Introduction to Computational Advertising

MS&E 239
Stanford University
Autumn 2010
Instructors: Andrei Broder and Vanja Josifovski
General course info

- Course Website: [http://www.stanford.edu/class/msande239/](http://www.stanford.edu/class/msande239/)
- TA: PranavDandekar (Office hours: TBD)
- Course email lists
  - Staff: [msande239-aut1011-staff](mailto:msande239-aut1011-staff)
  - All: [msande239-aut1011-students](mailto:msande239-aut1011-students)
  - Please use the staff list to communicate with the staff
- Lectures: 10am ~ 12:30pm Fridays in Gates B12
- Office Hours: by appointment – preferably right after the lecture
Questions?

We welcome questions and suggestions about all aspects of the course: msande239-aut1011-staff
Course Overview (subject to change)

1. 09/24 Intro
2. 10/01 Textual advertising basics
3. 10/08 Marketplace and economics
4. 10/15 Sponsored search
5. 10/22 Contextual advertising
6. 10/29 Reactive methods for ad selection
7. 11/05 Display advertising
8. 11/12 Targeting
9. 11/19 Emerging formats (Mobile etc)
10. 12/03 Project Presentations
Lecture 2: Marketplace & Economics
Disclaimers & acknowledgements

• This talk presents the opinions of the authors. It does not necessarily reflect the views of Yahoo! Inc or any other entity.

• Algorithms, techniques, features, etc mentioned here might or might not be in use by Yahoo! Or any other company.

• Some of the slides in this lecture are based on courseware generously donated by: Ashish Goel, Michael Schwartz, David Pennock, Prabhakar Raghavan, and others.

• Second part of this lecture is based on Edelman, Ostrovsky, and Schwarz, Internet Advertising and the Generalized Second Price Auction, 2005 which is required reading!
What is advertising? Why buy it?
"Half the money I spend on advertising is wasted; the trouble is I don't know which half."

John Wanamaker, ~ 1875
Advertising as information

- “I do not regard advertising as entertainment or an art form, but as a medium of information….” [David Ogilvy, 1985]
- “Advertising as Information” [Nelson, 1974]
- Irrelevant ads are annoying; relevant ads are interesting
  - Vogue, Skiing, etc are mostly ads and advertorials

Finding the best textual ad is an information retrieval problem with multiple, possible contradictory utility functions
The ad explains “The Vertue of the COFFEE drink” what coffee is, how it grows, how it cures numerous maladies, including Dropsy, Gout, and Scurvy, …
What are advertisers buying?

- “...in an information-rich world, the wealth of information means a dearth of something else: a scarcity of whatever it is that information consumes. What information consumes is rather obvious: it consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention and a need to allocate that attention efficiently among the overabundance of information sources that might consume it.”

Herbert Simon, 1971
Advertisers are buying attention!
What kind of users are advertisers looking for?

- **Qualified**
  - Selection of users by business rules on concrete attributes
  - For example: car model sold only in USA → US residents, appropriate income

- **Receptive**
  - Interest level of the user in the advertiser's message and the user's willingness to absorb it
  - For example: people interested in skiing ads are often interested in biking ads

- **Responsive**
  - Propensity of the user to respond in a desired way to the advertiser's message, within a relatively short period of time
  - Responsiveness is the goal of performance advertisers, but short term responsiveness is less important for brand advertisers.
  - For example: likelihood of clicks, conversions, or brick and mortar purchases
Advertising as a market for attention

- A good theory of attention is missing
- Attention is not like other goods – it is like finding a mate – the match is important on both sides!
  - Usual goods:
    - Car sellers do not care who buys the cars!
  - Attention
    - Advertisers want the attention of certain people
    - People are only open to certain ads (whether or not in the market for the advertised good)
Sponsored search advertising
The actors: Publishers, Advertisers, Users, & “Ad network”

- Each actor has a separate utility function
  - User utility (relevance of the ad)
  - Advertiser utility (ROI)
  - Publisher utility (Revenue)
• **Sponsored search:**
  • Publisher = the owner of the search results page (SERP)
  • Usually the publisher (the owner of the SERP) and the “matcher” are the same (Google, Bing) but can be different (MS provides both algo search and paid search results to Yahoo)
  • WLOG, we’ll assume the former for this lecture
Interactions in Sponsored Search

- **Advertisers:**
  - Submit ads associated to certain bid phrases
  - Bid for position
  - Pay CPC

- **Users**
  - Make queries to search engine, expressing some intent

- **Search engine**
  - Executes query against web corpus + other data sources
  - Executes query against the ad corpus
  - Displays a Search Results Page (SERP) = integration of web results, other data, and ads
Back to the ad discussed in Lecture 1
Visible and invisible parts

Title

Creative

Display URL

Tutorial at SIGIR 2010

Information Retrieval Challenges in Computational Advertising

research.yahoo.com/tutorials/sigir

Bid phrase: sigir 2010
Bid: $1

Landing URL:
http://research.yahoo.com/tutorials/sigir10_compadv/
Destination: the landing page

Tutorial on Information Retrieval Challenges in Computational Advertising
In conjunction with the 33rd ACM SIGIR Conference
19-23 July, 2010 - Geneva, Switzerland

OVERVIEW

Web advertising supports a large swath of the Internet ecosystem. It brings revenue to countless publishers that rent space on their pages for advertising: from small mom-and-pop shops to major Internet companies. It also provides valuable traffic to numerous commercial Web sites and has fueled the development of Web search engines. Today, Web advertising is increasingly impacting the world outside the Internet by shaping the...
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home [ACM SIGIR 2010]
Deadline for the Elsevier 2010 App Challenge extended to July 9th (read more > ... 33rd Annual ACM SIGIR Conference. 19-23 July 2010, Geneva, Switzerland. Cochairs: Stephane ...
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ACM SIGIR Special Interest Group on Information Retrieval ...
ACM SIGIR addresses issues ranging from theory to user demands in the application of ...
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SAPMIA 2010 - Social, Adaptive and Personalized Multimedia ... SIGIR 2010 (Geneva, Switzerland) - 33rd Annual ACM SIGIR Conference on Research & Development on Information ...
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  Time information Organization
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Feb 17, 2010 ... SIGIR 2010 welcomes contributions related to any aspect of IR theory and foundation, techniques, and applications. Relevant topics include ...
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**ACM SIGIR Special Interest Group on Information Retrieval

Home Page ★
Get ready for SIGIR 2010 in Geneva! SIGIR members Ricardo Baeza-Yates (Yahoo! Research), Jeffrey Dean (Google Inc.), Urs Hoelzle (Google Inc.), David Karger ...
www.sigir.org/ - Cached - Similar
### Details

<table>
<thead>
<tr>
<th>Status</th>
<th>Max. CPC</th>
<th>Clicks</th>
<th>Impr.</th>
<th>CTR</th>
<th>Avg. CPC</th>
<th>Cost</th>
<th>Avg. Pos.</th>
</tr>
</thead>
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<tr>
<td><strong>Below first page bid</strong></td>
<td>$0.40</td>
<td>20</td>
<td>2,449</td>
<td>0.82%</td>
<td>$0.16</td>
<td>$3.19</td>
<td>1.2</td>
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<td>647</td>
<td>0.46%</td>
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<td>$0.16</td>
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<td>$1.00</td>
<td>0</td>
<td>108</td>
<td>0.00%</td>
<td>$0.00</td>
<td>$0.00</td>
<td>1.1</td>
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<td>$0.00</td>
<td>$0.00</td>
<td>1</td>
</tr>
<tr>
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<td>$1.00</td>
<td>0</td>
<td>3</td>
<td>0.00%</td>
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<tr>
<td><strong>Rarely shown due to low quality score</strong></td>
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<td>$0.00</td>
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CPC costs

Quarterly Keyword Distribution by CPC, October 2004-October 2005

2005

2009

U.S. Average Search CPC by Category, September and October

<table>
<thead>
<tr>
<th>Category</th>
<th>CPC Sep ($)</th>
<th>CPC Oct ($)</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automotive</td>
<td>0.47</td>
<td>0.50</td>
<td>6.3</td>
</tr>
<tr>
<td>Finance</td>
<td>1.80</td>
<td>1.63</td>
<td>-9.4</td>
</tr>
<tr>
<td>Retail</td>
<td>0.40</td>
<td>0.43</td>
<td>7.5</td>
</tr>
<tr>
<td>Travel</td>
<td>0.55</td>
<td>0.54</td>
<td>-1.8</td>
</tr>
</tbody>
</table>

Source: efficientfrontier.com (Via ClikZ.com)
Other twists

- Advertisers can specify budgets
- Budgets can be implemented via
  - Spend it quickly till out of money
  - Spend it slowly till end-of-day
  - Spend it as the SE sees fit (Studied in theory but dubious – engine can use this nefariously to manipulate the price paid by other advertisers)
- There are sometimes “reserve prices” = minimum cost to be shown on a given kw (depends on kw)
- There are sometimes “minimum bids” = minimum bid required to participate in action (could depend on advertiser and keyword)
From the search engine perspective: Three sub-problems

1. **Ad retrieval**
   Match to query/context

2. **Ordering** the ads

3. **Pricing** on a click-through

Note: This is the *execution* order. The *design* order is the opposite:

MARKET DESIGN

COMPUTATIONAL ADVERTISING

SYSTEMS
1. Ad retrieval
Retrieve ads (previous lecture discussion)

- Advertisers bid on keywords/phrases
- For a given query the engine provides:
  - “Exact match” → The advertiser bid on that specific query a certain amount
  - “Advanced match” (AM) or “Broad match” → The advertiser did not bid on that specific keyword, but the query is deemed of interest to the advertiser.
    - Needed to ensure volume + new/rare queries (advertise on the tail queries)
- A lot more in Lecture 2 and 4
- We will ignore AM for now and assume that all ads have bids.
2. Ordering & pricing the ads
An introduction to game theory, auction theory and mechanism design
Definitions

- **Game theory** = branch of applied mathematics that formalizes strategic behavior in the form of **games** =
  - Set of players.
  - A set of *strategies* available to those players (each has its own set)
  - A specification of *payoffs* for each player for each combination of strategies.

→

- Each player’s payoff depends on the strategy chosen by every other player!
More on game theory

- Study of strategic interactions between two or more *rational* players
- Classic tool for studying markets -- captures competition as well as collaboration
Definitions (cont)

- **Auction theory** = branch of game theory that deals with how participants (players) act in auction markets
- **Mechanism design** = the study of designing the rules of a game or economic system (in particular auctions) to achieve a specific outcome.
2-person games and payoff matrices
Rock-scissors-paper (Single instance)

Payoffs to A.
Definition: Dominant strategy

- Strategy = a complete definition of how a player will play a game.
- Strategy X (for a player) dominates another strategy Y
  - if for all choices by other player(s), X yields at least as much payoff as Y.
- Strategy X is dominant if it dominates all other strategies.
Rock-scissors-paper discussion

- **What every 5-year old knows:**
  - There is no *pure* (= deterministic) dominant strategy for one shot RSP
  - But there is a *mixed* (= randomized) strategy that is optimal for both players.

- **What many 5-year olds wrongly believe:**
  - There is a pure dominant strategy for the repeated RSP game
    - E.g. Do whatever your dad did two rounds ago, go rrsspprr.., etc -- You just have to find it! 😊
Optimal (mixed) strategy

Each player picks one of the 3 choices uniformly at random.

<table>
<thead>
<tr>
<th></th>
<th>Rock</th>
<th>Scissors</th>
<th>Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>Scissors</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Paper</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>
Nash equilibria

- This pair of strategies is an example of a *Nash equilibrium*:
- **Nash equilibrium** = choice of strategies in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing his own strategy unilaterally.
- **Pure strategy** = deterministic definition of how a player will play a game
- **Mixed strategy** = an assignment of probabilities to each pure strategy -- the players throw coins to pick the strategy they follow.
Nash equilibria (cont)

- A game could have **many** Nash equilibria …
- … or **none**, if players must follow pure strategies.
- **Famous Nash theorem**

In every n-player game in which every player has finitely many pure strategies there exists a set of mixed strategies that forms a Nash equilibrium.
Relation to our setting (preview)

- Advertisers make bids (their moves)
- Advertiser seek attention and volume – prefer higher positions (not 100% true)
- Engines order ads and price clicks according to some rules known to all bidders
- The bidders can all keep reacting to each other – does this ever end?
Auctions
Types of auctions

- **First-price sealed-bid (FPSB) →**
  1. Bidders place their bid in a sealed envelope
  2. Simultaneously give them to the auctioneer.
  3. Highest bidder wins, pays his bid.

- **Second-price sealed-bid auctions (Vickrey auctions) →**
  1. Bidders place their bid in a sealed envelope
  2. Simultaneously give them to the auctioneer.
  3. Highest bidder wins, pays price equal to the second highest bid.

- **Open Ascending-bid auctions (English auctions) →**
  1. Price is steadily raised by the auctioneer
  2. Bidders drop out once the price becomes too high.
  3. Eventually there is only one bidder who wins the auction at the current price.

- **Open Descending-bid auctions (Dutch auctions) →**
  1. Price starts at infinity and is steadily lowered by the auctioneer
  2. The first bidder to accept the current price, wins
  3. Pays the current price.

- Where is E-bay on this spectrum? (discussion)
Second price auction (Vickrey auction)

- All buyers submit their bids privately
- Buyer with the highest bid wins; pays the price of the second highest bid

$150
$120
$90
$50

Only pays $120
Truthfulness (Incentive Compatibility) of Vickrey Auction

- An auction mechanism is **truthful**, if the dominant strategy for every player is to truthfully bid their own value.

- Telling the truth is optimal in second-price (Vickrey) auction
- Suppose your value for the item is $100; if you win, your net gain (loss) is $100 - price
- If you bid more than $100:
  - you increase your chances of winning at price > $100
  - you do not improve your chance of winning for < $100
- If you bid less than $100:
  - you reduce your chances of winning at price < $100
  - there is no effect on the price you pay if you do win
- Dominant optimal strategy: bid $100
  - Key: the price you pay is out of your control
- Vickrey’s Nobel Prize due in large part to this result!
Vickrey-Clark-Groves (VCG)

- Generalization of Vickrey
- Works for arbitrary number of goods, including allowing combination bids
- Auction procedure:
  - Collect bids
  - Allocate goods to maximize total social value (goods go to those who claim to value them most) = maximum weighted matching
  - Payments: Each bidder $b$ pays his externality = $(\text{max TSV without } b\text{'s participation}) – (\text{max TSV for everyone else when } b\text{ participates})$
  - NB: (max TSV for everyone else when $b$ participates) = max weighted matching without $b$ & without $b$’s items.
- Incentive compatible (truthful) = all the bidders do best when they bid their true value i.e. reveal their private information
VCG example

Max matching = 40 → A gets a2, B gets a1
Max matching without Bob = 25
Max matching without Bob, without a1 = 20
• Bob pays 5
Max matching without Alice = 20
Max matching without Alice, without a2 = 20
• Alice pays 0

Homework: Prove that seller always gets at least the reserve price for every item actually sold.
VCG Truthfulness Informal
“Proof” (not really!)

- Max matching without Alice does not depend on her bids
- Max matching without Alice and her assigned apple does not depend on her bids
  - Price paid by Alice for her apple does not depend on her bid
    - Should not bid more than her value – might pay too much!
    - Should not bid less – might not get it!
How does the sponsored search auction work?

- **Search engines**
  - run keyword auctions to sell available inventory of ad positions

- **Advertisers**
  - submit bids which indicate their willingness-to-pay per click
    - for example, bid of $2.10 per click for the keyword “laptop”

- **The search engine orders the ads in descending order**
  - Bid is a key determinant of ad position
  - Other factors such as CTR are also factored in
    - More on the exact mechanism later
Sponsored search auctions

The space next to search results is sold in an auction with sealed (max) bids

search “las vegas travel”, Yahoo!

SPONSOR RESULTS

- Expedia.com: Save on Travel to Las Vegas - Plan your flights, vacation packages, rental cars, cruises & more. Do it all at ersites.expedia.com
- Las Vegas Rooms Up to 75% Off - Find deep discounts on Las Vegas vacation packages. We book directly with all major hotels. www.tripreservations.com
- Go Skydiving on Your Las Vegas Vacation - Try a tandem skydive on your next vacation to Las Vegas. Las Vegas Hilton or Paris Las Vegas for hotel and skydive packages. www.parkplace.com

TOP 20 WEB RESULTS out of about 2,050,000

1. Las Vegas Leisure Guide - information and reservations for hotels, shows, attractions. www.pcap.com/lasvegas.htm cached | more results from Nevada > Las Vegas > Local Travel
- More sites about: Nevada > Las Vegas > Local Travel
2. Going To Las Vegas - tips for the Vegas-bound traveler including where to find the best shows, restaurants, hotels, & attractions. www.gobingtolasvegas.com

“las vegas travel” auction

1. Expedia.com: Save on Travel to Las Vegas
   Planning a trip to Las Vegas? Find the trip you're looking for on Expedia.com. www.expedia.com
   (Advertiser's Max Bid: $3.01)
2. Las Vegas Rooms Up to 75% Off
   Find deep discounts and last minute deals on Vegas hotels. www.tripreservations.com
   (Advertiser's Max Bid: $2.94)
3. Go Skydiving on Your Las Vegas Vacation
   Try a tandem skydive on your next vacation to Las Vegas. Las Vegas Hilton or Paris Las Vegas for hotel and skydive packages. www.parkplace.com
   (Advertiser's Max Bid: $2.93)
4. Las Vegas Hotel and Casino Specials
   Check out the official Web site of Bally's Las Vegas and find casino specials. From just $39/night. www.parislasvegas.com
   (Advertiser's Max Bid: $1.01)
5. Book Las Vegas Travel Reservations
   BookVegas.com - the number one Las Vegas resource for your Vegas vacation. We provide the latest from Las Vegas, including shows, restaurants, 40 tours, airfare and car rentals! www.bookvegas.com
   (Advertiser's Max Bid: $1.00)
Unique Features of the Market for Internet Ads

- Bidding takes place continuously
- The search engines effectively sell flows (clicks/hour)
- Not unlike electricity markets unused capacity is wasted
- On the other hand, user utility might be impaired by excessive advertisement
No Obvious Definition of a "Unit" of Advertisement

1. Advertiser's perspective: transaction is a "unit"
   ➔ Pricing model: pay per transaction (CPT/CPA)

2. Search engine's perspective: exposure is a "unit" (CPM)
   ➔ Pricing model: pay per exposure

3. Middle ground: click is a "unit"
   ➔ Pricing model: pay per click (CPC)

- All three pricing models are widely used
- Pay per click dominates sponsored search
- (Remember also the risk assumption discussion)
Early Internet Advertising

- In 1994 *history begins*:
  - Per-impression pricing, mostly referral to services, insane prices.
  - Person to person negotiations
  - Sometime unclear who should pay: search engine for being able to provide extra services (e.g. maps) or provider for getting traffic/exposure.
  - Even smallest contracts are large (> $10K)
  - Key word targeting not available
  - Entry slow, "unused capacity"
Generalized First-Price Auctions

- 1997 *auction revolution* by Overture (then GoTo.com, created at Idealab)
- Pay per-click for a particular keyword
  - Initially crazy idea, meant to combat search spam
  - Search engine “destination” that ranks results based on who is willing to pay the most
  - With algorithmic SEs out there, who would use it?
  - Commercial web sites would! (Much better than to depend on ranking!)
- Results:
  - Much better targeting & much smaller contracts
  - Links arranged in descending order of bids
  - Pay your bid (First price)
  - Overture became a platform for Yahoo! and MSN-- Imperfect mechanism: unstable due to dynamic nature of the environment
Example

- Two slots and three bidders.
  - First slot 200 clicks per hour
  - Second slot gets 100 clicks per hour.
  - Bidders 1, 2, and 3 have values per click of $10, $4, and $2, respectively.

- Pay-off for bidder $i = \# \text{ clicks} \times (\text{value}(i) - \text{CPC})$

- There is no pure strategy equilibrium in the one-shot version of the game (one hour). If bidders best respond to each other, they will want to revise their bids as often as possible. (HW, [0pt]: Prove this fact)
Generalized First Price: 14 hours in the “life of the top bid” -- real data on a particular kw! (not previous example)
GFP: one week of changes in top bid
Generalized Second-Price (GSP) Auctions

- 2002 GSP implemented by Google
- Yahoo!/Overture and others switched to GSP
- Two way of generalizing:

  - **Bid ranking:** Order the ads by their bids. Rename ads so ad $i$ ends in position $i$. Bidder in position $i$ pays $bid(i+1)$.

  - **Revenue ranking:** Order the ads by expected revenue in position 1 assuming maximum bids, that is by $b(j)\times ctr(j)$. Rename ads so ad $i$ ends in position $i$. Bidder in position $i$ pays $bid(i+1)\times ctr(i+1)/ctr(i)$.

Note that bidder $i$ pays less than $bid(i)$ since $bid(i)\times ctr(i) > bid(i+1)\times ctr(i+1)$

If all CTRs are the same, revenue ranking is the same as bid ranking!
Example

- Same example under GSP mechanism with bid ranking

- Two slots and three bidders.
  - First slot 200 clicks per hour
  - Second slot gets 100.
  - Bidders 1, 2, and 3 have values per click of $10, $4, and $2, respectively.

- If all advertisers bid truthfully, then bids are $10, $4, $2.
  - Payments for slot one and two are $4 and $2 per click.
  - Total payment of bidder 1 is $800 = $1200 pay-off
  - Total payment of bidder 2 is $200 = $200 pay-off
  - In this example truth-telling is an equilibrium because no bidder can benefit by changing his bid.
Definitions for GSP

\(N\) objects and \(K\) bidders (advertisers)

\(\alpha_i\) expected number of clicks in position \(i\)

\(s_k\) the value per click to bidder \(k\)

\((\alpha_i s_k - \text{payments})\) = bidder \(k\)'s payoff from position \(i\)

\(\alpha_1 > \alpha_2 ... > \alpha_N\)

\(b(j)\) the bid of \(j\)-th highest bidder at a given time

\(g(j)\) the identity \(j\)-th highest bidder

\(g(1)\) gets the top slot, \(g(2)\) the second slot etc

\(p(i) = \alpha_i b^{(i+1)}\) is total payment of bidder \(g(i)\)

\(\alpha_i (s^{(i)} - b^{(i+1)})\) is \(g(i)\)'s payoff
GSP in the original Overture system = Bid Ranking

“las vegas travel” auction bidders

1. **Expedia.com: Save on Travel to Las Vegas**
   (Advertiser's Max Bid: $3.01)

2. **Las Vegas Rooms Up to 75% Off**
   Find deep discounts and last minute deals on Vegas hotels. www.tripreservations.com
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   (Advertiser's Max Bid: $2.93)

4. **Las Vegas Hotel and Casino Specials**
   Check out the official Web site of Bally's Las Vegas casino specials. From just $39/night. www.parx.com
   (Advertiser's Max Bid: $1.01)

5. **Book Las Vegas Travel Reservations**
   BookVegas.com - the number one Las Vegas resource for hotels, restaurants, 40 tours, airfare and car rentals!
   (Advertiser's Max Bid: $1.00)

---

“las vegas travel”, bid ranking

<table>
<thead>
<tr>
<th>SPONSOR RESULTS</th>
<th>pays $2.95 per click</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expedia.com: Save on Travel to Las Vegas flights, vacation packages, rental cars, casino gambling, gambling &amp; more...</td>
<td>pays $2.94</td>
</tr>
<tr>
<td>Las Vegas Rooms Up to 75% Off packages. We book directly with all major hotels</td>
<td>pays $1.02</td>
</tr>
<tr>
<td>Go Skydiving on Your Las Vegas Vacation in 30 seconds. Free shuttle from your hotel. Find your hotel on map!</td>
<td>... bidder i pays bid_i+1+.01</td>
</tr>
</tbody>
</table>

---

TOP 20 WEB RESULTS out of about 5,890,000:

1. **Las Vegas Leisure Guide**
   Las Vegas guide for tourists and locals. Find a complete list of Las Vegas entertainment, information and reservations for hotels, shows, attractions, restaurants. www.pcap.com/lasvegas.htm cached | more results from www.pcap.com | More sites about: Nevada > Las Vegas > Local Travel
2. **Going To Las Vegas**
   Vegas guide for tourists and locals. Find a complete list of Las Vegas entertainment, information and reservations for hotels, shows, attractions, restaurants. www.pcap.com/lasvegas.htm cached | more results from www.pcap.com | More sites about: Nevada > Las Vegas > Local Travel
GSP as implemented by Google = revenue ranking

- Assume that each advertiser $i$ has a certain attraction factor $adv_i$ that impacts its click-through rate (CTR)
- Key “separability” assumption:
  \[ CTR_i = adv_i \times pos_i \]
- Bid$_i$ gets scaled by $adv_i$
- Advertiser $i$ is charged
  \[ price_i = \frac{1}{adv_i} \times bid_i = \text{Enough to keep it in position } i \]
- Notes
  - Last position may require special handling – usually there is min “floor price” = a form of reserve
Revenue ordering

“las vegas travel” bidders and CTR

1. **Expedia.com: Save on Travel to Las Vegas**
   Planning a trip to Las Vegas? Find the trip you're looking for! Don't just travel. Travel Right. Expedia.com. 
   (Advertiser's Max Bid: $3.01) $3.01 \times 0.1 = 0.301$

2. **Las Vegas Rooms Up to 75% Off**
   Find deep discounts and last minute deals on Vegas hotels. 
   www.tripreservations.com 
   (Advertiser's Max Bid: $2.94) $2.94 \times 0.2 = 0.588$

3. **Go Skydiving on Your Las Vegas Vacation**
   Try a tandem skydive on your next vacation to Las Vegas. 
   www.lvgravityzone.com 
   (Advertiser's Max Bid: $2.93) $2.93 \times 0.1 = 0.293$

4. **Las Vegas Hotel and Casino Specials**
   Check out the official Web site of Bally’s Las Vegas casino specials. From just $39/night. 
   www.parislasvegas.com 
   (Advertiser's Max Bid: $1.01) $1.01 \times 0.1 = 0.101$

5. **Book Las Vegas Travel Reservations**
   BookVegas.com - the number one Las Vegas reserving site, restaurants, 40 tours, airfare and car rentals! 
   (Advertiser's Max Bid: $1.00) $1.00 \times 0.1 = 0.100$

“las vegas travel” revenue ordering

- **TripReservations**
  pays $3.01 \times 0.1 \times 0.2 + 0.01 = 1.51$
  per click

- **Expedia**
  pays $2.93 \times 0.1 \times 0.1 + 0.01 = 2.94$

- **LVGravityZone**
  pays $bid_{i+1} \times CTR_{i+1} / CTR_i + 0.01$

etc...

We translated everything to RPS and back!
Is Google pricing = VCG?

Put Nobel Prize-winning theories to work.
Google’s unique auction model uses Nobel Prize-winning economic theory to eliminate the winner’s curse – that feeling that you’ve paid too much. While the auction model lets advertisers bid on keywords, the AdWords™ Discounter makes sure that they only pay what they need in order to stay ahead of their nearest competitor.

Well, not really …
Generalized Second-Price and VCG Auctions

- GSP is not VCG -- GSP has no dominant strategies
- Truth-telling is generally not an equilibrium
- With only one slot, VCG and GSP are identical
- With several slots, the mechanisms are different:
  GSP charges bidder i the bid of bidder i+1 (In practice + $0.01)
  VCG charges bidder i for his externality
Truth-telling is not a dominant strategy under GSP

Proof: Example with three bidders and two slots

- Per click values are $10, $4, and $2
- CTR's are 200 and 199
- (Assume all ads are equally attractive)
- If all bid truthfully bidder 1 bids $10 and pays $4 so his payoff is:
  \[(10-4) \times 200 = 1200\]
- If bidder 1 bids $3 (and pays $2) his payoff is:
  \[(10-2) \times 199 = 1592 > 1200\]
GSP equilibrium

- What are Nash equilibrium strategies? There are many!

- Locally envy-free equilibrium [Edelman, Ostrovsky, Schwarz 2005]

  See also Symmetric equilibrium [Varian 2006]

  Fixed point where bidders don’t want to move ↑ or ↓
  - Bidders first choose the optimal position for them: position i
  - Within range of bids that land them in position i, bidder chooses point of indifference between staying in current position and swapping up with bidder in position i-1
Let us compute VCG payments for the example considered before.

- Two slots and three bidders.
  - First slot 200 clicks per hour
  - Second slot gets 100.
  - Bidders 1, 2, and 3 have values per click of $10, $4, and $2, respectively.

The second bidder's payment is $200, as before (externality imposed on 3 who loses $200 = value for him of the slot he does not get!)

However, the payment of the first advertiser is now $600:
- $200 for the externality that he imposes on bidder 3 (by forcing him out of position 2) +
- $400 for the externality that he imposes on bidder 2 (by moving him from position 1 to position 2 and thus causing him to lose (200-100)=100 clicks per hour).

Note that in this example, revenues under VCG are lower than under truth telling equilibrium of GSP!
Adaptation of VCG to sponsored search

The higher the bid, the better the position

The last bidder to get a slot pays same as in GSP

\[ p_{V,(i)} \text{ total payment of bidder in position } i \text{ under VCG} \]

\[ p_{V,(i)} = (\alpha_i - \alpha_{i+1})b(i+1) + p_{V,(i+1)} \]

Remark. If all advertisers were to bid the same amounts under the two mechanisms, then each advertiser’s payment would be at least as large under GSP as under VCG.
Static GSP and Locally Envy-Free Equilibria

- At what values can GSP bids stabilize?
- Intuitively, these restrictions should apply:
  1. All bidders play static best response
  2. Locally envy free equilibrium = No bidder wants to switch with a bidder right above him
Lemma

- The outcome of any locally envy-free equilibrium of auction GSP is a stable assignment ➔ no one does better by unilaterally changing his bid.

- Step 1 shows that locally envy-free equilibrium yields an assortative match (the higher bidders get higher positions).

- Step 2 shows that a bidder in position i does not want to "trade places" with bidders above him.
Part 1: Locally envy free implies assortative match (higher value $\rightarrow$ higher position)

$s_i$ value of player in position $i$

$\alpha_i s_i - p_i \geq \alpha_{i+1} s_i - p_{i+1}$ (no wish to move one down)

$\alpha_{i+1} s_{i+1} - p_{i+1} \geq \alpha_i s_{i+1} - p_i$ (... or one position up)

$\Rightarrow \alpha_i s_i - \alpha_i s_{i+1} + \alpha_{i+1} s_{i+1} \geq \alpha_{i+1} s_i$

Since $\alpha_i > \alpha_{i+1}$, we have $s_i \geq s_{i+1}$
Part 2: No player wants to re-match with position above

Suppose bidder $k$ is considering re-matching with position $m < k - 1$

\[ \alpha_k s_k - p_k \geq \alpha_{k-1} s_k - p_{k-1} \]
\[ \alpha_{k-1} s_{k-1} - p_{k-1} \geq \alpha_{k-2} s_{k-1} - p_{k-2} \]
\[ \vdots \]
\[ \alpha_{m+1} s_{m+1} - p_{m+1} \geq \alpha_m s_{m+1} - p_m. \]

Since $\alpha_j > \alpha_{j+1}$ for any $j$, and $s_j > s_k$ for any $j < k$, the above inequalities remain valid after replacing $s_j$ with $s_k$.

Doing that, then adding all inequalities up, yields $\alpha_k s_k - p_k \geq \alpha_m s_k - p_m$. Thus advertiser $k$ cannot re-match profitably with position $m$, and we are done.
Summary
Key points

- Rudiments of Game theory, Auction theory, Mechanism Design
- Ordering of ads is the result of a complex interplay between IR considerations and Econ consideration
- GFP is not stable
- GSP is not truthful
- VCG is truthful and stable but not really used
- Sponsored Search mechanisms are still being tweaked → good area of research
Sponsored Search Economics Research

- Key papers, survey, and ongoing research workshop series
  - Varian, Position Auctions, 2006
  - Workshops on Sponsored Search Auctions (1-6)
  - We’ll put papers and pointers on web site.
Thank you!

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