

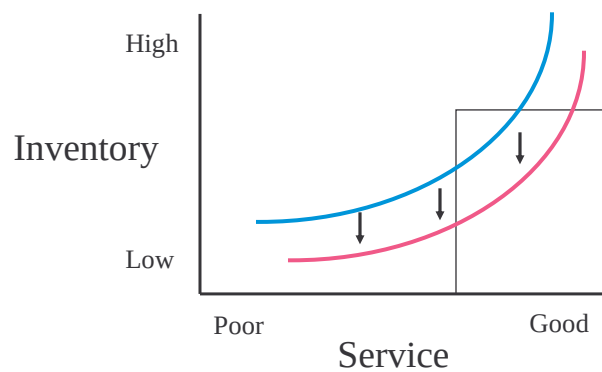
Supply Chain Analysis Tools

MS&E 262
Supply Chain Management

April 14, 2004

Motivation

Inventory/Service Trade-off Curve



Analytical tools help lower the curve!

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Outline

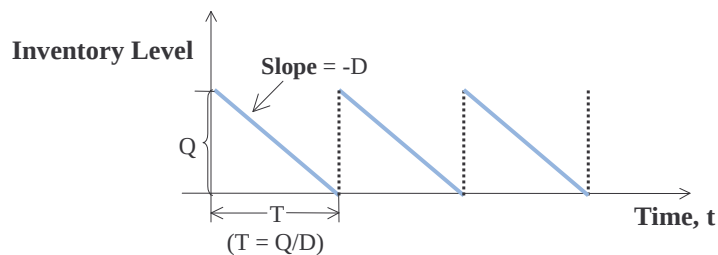
- Analytical Tools
 - EOQ
 - Newsvendor
 - Lot Size Reorder (Q,R) Model
 - Periodic Review (T,S) Model
 - Random Lead Times
- Supply Chain Improvements
 - Risk Pooling/Consolidation Example
 - Multi-echelon Example
 - Other Improvements

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Analytical Tools

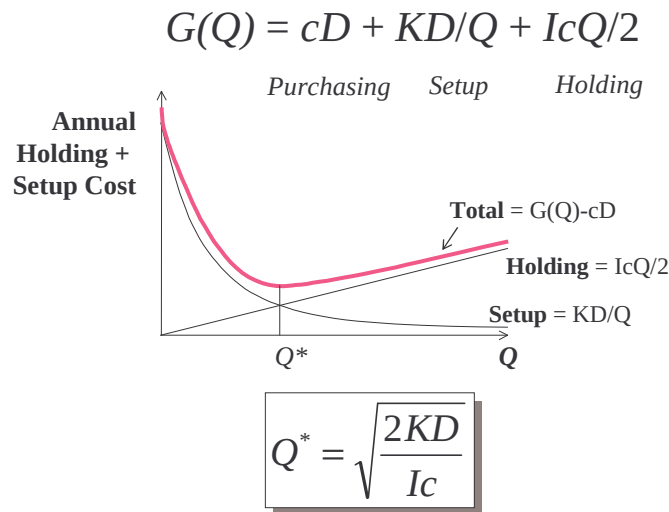
Economic Order Quantity Model

- How much to order/produce?
 - Fixed order cost of \$ K
 - Inventory holding cost of \$ $h = \$ I_c$
 - Shortages prohibited
 - Deterministic (constant) demand rate per year, D



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EOQ Model - Derivation



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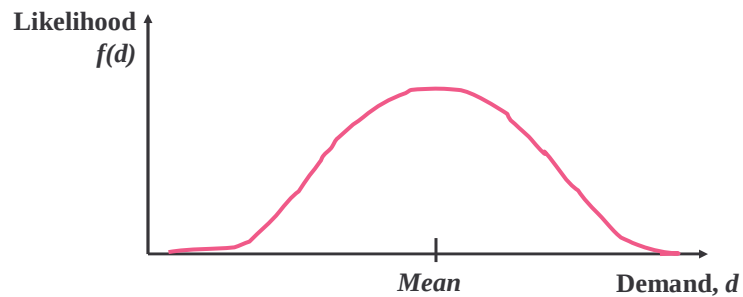
EOQ Model – Sensitivities/Shortcomings

- “40 – 20 – 02” rule
 - 40 % error in an input parameter results in 20 % error in Q
 - The result is a 2 % increase in the costs, $G(Q)$
- ➡ *Cost function is relatively insensitive to errors in Q*
- Shortcomings of EOQ Model?
 - Zero lead time (easily extended to fixed lead time)
 - Infinite production rate (finite production rate, P , if $P > D$)
 - No shortages allowed (easily extendable)
 - **Constant, deterministic demand rate**

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Newsvendor Model

- How much to order/produce?
 - Underage cost/unit c_u
 - Overage cost/unit c_o



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Newsvendor Model (cont.)

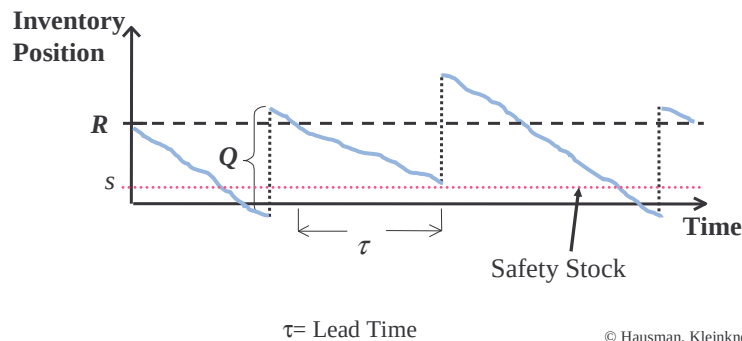
$$F(Q^*) = \frac{c_u}{c_u + c_o}$$

- Shortcomings of Newsvendor Model?
 - No consideration of positive lead times
 - One shot model (can be extended to multiple periods)
 - **No setup cost for placing orders included**

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Lot Size – Reorder Point (Q,R) Model

- We need to decide two things:
 - How much to order each time we place an order (Q)?
 - At which reorder point (R) do we place an order?



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(Q,R) Model – Notations

- | | |
|----------------------------|-----------|
| • Average demand rate/year | λ |
| • Setup cost | K |
| • Variable cost | c |
| • Holding cost | $h=Ic$ |
| • Order quantity | Q |
| • Reorder point | R |
| • Lead time | τ |
| • Safety stock | s |

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(Q,R) Model – Unit Shortage Cost p

- The expected total annual cost is

$$G(Q, R) = \underbrace{\frac{K\lambda}{Q}}_{\text{Setup}} + \underbrace{Ic \left(\frac{Q}{2} + R - \lambda\tau \right)}_{\text{Holding}} + \underbrace{p \left(\frac{\lambda}{Q} \right) \int_R^{\infty} (x - R) f(x) dx}_{\text{Shortage}}$$

n Defining $n(R) = E$ [# of units short in a cycle] we obtain

$$Q = \sqrt{\frac{2\lambda(K + pn(R))}{Ic}}$$

$$F(R) = 1 - \frac{QIc}{p\lambda}$$

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(Q,R) Model – Unit Shortage Cost p (cont.)

- For normally distributed demand, define

$$L(z) = \int_z^{\infty} (t - z) \phi(t) dt$$

where $z = (R - \mu_L) / \sigma_L$.

- Hence

$$n(R) = \sigma_L L(z)$$

- Use the approximate solution:**

- $Q = \text{EOQ}$
- Obtain z from tables for $L(z)$
- $R = \mu_L + z\sigma_L$

μ_L : mean lead time demand; σ_L : standard deviation of lead time demand

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(Q,R) Model – Service Level Approaches

- **Type 1:**

Bad

- $\alpha = \text{Prob}(\text{no stock-out in lead time})$

- **Type 2:**

Better

- $\beta = \text{Proportion of demand met from on-hand stock}$

- Recall $n(R) = E[\# \text{ of units short in cycle}]$

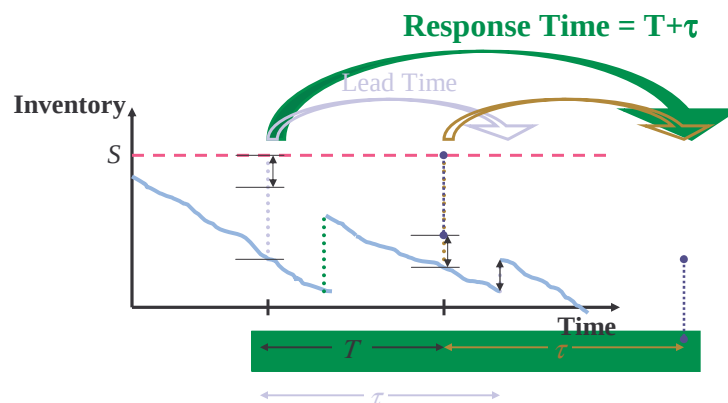
$$\frac{n(R)}{Q} = \frac{\sigma_L L(z)}{Q} = 1 - \beta$$

$$L(z) = \frac{(1 - \beta)Q}{\sigma_L}$$

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Periodic Review – (T,S) Model

- Review every T units
- Order up to S units at every review



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(T,S) Model (cont.)

$$T = EOQ/\lambda$$

$$S = \mu_{\tau+T} + z \sigma_{\tau+T}$$

Where

$\mu_{\tau+T}$ = mean demand over $\tau+T$ periods

$\sigma_{\tau+T}$ = standard deviation of demand over $\tau+T$ periods

z see (Q,R) model

Hence

$$\text{Safety Stock} = z \sigma_{\tau+T}$$

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Lead Time Demand Variability

- **Expectation of Sum** = Sum of Expectations
- **General Variance Formula** (Lead time = τ periods, demand in period $i = d_i$)

$$\sigma_{d_{LT}}^2 = \sum_{i=1}^{\tau} \sigma_{d_i}^2 + 2 \sum_{i < j} COV(d_i, d_j), \text{ where } d_{LT} = \sum_{i=1}^{\tau} d_i$$

- Variance of Sum = Sum of Variances (for independent variables)
- *Example:* Lead time demand*

$$\text{Mean} \quad \mu_L = \tau \mu$$

$$\text{Variance} \quad \sigma_L^2 = \tau \sigma^2$$

* assuming independence between periods

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Random Lead Times

- If lead time is random, with mean τ and variance s^2
- And demand in time t has mean μt and variance $\sigma^2 t$

Then the demand during (random) lead time has *

Mean	$\mu_L = \tau\mu$
Variance	$\sigma_L^2 = \mu^2 s^2 + \tau\sigma^2$

*Assuming orders do not cross and successive lead times are independent

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Lead Time Example

- Supplier's Production Time = 3 weeks
- Transportation Time from Supplier = 4 weeks
- End Product Demand, per week $\sim N(\mu, \sigma^2)$
- Case 1: No variability in transportation time $\sim N(4, 0)$ weeks
 - $\tau = 7$ weeks
 - $\mu_L = 7\mu, \sigma_L^2 = 7\sigma^2$
- Case 2: Transportation Time from Supplier $\sim N(4, 0.81)$ weeks
 - $\tau = 7$ weeks
 - $\mu_L = 7\mu, \sigma_L^2 = \mu^2(0.81) + 7\sigma^2$

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Supply Chain Improvements

- Uniform vs. Non-uniform Service Levels
- Risk Pooling/Consolidation ★
- Multi-Echelon Analysis ★
- Postponement ★
- Lead-time Reduction ★
- Review Period Reduction ★
- Variable Lead-time

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Supply Chain Improvements

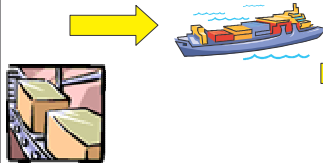
Risk Pooling/Consolidation

- What is meant by “Risk Pooling”?
- Example
 - Laser Printer Supply Chain

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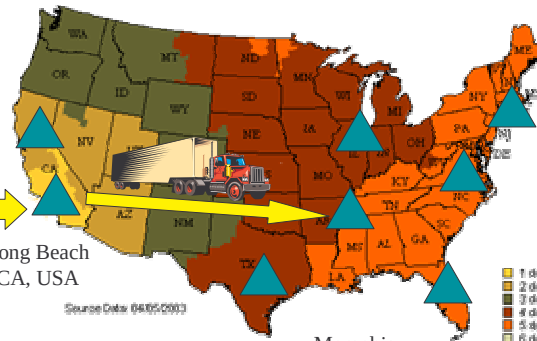
Laser Printer: Finished Goods Logistics

Penang, Malaysia



Long Beach
CA, USA

Source Data: 04/05/2003



Memphis
TN, USA

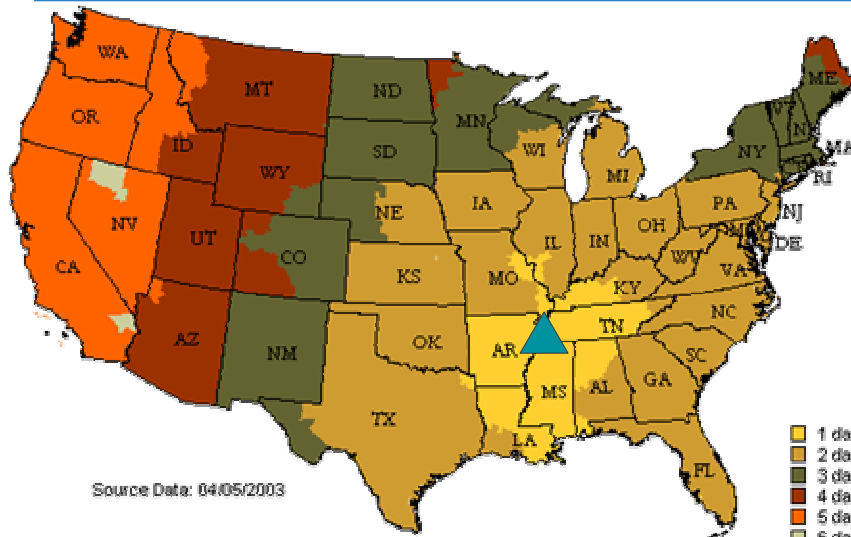


Represents a DC location for distributor "D1"

<http://www.ups.com/maps>

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UPS Ground Map for "Memphis, TN"



Source Data: 04/05/2003

<http://www.ups.com/maps>

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Laser Printer's Distributor Network

- Assume the following distributor network:
 - 5 Independent Distributors (D1, D2, D3, D4, D5)
 - Each distributor operates 8 DCs across the US
- Who are the distributors' customers?
- Who owns the printer inventory?
- Relevant inventory metrics for a DC?

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Laser Printer's Distributor Network

- Opportunity for Risk Pooling
 - For any particular distributor?
 - For any particular location (e.g., Memphis, TN)?
 - For the original equipment mfg (OEM)?

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- 1. Demands at the multiple DCs are statistically independent.
- 2. The means and standard deviations of demand for the multiple product DCs are identical.
- 3. The leadtimes for the multiple DCs are identical/constant.
- 4. The review periods at the DCs are identical.
- 5. The safety factors for the DCs are identical.
- 6. All DCs have the same inventory value.

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Let:

- σ_i = standard deviation of demand per period at DC i ;
- L_i = lead time for DC i ;
- T_i = review period for DC i ;
- z_i = safety factor for DC i ;
- n = number of DCs in a region (e.g., DCs in Memphis, TN).

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Laser Printer's Distributor Network

$$\text{Safety Stock at DC}_i = z_i a_i \sqrt{T_i + L_i}$$

$$\text{Total System Safety Stock} = \sum_{i=1}^n z_i a_i \sqrt{T_i + L_i}$$

$$\text{Safety Stock, Unpooled}_{Total} = n z a \sqrt{T + L}$$

$$\text{Safety Stock, Pooled}_{Total} = z a_{Pooled} \sqrt{T + L}$$

$$a_{Pooled} = \sqrt{\sum_i a_i^2} = a \sqrt{n}$$

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Laser Printer's Distributor Network

$$\text{Reduction Effect through Pooling: } \frac{\text{SafetyStock,Unpooled}_{Total} - \text{SafetyStock,Pooled}_{Total}}{\text{SafetyStock,Unpooled}_{Total}}$$

$$1 - \frac{z a_{Pooled} \sqrt{T + L}}{n z a \sqrt{T + L}} = 1 - \frac{a \sqrt{n}}{n a} = 1 - \frac{1}{\sqrt{n}}$$

Reduction Effect in Memphis (n = 5): 55.3%

Recall, the Distributors operate 8 separate DCs across the US

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Laser Printer's Distributor Network

What Happened?

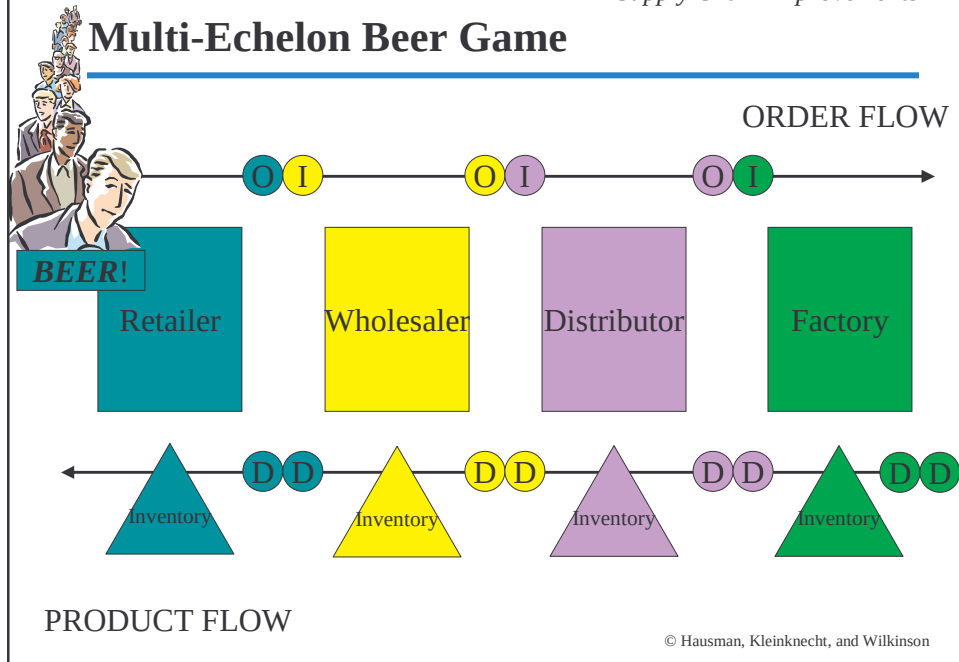
- Benefit to the OEM in a particular region?
- Cost Savings attributed to Risk Pooling?
- Benefits, other than logistics?

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Multi-Echelon Supply Chain Analysis

- Concept
 - Interactions of various levels in supply chain
 - Levels are referred to as *echelons*
- Example
 - “Beer Game”

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Multi-Echelon Beer Game

What Happened During Play?

- Communication?
- Lead Time?
- Holding vs. Backlogging Costs?
- Time Horizon/Duration?

Multi-Echelon Beer Game

Applying Inventory Theory:

- Holding vs. Backlogging Costs? ($p = \$1/wk$, $h = \$0.50/wk$)
- Lead Time Effects?
- Information Sharing?
- What policy could you play?

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Postponement/Delayed Differentiation

- Concept
 - Delay product differentiation until as late as possible in the production process
- Examples
- Advantages?
- Trade-offs?

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Lead Time Reduction

- Advantages?
- Trade-offs?
- How to do analysis?

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Review Period Reduction

- Advantages?
- Trade-offs?
- How to do analysis?

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Summary

- Avoid “black box” approach
- Understand underlying assumptions
- Perform sensitivity analysis on different parameters
- At worst, simulate the system!

Analytical tools can help significantly improve
supply chain performance!

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