Week 6: 2-Degree-of-Freedom Kinematics

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kinematics
in multiple degrees
of freedom
Hapkit kinematics (1-DOF)

\[
\frac{r_{\text{pulley}}}{r_{\text{sector}}} \theta_{\text{pulley}} = \frac{r_{\text{sector}}}{r_{\text{sector}}} \theta_{\text{sector}}
\]

\[
x_{\text{handle}} = \frac{r_{\text{pulley}}}{r_{\text{sector}}} \theta_{\text{pulley}}
\]
joint variables

Be careful how you define joint positions

Absolute

Relative
forward kinematics for higher degrees of freedom

fwd kinematics: from joint angles, calculate endpoint position
serial structures
absolute forward kinematics

\[ x = L_1 \cos(\theta_1) + L_2 \cos(\theta_2) \]
\[ y = L_1 \sin(\theta_1) + L_2 \sin(\theta_2) \]
relative forward kinematics

\[ x = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) \]
\[ y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) \]
Inverse Kinematics

• Using the end-effector position, calculate the joint angles necessary to achieve that position

• There can be:
  – No solution (workspace issue)
  – One solution
  – More than one solution
example

• Two possible solutions

• Our devices will be simple enough that you can just use geometry for inverse kinematics
parallel structures
four-bar linkage

- commonly used 1-DOF mechanism
- relationship between input link angle and output link angle can be computed from geometry

Types of four-bar linkages, \( s = \) shortest link, \( l = \) longest link
five-bar linkage

• commonly used 2-DOF mechanism
• relationship between input link angle and output link angle can be computed from geometry

example:
pantograph

**Definition 1:** a mechanical linkage connected in a manner based on parallelograms so that the movement of one pen, in tracing an image, produces identical movements in a second pen.

**Definition 2:** a kind of structure that can compress or extend like an accordion.
pantograph example

A Polygraph is a device that produces a copy of a piece of writing simultaneously with the creation of the original, using pens and ink. Famously used by Thomas Jefferson ~1805.

Typically uses a pantograph mechanism: a five-bar linkage with parallel bars such that motion at one point is reproduced at another point.
pantograph haptic device

Xiyang Yeh, ME 327 2012
http://charm.stanford.edu/ME327/Xiyang
pantograph haptic device

Sam Schorr and Jared Muirhead, ME 327 2012
http://charm.stanford.edu/ME327/JaredAndSam
A. Design

In order to render virtual environments using Graphkit, we designed a 2-DOF haptic device based on the pantograph haptic device created as a customization project for Hapkit 3.0. This device allows for a planar 2-DOF workspace, suitable for various applications. Details of the design and control of Graphkit are provided in Section B.

B. Software and Control

Connected using alligator clips as shown in Figure 2(a), two Hapkits are modified by replacing their handles with the new handles. In order to electrically connect the two Hapkits, each previously made their own 1-DOF device, can work one with a "left kit" and one with a "right kit" who have kits are divided into "left kits" and "right kits". Two students, one Hapkit board is chosen to be the "Master" and the other "Slave".

Graphkit is made from two original Hapkits 3.0, by reusing the capstan drive mechanisms, and allowing practical for drawing (Figure 2). In order to render virtual environments using Graphkit, Graphkit takes advantage of Hapkit 3.0's modular design and reusing the capstan drive mechanisms, and allowing practical for drawing (Figure 2).
Thus, users have limited freedom to obtain, modify, and learn software [2], [4], [8]–[10]. Similarly, most haptic hardware source hardware has not seen the same growth as open source design tools and complex manufacturing processes, open. However, with the increased development of proprietary solutions such as RepRap [6], and robotics projects [7].

Electronics platforms such as Arduino [5], open 3-D printing Free hardware designs have also had significant impact like GNU/Linux, Apache and R [2]–[4]. Open source and technology companies do business, and resulted in products. Mentions have aided in academic research, changed the way.

A. Motivation

Anyone to make, modify, distribute and use that thing. " Available to, and usable by, the public in a way that allows electrical or mechanical – whose design information is well as tutorials for customization are available at http://hapkit.stanford.edu.

Engineering students. Open source designs for all devices as Pantograph mechanism was used in an undergraduate class to transform them into a two-degree-of-freedom device. The Hapkits, a user acquires a small number of additional parts mechanism. These designs are modular; after building two elements along with a novel kinematic design to form a serial.

Pantograph mechanism, and the other uses customized Hapkit designs that evolved from the original Hapkit 3.0. One uses educational applications. In this paper we present a family of open source kinesthetic haptic devices that build upon researchers, educators, and students to share designs arising.

free hardware for educational haptics that permits advancement through communities of practice. We propose customize, and improve designs, thus enabling technological.

The Open Source and Free (as in “Libre”) software move—Open Hardware is a thing – a physical artifact, either. From our group, Hapkit 3.0 [11], uses 3-D printed structural elements from our group, Hapkit 3.0 [11], [14]–[22], the most recent design of the Haptic Paddle in 1997, numerous other universities have made their own version of the device [11], [14]–[22], of the Haptic Paddle [13], a one-degree-of-freedom (1-DOF).

B. Prior Work

3-D Printed Kinesthetic Haptic Devices∗

I. INTRODUCTION

∗This work was supported in part by National Science Foundation grant.
while constraining ourselves to the following: the workspace of the handles (workspace (shown in Figure 4(b)) and minimized the length 3.0's design [11]. Additionally, we maximized the device's motion by using the capstan drive transmission from Hapkit components from moving. Haplink also maintains low friction between both motors are grounded, preventing the highest mass of the device. We refer to the first Hapkit Sector Pulley in the series as Hapkit A and the second one as Hapkit B.

In order to control Haplink, we use our original Hapkit Board sensor, grounded and located under the magnet attached to the shaft of motor B. In order to render virtual environments, the control loop is coupled serial drive mechanism and 3-D printed structural components and the ability of users to design their own end-effectors. Starting from a single Hapkit 3.0 device, the structural components and the ability of users to design their own end-effectors. Starting from a single Hapkit 3.0 device, the 2-DOF kinesthetic haptic device that uses a coupled serial drive mechanism and 3-D printed structural components. Haplink is controlled with a single Hapkit Board.

Fig. 4. Workspace achieved by (a) Graphkit with 80 mm, deg and 3.0, and a square with an area of at least 100 mm, and (b) Graphkit with 126 mm and (b) mm, mm, and 50 deg and 88 mm.

Fig. 5. Haplink: A 2-DOF kinesthetic haptic device that uses a coupled serial drive mechanism and 3-D printed structural components. (a) Built Haplink. (b) Kinematic model of Haplink. The dimensions of the final configuration are given in Figure 4(b).

Haplink maintains major aspects of the 3-D printed structural components and the ability of users to design their own end-effectors. Starting from a single Hapkit 3.0 device, the structural components and the ability of users to design their own end-effectors. Starting from a single Hapkit 3.0 device, the 2-DOF kinesthetic haptic device that uses a coupled serial drive mechanism and 3-D printed structural components. Haplink is controlled with a single Hapkit Board.
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The shaft of motor B.

Sensor, grounded and located under the magnet attached to located with Hapkit A and we added one magnetoresistive sensors are connected to a single Hapkit board, which drives both motors. Figure 5(b) shows a model of Hapkit.
Kinematics

The forward kinematic equations are:

\[
\begin{bmatrix}
  p_x \\
  p_y
\end{bmatrix} = \begin{bmatrix}
  -l_a \sin(\tilde{\theta}_a) + c_x \\
  l_a \cos(\tilde{\theta}_a) + c_y
\end{bmatrix}
\]

\[
\begin{bmatrix}
  r_x \\
  r_y
\end{bmatrix} = \begin{bmatrix}
  -l_b \sin(\tilde{\theta}_a + \tilde{\theta}_b) + p_x \\
  l_b \cos(\tilde{\theta}_a + \tilde{\theta}_b) + p_y
\end{bmatrix}
\]

where:

\[
\tilde{\theta}_a = \theta_a + \Delta \theta_a
\]

\[
\tilde{\theta}_b = \theta_b + \Delta \theta_b
\]

\[
\theta_{ma} = -\frac{r_a}{r_{ma}} \theta_a
\]

\[
\theta_{mb} = -\frac{r_b}{r_{mb}} \theta_b
\]