Exercise 1 Two different types of parts arrive at a machining center, in two independent streams: Arrival rates are \( \lambda_1 = 4.5, \lambda_2 = 0.045 \), and service rates are \( \mu_1 = 10, \mu_2 = 0.1 \), for the two types respectively.

Assume that the arrivals are Poisson, and the service times exponential. Calculate the average waiting times (delay in queue) for each type of part, and the overall average, under the following three service regimes:

(a) A single server, serving under FCFS.
(b) A single server, giving priority to type 1 over type 2.
(c) Two dedicated servers, one for each type, working at speeds \( s_1, s_2 \), where \( s_1 + s_2 = 1 \) and the speeds are chosen so that both servers work with the same traffic intensity (note — the new service rates are \( s_i \mu_i, i = 1, 2 \)).

Exercise 2 A three machine plant is described below. The input rate is \( \lambda = 100 \), of which \( q_1 = 0.40 \) go to machine 1 for first stage processing, and \( q_2 = 0.60 \) go to machine 2 for first stage processing (these proportions are to do with product mix and are fixed).

30\% of the product of machine 1 needs to go through machine 2 as well, \( p_2^1 = 0.30 \). The product from stage 1 goes to machine 3 to undergo stage 2. The processes entail some rework given by \( p_2^2 = 0.20, p_3^3 = 0.10 \). Also, some of the product of machine 3 is returned to machine 2, for repeat of stage 1, \( p_2^3 = 0.05 \). Some other routings also occur, to summarize

\[
P = \begin{bmatrix}
0 & .3 & .7 \\
.05 & .20 & .75 \\
0 & .05 & .10 
\end{bmatrix}
\]

It is also proposed to have processing rates

\[
\mu^T = \begin{bmatrix}
40 & 100 & 130 
\end{bmatrix}
\]

The coefficient of variability of the arrival stream is \( \alpha_A = 0.15 \), the coefficient of variability of the machine processing times is \( \alpha_X = 0.25 \) for all three machines.

(a) Find the flow rates for this proposed network and classify the machines into oversaturated bottlenecks, balanced bottlenecks, and nonbottlenecks.

(b) treat the station closest to saturation but non bottleneck as a bottleneck and write the properties of the diffusion aproximation for this system: These include the variance covariance matrix of the netput process, and the diffusion approximations to the queue length processes.

(c) Increase the processing rate of the most saturated bottleneck machine until you obtain two balanced bottlenecks, and manipulate \( \lambda \) so that the balanced bottlenecks will both have \( \rho = 0.95 \). Obtain again the fluid flows, and the netput matrix of covariances.

(d) Find the means and covariances of the two dimensional reflected Brownian motion obtained in (c)