ASSIGNMENT 2
Discuss Session January 30, 2015

1. Exercise 2.2, Course Monograph, Chapter 2, page 54.

2. Exercise 2.6, Course Monograph, Chapter 2, page 54.


4. Consider the SDP problem

\[
\begin{aligned}
& \text{minimize} \quad C \cdot X \\
& \text{s.t.} \quad A_i \cdot X = b_i, \quad i = 1, ..., m, \\
& \quad Q_j \cdot X = 0, \quad j = 1, ..., q, \\
& \quad X \succeq 0,
\end{aligned}
\]

where coefficient matrices \( Q_j, j = 1, \ldots, q \), are positive semidefinite.

a) Suppose that there is an optimal solution \( X^* \) with zero duality gap, show that there must be an optimal solution matrix with its rank \( r \) satisfying \( r(r+1)/2 \leq m \). (Note that the bound is independent of \( q \).)

b) Using the above result to show that the quadratic problem

\[
\begin{aligned}
& \text{minimize} \quad x^T Q x + 2 c^T x \\
& \text{s.t.} \quad A x = 0, \\
& \quad \| x \|^2 = 1
\end{aligned}
\]

is an SDP problem, where given \( Q \) is an \( n \)-dimensional symmetric matrix and \( A \) is an \( m \times n \) matrix with \( m < n \).

5. Exercise 2.4, Course Monograph, Chapter 2, page 54.

6. Now you have down loaded SEDUMI1.05, DSDP5.8, and/or CVX, and use them to solve the following localization SDP problems:

a) The SDP problem

\[
\begin{aligned}
& \text{minimize} \quad C \cdot X \\
& \text{s.t.} \quad (e_i - e_j)(e_i - e_j)^T \cdot X = 1, \quad 1 \leq i < j \leq 3, \\
& \quad X \succeq 0 \in S^3,
\end{aligned}
\]
and check the solution matrix rank when (i) $C = I$, (ii) $C = -I$, and (iii) $C = -e_3e_3^T$.

b) The SDP problem

$$\begin{array}{l}
\text{minimize} \quad C \bullet X \\
\text{s.t.} \quad e_i e_i^T \bullet X = 1, \\
\quad (e_i - e_{i+1})(e_i - e_{i+1})^T \bullet X = 1, \quad i = 1, 2, \\
\quad X \succeq 0 \in S^3,
\end{array}$$

and check the solution rank when (i) $C = I$, (ii) $C = -I$, and (iii) $C = -e_3e_3^T$.

Here $e_i \in \mathbb{R}^3$ is the vector of all zeros except 1 at the $i$th position.