Exercises 1.7 Exercises

Exercise 1.5. Consider a linear optimization problem with absolute values of the following form:

\[ \begin{align*}
\text{minimize} & \quad c^T x + d^T |y| \\
\text{subject to} & \quad A x + B y \leq b \\
& \quad y_i = |z_i|,
\end{align*} \]

Assume that all entries of \( B \) and \( d \) are nonnegative.

(a) Provide two different linear programming formulations of the above problem.

(b) Show that the original problem and the two reformulations are equivalent in the sense that either all three are feasible, or all three have the same optimal solution.

(c) Provide an example to show that if \( B \) has negative entries, the problem may have a local minimum that is not a global minimum. (It will be seen in Chapter 2 that this is never the case in linear programming problems.

Exercise 1.6. Provide a linear programming formulation of the rocket control problem discussed at the end of Section 1.3.

Exercise 1.7. The (moment) programming problem Suppose that \( Z \) is a random variable taking values in the set \( 0, 1, \ldots, K \), with probabilities \( p_0, \ldots, p_K \), respectively. We are given \( \sum_{k=0}^{K} k^4 p_k \) of \( Z \) and would like to obtain upper and lower bounds on the value of the fourth moment \( E[Z^4] = \sum_{k=0}^{K} k^4 p_k \) of \( Z \). Show how linear programming can be used to approach this problem.

Exercise 1.8. (Road lighting) Consider a road divided into segments that is illuminated by \( m \) lamps. Let \( P_i \) be the power of the \( i \)-th lamp. The illumination \( I_i \) of the \( i \)-th segment is assumed to be \( \sum_{i=1}^{m} a_{ij} P_j \), where \( a_{ij} \) are known coefficients. We are interested in choosing the illumination \( I_i \) so that the illuminations \( I \) are close to the desired illuminations \( I_d \). Provide a reasonable linear programming formulation of this problem. Note that there is no more than one possible formulation.

Exercise 1.9. Consider a school district with \( J \) neighborhoods. Each neighborhood has a capacity of \( C_j \) students for grade \( j \). Let \( g_j \) be the number of students in grade \( j \). We want to assign students to schools in a way that minimizes the total distance traveled by all students. (You may ignore the fact that numbers of students must be integer.)

Exercise 1.10. (Production and inventory planning) A company must determine the number of units of a product to produce at the end of the month. Material produced during

Figure 8.10: The function \( f(x) \) of Exercise 1.10.

minimize \( 2x_1 + 3|x_2 - 1| \leq 5 \),

subject to \( 2x_1 + 2|x_2 - 1| \leq 5 \),

and reformulate it as a linear programming problem.
Exercises 1.16: A number of an entity is any number of a million dollars. The model of a company's production process with the following constraints:

1. The company cannot produce more than 3 units of product A in a month.
2. The company cannot produce more than 4 units of product B in a month.
3. The company cannot produce more than 5 units of product C in a month.
4. The company cannot produce more than 6 units of product D in a month.

Each month, the company can produce up to 10% of the total production capacity of the previous month. The total production capacity for the previous month is 100 units. The company has a budget of $500 million for production during the month.

The company currently has $20 million in cash and $30 million in debt. The company needs to raise $10 million in additional funding to continue production for the next month.

Exercises 1.17: Consider the problem of maximizing profit. The company has two different products, A and B. The profit function is given by:

\[ \text{Profit} = 3A + 2B \]

Subject to:

\[ A \geq 0, \quad B \geq 0 \]

Constraints:

\[ A + B \leq 10 \]

\[ 2A + 3B \leq 15 \]

\[ A + 2B \leq 12 \]

\[ 3A + B \leq 18 \]

Exercises 1.18: Consider the problem of minimizing a function. The function is given by:

\[ f(x, y) = x^2 + y^2 \]

Subject to:

\[ x^2 + y^2 \leq 1 \]

Exercises 1.19: Consider the problem of maximizing profit. The company has three different products, A, B, and C. The profit function is given by:

\[ \text{Profit} = 4A + 3B + 2C \]

Subject to:

\[ A + B + C \leq 10 \]

\[ 2A + 2B + 3C \leq 15 \]

\[ A + 2B + C \leq 12 \]

\[ 3A + B + 2C \leq 18 \]

Exercises 1.20: Consider the problem of minimizing a function. The function is given by:

\[ f(x, y) = x^2 + y^2 \]

Subject to:

\[ x^2 + y^2 \leq 1 \]

Exercises 1.21: Consider the problem of maximizing profit. The company has two different products, A and B. The profit function is given by:

\[ \text{Profit} = 3A + 2B \]

Subject to:

\[ A \geq 0, \quad B \geq 0 \]

Constraints:

\[ A + B \leq 10 \]

\[ 2A + 3B \leq 15 \]

\[ A + 2B \leq 12 \]

\[ 3A + B \leq 18 \]

Exercises 1.22: Consider the problem of minimizing a function. The function is given by:

\[ f(x, y) = x^2 + y^2 \]

Subject to:

\[ x^2 + y^2 \leq 1 \]

Exercises 1.23: Consider the problem of maximizing profit. The company has three different products, A, B, and C. The profit function is given by:

\[ \text{Profit} = 4A + 3B + 2C \]

Subject to:

\[ A + B + C \leq 10 \]

\[ 2A + 2B + 3C \leq 15 \]

\[ A + 2B + C \leq 12 \]

\[ 3A + B + 2C \leq 18 \]

Exercises 1.24: Consider the problem of minimizing a function. The function is given by:

\[ f(x, y) = x^2 + y^2 \]

Subject to:

\[ x^2 + y^2 \leq 1 \]

Exercises 1.25: Consider the problem of maximizing profit. The company has two different products, A and B. The profit function is given by:

\[ \text{Profit} = 3A + 2B \]

Subject to:

\[ A \geq 0, \quad B \geq 0 \]

Constraints:

\[ A + B \leq 10 \]

\[ 2A + 3B \leq 15 \]

\[ A + 2B \leq 12 \]

\[ 3A + B \leq 18 \]

Exercises 1.26: Consider the problem of minimizing a function. The function is given by:

\[ f(x, y) = x^2 + y^2 \]

Subject to:

\[ x^2 + y^2 \leq 1 \]

Exercises 1.27: Consider the problem of maximizing profit. The company has three different products, A, B, and C. The profit function is given by:

\[ \text{Profit} = 4A + 3B + 2C \]

Subject to:

\[ A + B + C \leq 10 \]

\[ 2A + 2B + 3C \leq 15 \]

\[ A + 2B + C \leq 12 \]

\[ 3A + B + 2C \leq 18 \]

Exercises 1.28: Consider the problem of minimizing a function. The function is given by:

\[ f(x, y) = x^2 + y^2 \]

Subject to:

\[ x^2 + y^2 \leq 1 \]

Exercises 1.29: Consider the problem of maximizing profit. The company has two different products, A and B. The profit function is given by:

\[ \text{Profit} = 3A + 2B \]

Subject to:

\[ A \geq 0, \quad B \geq 0 \]

Constraints:

\[ A + B \leq 10 \]

\[ 2A + 3B \leq 15 \]

\[ A + 2B \leq 12 \]

\[ 3A + B \leq 18 \]

Exercises 1.30: Consider the problem of minimizing a function. The function is given by:

\[ f(x, y) = x^2 + y^2 \]

Subject to:

\[ x^2 + y^2 \leq 1 \]

Exercises 1.31: Consider the problem of maximizing profit. The company has three different products, A, B, and C. The profit function is given by:

\[ \text{Profit} = 4A + 3B + 2C \]

Subject to:

\[ A + B + C \leq 10 \]

\[ 2A + 2B + 3C \leq 15 \]

\[ A + 2B + C \leq 12 \]

\[ 3A + B + 2C \leq 18 \]

Exercises 1.32: Consider the problem of minimizing a function. The function is given by:

\[ f(x, y) = x^2 + y^2 \]

Subject to:

\[ x^2 + y^2 \leq 1 \]

Exercises 1.33: Consider the problem of maximizing profit. The company has two different products, A and B. The profit function is given by:

\[ \text{Profit} = 3A + 2B \]

Subject to:

\[ A \geq 0, \quad B \geq 0 \]

Constraints:

\[ A + B \leq 10 \]

\[ 2A + 3B \leq 15 \]

\[ A + 2B \leq 12 \]

\[ 3A + B \leq 18 \]

Exercises 1.34: Consider the problem of minimizing a function. The function is given by:

\[ f(x, y) = x^2 + y^2 \]

Subject to:

\[ x^2 + y^2 \leq 1 \]

Exercises 1.35: Consider the problem of maximizing profit. The company has three different products, A, B, and C. The profit function is given by:

\[ \text{Profit} = 4A + 3B + 2C \]

Subject to:

\[ A + B + C \leq 10 \]

\[ 2A + 2B + 3C \leq 15 \]

\[ A + 2B + C \leq 12 \]

\[ 3A + B + 2C \leq 18 \]

Exercises 1.36: Consider the problem of minimizing a function. The function is given by:

\[ f(x, y) = x^2 + y^2 \]

Subject to:

\[ x^2 + y^2 \leq 1 \]