CHAPTER 11
CURRENCY AND INTEREST RATE FUTURES

Answers to end-of-chapter exercises

ARBITRAGE IN THE CURRENCY FUTURES MARKET

1. Consider the following:
   Spot Rate: $ 0.65/DM
   German 1-yr interest rate: 9%
   US 1-yr interest rate: 5%

   a. Calculate the theoretical price of a one year futures contract.
   b. What would you do if the futures price was quoted at $0.65/DM in the market place? Where would you borrow? Lend? Calculate the gain on a $100 million arbitrage transaction.
   c. What would you do if the future price was quoted at $0.60/DM in the market place? Where would you borrow? Lend? Calculate the gain on a $100 million arbitrage transaction.

   SOLUTIONS:
   a. \( F = S \times (1 + r_S) / (1 + r_{FC}) = 0.65 \times (1.05) / 1.09 = 0.626/DM \)
   b. Borrow $ at 5%; Exchange into DM at spot rate; Invest in DM at 9%; Sell forward at $.65/DM. Earn interest differential on nominal amount with no loss or gain on currency. Gain = $100,000,000 \times (.09 - .05) = $4,000,000.
   c. Borrow DM at 9%; Exchange into $ at spot rate; Invest in the US at 5%; Buy forward at $.60/DM. Gain on currency more than offsets negative interest rate differential. Gain = 100,000,000 \times (.05 - .09) + 100,000,000 \times (1/0.60 - 1/0.65) = $882,051

2. Consider the following prices:
   Spot Rate: Yen 100/$
   1-yr US interest rate: 5%
   Futures price: Yen 97.62/$

   a. What value of the one-year Japanese interest rate will remove arbitrage incentives conditional on the spot rate, futures price, and US interest rate?
b. If the yen interest rate is higher than the one found above, what would you do to take advantage of arbitrage opportunities?

c. If the yen interest rate is lower than the one found above, what would you do to take advantage of arbitrage opportunities?

SOLUTIONS:

a. The exchange rate is expressed in FC/$. Adjust formula to calculate the futures price to take this into consideration.

\[
F = S \times \left(1 + \frac{i_{yen}}{1 + i_S}\right) \\
i_{yen} = \frac{F}{S} \times (1 + i_S) - 1 \\
i_{yen} = \frac{97.62}{100} \times (1.05) - 1 \\
i_{yen} = 2.5\%
\]

b. Borrow US$ at \(i_S\); Buy yen at spot rate; Invest in yen securities at \(i_{yen}\); Sell yen forward for US$.

c. Borrow in yen at \(i_{yen}\); Sell yen at spot rate for US$; Invest in the US$ securities at \(i_S\); Buy yen forward.

ARBITRAGE IN THE INTEREST RATE FUTURES MARKET

3. Suppose the interest rate futures contract for delivery in three months is currently selling at 110. The deliverable bond for that particular contract is a 25-year bond, currently traded at 100 with a coupon rate of 10%. The current 3-month rate is 7%.

a. Is there any arbitrage opportunity? If yes, what would you do and what would be your potential gain from an arbitrage transaction?

b. What is the theoretical price of the futures contract?

c. Suppose the price was 95 instead of 110. What would you do to take advantage of arbitrage opportunities?

SOLUTIONS:

a. Yes, there is an arbitrage opportunity. Here is how:

Sell Futures contracts at 110; Purchase the bond at 100 Borrow 100 at 7%. 
Profit = Proceeds - Outlays
Profit = (Price of Bond + Accrued Interest) - (Principal Repayment + Interest Payment);
Profit = 110 + (100 * 10% / 4) - (100 - 100 * 7% / 4)
= 110 + 2.5 - 100 - 1.75;
Profit = 10.75

b. The correct price is determined so that there are no arbitrage opportunities.
0 = (F + 2.5) - (100 + 1.75); F = 101.75 - 2.5 = 99.25

c. Buy the futures at 95; Sell Bond at 100; Lend at 7% for 3 months.
Profit = (Principal + Interest Payment) - (Price of Bond + Accrued Interest); Profit = 100 + 1.75 - 95 - 2.5; Profit = 4.25

SPREAD RISK IN THEEUROCURRENCY MARKET

4. The Portfolio Manager of the WXYZ pension fund wants to protect herself against a decline in future interest rates. The fund’s planned short-term investments are placed in 3-month Eurodollar deposits at the LIBID rate. The current LIBID-LIBOR spread in the interbank market is 7.375-7.500%, and the current price of a CME futures contract (which settles on the basis of three-month Eurodollar LIBOR) is 92.50 reflecting a 7.500% interest rate.

a. How could the WXYZ fund use the futures market to hedge itself? What is the minimum interest that the firm locks in?

b. Suppose that at maturity, Eurodollar rates have fallen to 6.375-6.500% in the interbank market. Evaluate the hedge. What deposit rate has the fund secured?

c. Suppose that at maturity, Eurodollar rates have increased to 8.375-8.625% in the interbank market. Assume that the LIBID-LIBOR spread has widened because of greater interest rate and macroeconomic uncertainty. Now, evaluate the hedge. What deposit rate has the fund secured?

SOLUTIONS:

a. The fund manager should use the money to buy the CME futures contract at 92.50 to lock in the 7.50% interest rate.

b. In this case, the hedge caused a net gain and the locked-in deposit rate of 7.5% is higher than the Eurodollar deposit rate of 6.375% at maturity.

c. In this case, the hedge caused a net loss and the locked-in deposit rate of 7.5% is lower than the Eurodollar deposit rate of 8.375% at maturity.

FORWARD INTEREST RATES
5. Check today’s newspaper and locate values for today’s three-month, six-month, one-year, and two-year interest rate on government securities.

a. Calculate the market implied value of the three-month interest rate beginning three months from now.

b. Calculate the market’s implied value of the one-year interest rate beginning one year from now.

SOLUTIONS:

a. Suppose today's 3-month and 6-month rates were 5.25% and 5.50% respectively, expressed as per annum rates. Then $i(3,3)$, which is the implied 3-month rate starting 3-months from now, can be found by solving:

$$(1 + i(0,6)/2) = (1 + i(0,3)/4) \times (1 + i(3,3)/4)$$

The solution is $i(3,3) = 5.6755\%$. Note that it was necessary to divide the 6-month rate by 2 and the 3-month rate by 4 to find the actual *per period* return. A more accurate calculation would count the actual number of days in the 3-month and 6-month period and take account of whether the interest rate convention for the security chosen required a 360 day or 365 day year. We ignore these real world considerations and take 3 months as 1/4 year and 6 months as 1/2 year.

b. Suppose today's 1-year and 2-year rates were 6.00% and 6.50% respectively, expressed as per annum rates. Then $i(1,1)$, which is the implied 1-year rate starting 1-year from now, can be found by solving:

$$(1 + i(0,2))^2 = (1 + i(0,1)) \times (1 + i(1,1))$$

This formula is the same as equation 11.2 in the text on page 376. The solution is $i(1,1) = 7.0024\%$.

Note the intuition of this result. In order to equalize the return on a two year investment that earns 6.5% per year, a one year investment at 6.0% would have to be followed by a second investment at about 7.0%. The average of 6.0% and 7.0% is about 6.5%.
Exercise 11.1

11.1.a  \( F = S \left( 1 + r_S \right) / \left( 1 + r_{DM} \right) = 0.65 \left( 1 + 5\% \right) / \left( 1 + 9\% \right) = \$0.626/DM \)

11.1.b  Borrow $100 million at 5%  (pay $105M in a year)
       Exchange $100M into DM at spot rate $0.65/DM  (get DM153.85M now)
       Invest in DM at 9%  (get DM167.69M in a year)
       Sell DM forward at $0.65/DM  (get $109M in a year)

Therefore, we can earn **$4 Million** in a year ($109M - $105M).

11.1.c  Borrow DM1.538M at 9%  (pay DM1.677M in a year)
       Exchange DM into $1M at spot rate $0.65/DM  (get $1M now)
       Invest in $1M at 5%  (get $1.05M in a year)
       Sell US$ forward at $0.60/DM  (get DM1.75M in a year)

Therefore, we can earn **DM73,000** in a year (DM1.75M - DM1.677M).

Exercise 11.2

11.2.a  \( (¥97.62/\$ / ¥100/\$) * (1 + 5\%) - 1 = 2.501\% \)

11.2.b  Borrow US dollar at 5%, exchange US dollar into Yen at spot rate ¥100/$, invest in Yen at the higher Yen interest rate, and then sell Yen forward at futures price ¥97.62/$.

11.1.c  Borrow Yen at the lower Yen interest rate, exchange Yen into US$ at spot rate ¥100/$, invest in US$ at 5%, and then sell US$ forward at futures price ¥97.62/$.

Exercise 11.3

11.3.a  Yes, there is an arbitrage opportunity. Sell futures contracts at 110, purchase the bond at 100 and borrow 100 at 7% for 3 months.

Assume that the bond coupon is paid annually, then we have:
\[
\text{Profit} = \text{Proceeds} - \text{Outlays} \\
= (\text{Price of Bond + Accrued Interest}) - (\text{Principal Repayment + Interest Payment}) \\
= (110 + 100*10\%/4) - (100 + 100*7\%/4) \\
= 112.5 - 101.75 = 10.75
\]

11.3.b  The theoretical price of the futures contract should eliminate arbitrage opportunities.
Price of Futures Contract
= Principal Repayment + Interest Payment – Accrued Interest
= 100 + 100*7%/4 – 100*10%/4 = 101.75 – 2.5 = 99.25

11.3.c We can buy the futures contract at 95, sell bond at 100, and lend at 7% for 3 months.

Assume that the bond coupon is paid annually, then we have:
Profit = Proceeds – Outlays
= 100 + 100*7%/4 – 95 – 100*10%/4 = 101.75 – 97.5 = 4.25

Exercise 11.4

11.4.a The fund manager should buy CME futures contract at 92.5 to lock the 7.5% interest rate.

11.4.b When the Euro-dolloor deposits LIBID rate falls to 6.375%, then the hedge will lead to a net gain. The deposit that the fund has secured is 7.5%, higher than 6.375%.

11.4.c When the Euro-dolloor deposits LIBID rate increases to 8.375%, then then hedge will lead to a net loss. The deposit that the fund has secured is 7.5%, lower than 8.375%.