1. Let $K = S^n_+$. Then

$$
\psi_n(X, S) := n \ln(X \cdot S) - \ln \det(X) - \ln \det(S) \geq n \ln(n)
$$

for any $X \in S^n_{++}$ and $S \in S^n_{++}$.

2. Prove Lemma 1 in Lecture Note 9, that is, if $D \in S^n$ such that $0 \leq \|D\|_\infty < 1$ (that is, the largest absolute value of the eigenvalues of $D$ is less than 1), then

$$
-I \cdot D \leq -\ln \det(I + D) \leq -I \cdot D + \frac{\|D\|^2}{2(1 - \|D\|_\infty)}.
$$

3. Prove Lemma 3 in Lecture Note 9 (Hint: you may think $V$ is a diagonal matrix with positive diagonal entries).

4. What does the Gram matrix of the potential reduction algorithm look like when every matrix $A_i$ is a rank-one PSD matrix, that is, $A_i = a_i a_i^T$, at a primal interior point $X$ and dual interior point $(y, S)$.

5. Prove Theorem 2 in Lecture Note 10.