ASSIGNMENT 1

1. Prove that the dual cone of the $p$-order cone, $p = 1, \ldots, \infty$, is the $q$-order cone where $\frac{1}{q} + \frac{1}{p} = 1$.

2. (a). Exercise 1.3, Course Monograph, Chapter 1, page 27.
   (b). Exercise 1.8, Course Monograph, Chapter 1, page 28.

3. Let $X$ be a positive semidefinite matrix of rank $r$, and $A$ be any given symmetric matrix. Then, there is a decomposition of $X$

   \[ X = \sum_{i=1}^{r} x_i x_i^T, \]

   such that for all $i$,

   \[ x_i^T A x_i = A \bullet (x_i x_i^T) = A \bullet X/r. \]

4. Let $f(X) = \log \det(X)$ over positive definite matrix cone.
   (a). $f$ is a concave function.
   (b). find out $\nabla f(X)$ and $\nabla^2 f(X)$.

5. Exercise 1.11, Course Monograph, Chapter 1, page 28.

6. Download SEDUMI1.05, DSDP5.8, and/or CVX and install them in Matlab. Solve the SDP example on Lecture Note #1.