Bimatrix Games as LCP’s

The initial set up

Let $A$ and $B$ denote two $m \times n$ matrices.

(Payoff matrices for Players I and II, respectively.)

Let $\sigma_m = \{ x \in R_+^m : e^T x = 1 \}$ and $\sigma_n = \{ y \in R_+^n : e^T y = 1 \}$.

If $x \in \sigma_m$ and $y \in \sigma_n$, the expected losses of Players I and II are, respectively:

$$x^T A y \text{ and } x^T B y.$$ 

Let $\Gamma(A, B)$ denote the corresponding two person game.
Nash Equilibrium Point of $\Gamma(A, B)$

The pair $(x^*, y^*) \in \sigma_m \times \sigma_n$ is a \textit{Nash Equilibrium Point (NEP)} for $\Gamma(A, B)$ if

\[
(x^*)^TAy^* \leq x^TAy^* \quad \text{for all } x \in \sigma_m \\
(x^*)^TBy^* \leq (x^*)^TBy \quad \text{for all } y \in \sigma_n
\]

It is crucial to note that given $(x^*, y^*) \in \sigma_m \times \sigma_n$, each of the vectors $x^*, y^*$ is optimal in a simple linear program defined in terms of the other. The LP’s are:

\[
\text{minimize } (Ay^*)^Tx \quad \text{subject to } e^Tx = 1, \ x \geq 0
\]

and

\[
\text{minimize } (B^Tx^*)^Ty \quad \text{subject to } e^Ty = 1, \ y \geq 0
\]