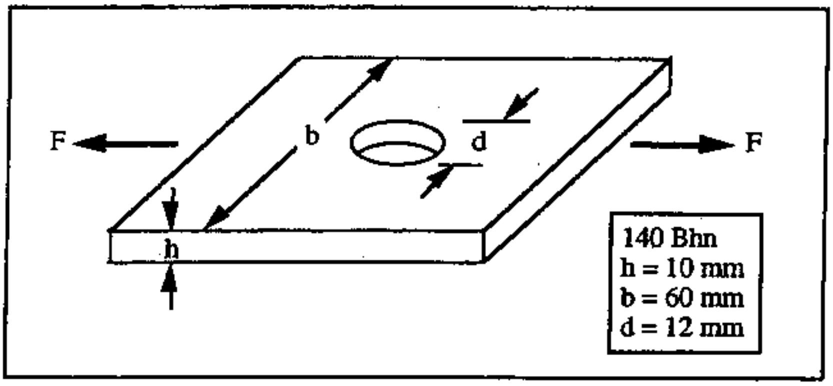
5. From the figure above, the stresses are marginal with respect to causing eventual failure. ■

SOLUTION (8.33)

Known: A cold-drawn rectangular steel bar has known hardness value and dimensions and is to have infinite life with 90% reliability and a safety factor of 1.3.

Find: Estimate the maximum tensile force that can be applied to the ends:
(a) if the force is completely reversed,
(b) if the force varies between zero and a maximum value.

Schematic and Given Data:



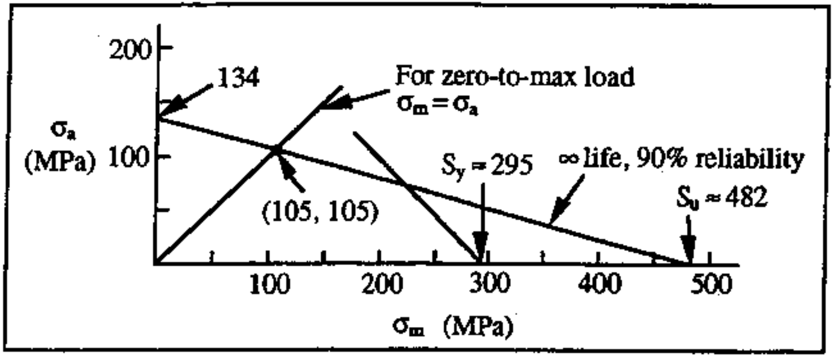
Assumption: The hole is symmetrically machined in the plate.

Analysis:

1. For 140 Bhn, $S_u \approx 0.5(140) = 70$ ksi or
   $S_u = 6.890(70) = 482.3$ MPa (Appendix A-1)
2. From Eq. (3.10a), $S_y \approx 525$ Bhn - 30,000
   $= 42{,}800$ psi $= 295$ MPa
   (May be higher for cold drawn, in any case, problem is not affected)
3. $S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]
   $S_n' = 0.5 S_u$ (Fig. 8.5)
   $C_L = 1$ (Table 8.1)
   $C_G = 0.8$ (Table 8.1)
   $C_s = 0.78$ (Fig. 8.13)
   $S_n = (0.5)(482.3)(1)(0.8)(0.78) = 150$ MPa ■
   But this is (conservatively) for 50% reliability. For 90% reliability back off 1.3 standard deviations (Fig. 6.17) of 8% or 10.4%. Therefore, S_n (90% reliability) $\approx 150(0.896) = 134$ MPa.
4. From Fig. 4.40, $K_t = 2.5$
   From Fig. 8.24, $q = 0.85$ (by extrapolation)
   Thus, $K_f = 1 + (K_t - 1)q$ [Eq. (8.2)]
   $= 1 + (1.5)(0.85) = 2.28$
5. $\sigma = \frac{F}{A} K_f = \frac{F}{(10)(48)}(2.28) = 0.00475\,F$
   or $F = 0.21\sigma$

6. For completely reversed load, $\sigma_{max} = 134$ MPa;
$F_{max} = 0.21\ \sigma_{max}$ or, with a safety factor of 1.3,
$F_{max} = (0.21)(134)/1.3 = 22$ kN ■

7.



8. From the figure above, for zero-to-max load,
$\sigma_{max} = 105 + 105 = 210$ MPa. Therefore, with a safety factor of 1.3,
$F_{max} = (0.21)(210)/1.3 = 34$ kN ■

SOLUTION (8.38)

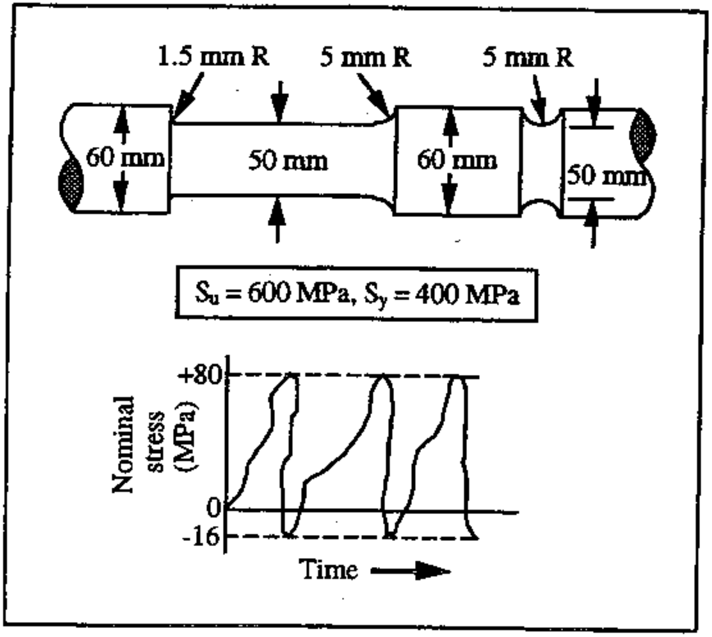
Known: A shaft is subjected to a fluctuating nominal stress. The shaft is made of steel having known S_u and S_y.

Find: Estimate the safety factor with respect to eventual fatigue failure if:

(a) the stresses are bending,

(b) the stresses are torsional.

Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to surface finish and critical fillet radii.

Analysis:

1. For bending stresses,

   $S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]

   $S_n' = 0.5S_u$ (Fig. 8.5)

   $C_L = 1$ (Table 8.1)

   $C_G = 0.9$ (Table 8.1)

   $C_s = 0.77$ (Fig. 8.13)

   $S_n = 0.5(600)(1)(0.9)(0.77) = 208$ MPa ■

2. Highest stress is at the 1.5 mm fillet where

   $D/d = 1.2$ and $r/d = 0.03$

   From Fig. 4.35, $K_t = 2.3$

   From Fig. 8.24, $q = 0.78$

   From Fig. (8.2), $K_f = 1 + (K_t - 1)q$

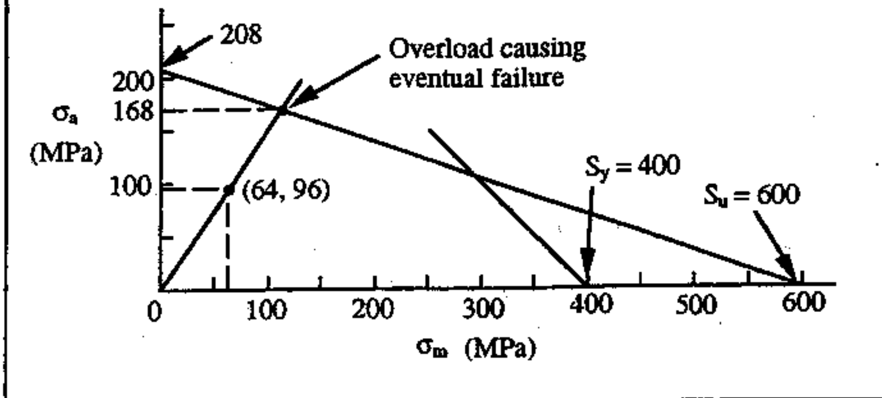
   $K_f = 1 + (1.3)(0.78) = 2.01$

3. At the fillet

   $$\sigma_m = 2.01\left(\frac{80 - 16}{2}\right) = 64 \text{ MPa}$$

   $$\sigma_a = 2.01\left(\frac{80 + 16}{2}\right) = 96 \text{ MPa}$$

4. Thus, for the bending stresses,



   SF = 168/96 = 1.8 ■

5. For torsional stresses,

   $S_n = S_n' C_L C_G C_s$

   $S_n' = 0.5S_u$

   $C_L = 0.58$

   $C_G = 0.9$

   $C_s = 0.77$

   $S_n = 0.5(600)(0.58)(0.9)(0.77) = 121$ MPa ■

6. From Fig. 4.35, $K_t = 1.78$

   From Fig. 8.24, $q = 0.81$

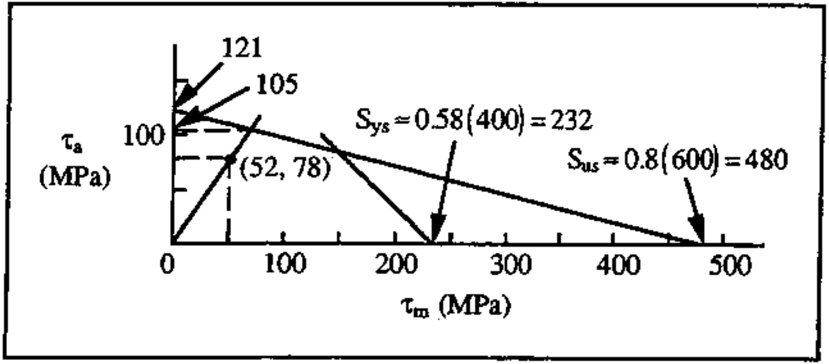
   $K_f = 1 + (1.78 - 1)(0.81) = 1.63$

7. At critical fillet,

$$\tau_m = 1.63\left(\frac{80 - 16}{2}\right) = 52 \text{ MPa}$$

$$\tau_a = 1.63\left(\frac{80 + 16}{2}\right) = 78 \text{ MPa}$$

8. Thus, for torsional stresses,



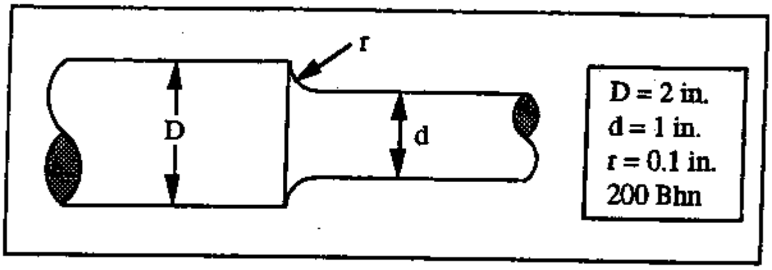
SF = 105/78 = 1.3 ■

SOLUTION (8.46)

Known: A stepped shaft having known dimensions was machined from AISI steel of known hardness. The loading is one of completely reversed torsion. During a typical 30 seconds of operation under overload conditions the nominal (Tc/J) stress in the 1-in.-dia. section was measured.

Find: Estimate the life of the shaft when operating continuously under these conditions.

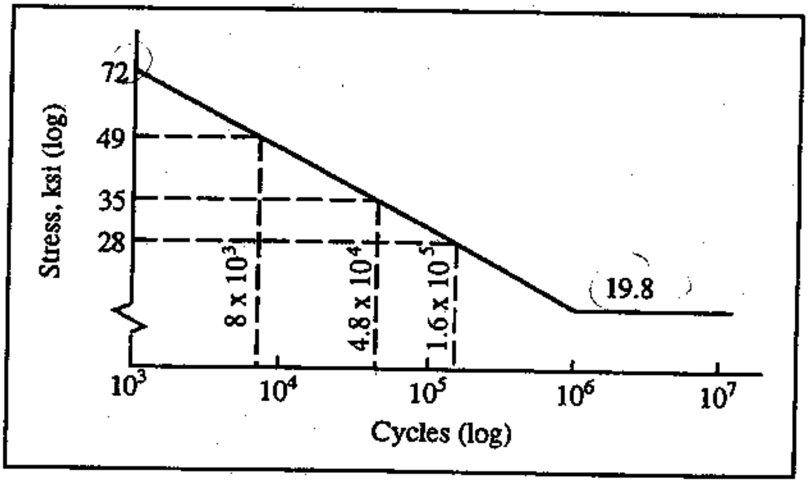
Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to the critical fillet geometry and surface finish.

Analysis:

1. At the fillet,
   from Fig. 4.35(c), $K_t = 1.46$
   from Fig. 8.24, $q = 0.86$
   Thus, using Eq. (8.2), $K_f = 1 + (0.46)(0.86) = 1.40$
2. $S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]
   $S_n' = 0.25$ Bhn (Fig. 8.5)
   $C_L = 0.58$ (Table 8.1)
   $C_G = 0.9$ (Table 8.1)
   $C_s = 0.76$ (Fig. 8.13)
   $S_n = 0.25(200)(0.58)(0.9)(0.76) = \underline{19.8 \text{ ksi}}$
3. From Table 8.1,
   10^3 cycle strength $= 0.9S_{us} = 0.9(0.8)S_u$
   $= 0.9(0.8)(0.5)\text{Bhn} = 0.9(0.8)(0.5)(200) = \underline{72 \text{ ksi}}$
4.



5. The 30 second test involves these stresses (in the fillet) above the endurance limit (see graph):
   1 cycles at $\tau_a = 35(1.4) = 49$ ksi
   $(N = 8 \times 10^3$ cycles)
   2 cycles at $\tau_a = 25(1.4) = 35$ ksi
   $(N = 4.8 \times 10^4$ cycles)

4 cycles at $\tau_a = 20(1.4) = 28$ ksi

$(N = 1.5 \times 10^5 \text{ cycles})$

$$\text{Life used in 30 seconds} = \frac{1}{8 \times 10^3} + \frac{2}{4.8 \times 10^4} + \frac{4}{1.6 \times 10^5} = 1.916 \times 10^{-4}$$

$$\text{Estimated life} = \frac{1}{1.916 \times 10^{-4}} = 5217 \text{ periods of 30 seconds}$$

Estimated life ≈ 43 hours

■