MS&E 341 – Homework #2  
Due: TH, April 26, 2001

I. Consider the market for natural gas in a small country. Assume that many firms produce natural gas, but that all natural gas, once produced, is purchased by only one firm, who has a monopoly in the wholesale market of natural gas. Assume that that monopolistic wholesaler also owns all the natural gas pipelines. Assume that there are many retailers for natural gas competing with each other, but each must buy natural gas from the single wholesaler. Once they buy that gas they must pay to have it transported through the pipelines. Assume that the monopolist maximizes profits. Assume that the supply of natural gas can be described by an upward sloping supply function (ignore any issues associated with the natural gas as a depletable resource).

a) Develop necessary conditions to show the profit maximizing wholesale price and profit maximizing price of natural gas pipeline transportation services.

b) Show that the equilibrium of part a) is not economically efficient, under the assumption that all other markets in the economy are operating competitively. Show that the consumer price is higher and the producer price is lower than would be economically efficient.

c) The government of the country now regulates the price that can be charged for natural gas pipeline transportation services, so as to force a reduction in the transportation charges. Show that this action will not influence the price that consumers pay for natural gas nor the price received by those who produce natural gas. Explain why this is so.

d) The government tries another approach and forces the monopolist to sell the pipeline to an entirely independent firm. That firm is allowed to operate as a monopolist and to set its own price for transportation services but is not allowed to enter into any negotiations about prices or quantities with the wholesale monopolist. Show that this action will increase the price that consumers pay for natural gas and decrease the price received by those who produce natural gas. Explain why this is so.

e) Beginning with the situation in part d), the pipeline monopolist and the wholesale monopolist are now allowed to make agreements with each other about their pricing rules, so this becomes a cooperative game. Show that in the solution to this game, both firms can make more profits than would be the case in part d). Show, in addition, that the consumer
price of gas and the producer price will be identical to those in part a) and c).

2. (This question looks at how incentives to innovate depend on market structure.)

Consider a market for widgets in which there is one producer and two different types of buyers. Widgets can be produced at a constant average cost (and constant marginal cost) of $M$, where $M$ is a positive constant strictly smaller than 1.0. The inverse demand function for the two different types of buyers are

\[ P_1 = 2 - 3y_1 \]
\[ P_2 = 1 - 1.5y_2 \]

where $y_1$ and $y_2$ are total quantities of widgets sold to buyer type 1 and type 2 respectively.

a. Assume that widgets are produced by a single firm and there are significant barriers to entry so the firm may act as a monopolist. The firm is able to segment the markets so that it can sell at different prices to the two groups of buyers. Assume that this market-segmenting monopolist can reduce its marginal costs by a small amount, to a new marginal cost, $m$. This can be thought of as an "incremental process innovation": the monopoly price with the process innovation lies above $M$. Calculate the change in profits due to the process innovation.

b. Now assume that this market cannot be segregated and the monopolist must sell at one price ($P$). Assume that the monopolist sells to both markets. Sketch the aggregate inverse demand function. Calculate the profits due to the process innovation.

c. Assume now that this industry is purely competitive and does not differentiate between different customer classes. An outside research laboratory develops the process innovation and licenses it to the competitive firms. What are the royalty revenues for the outside firm.

d. What can we conclude about the relative incentives to innovate and why they vary?

3. (This problem looks at incentives for location or, more generally, product variety or differentiation.)

Imagine a duopoly in which firms sell a homogenous product for consumers who live along a 100-mile highway. We imagine that consumers are distributed uniformly along the highway at a density of
ten consumers per mile. Each consumer has a reservation price of $100 for one unit of the good, and will buy either zero or one unit only. There is a transportation cost of getting to and from the store that must be added to the price of the good when comparing against the reservation price. The good cost each of the two firms $1 per unit to produce and bring to the store. If the firm is located \( d \) miles from a consumer, the consumer pays \( \left( \frac{d}{50} \right)^2 \) for the round trip from home to the store and back. The two firms locate their store (which serves to differentiate their products in the minds of consumers, unless located in the same place) and then compete with Bertrand conjectures.

(a) Suppose each store is located at opposite extremes of the highway. What are the equilibrium prices in this case?

(b) Suppose both stores are located right at mile 50. What are the equilibrium prices in this case?

(c) Suppose each firm decides first where to locate its store, and then compete in prices. What is the optimal location for each store? What are the equilibrium prices?

(d) Suppose the social planner decides where to locate the stores. What will be the optimal location for each store and the equilibrium prices?

4. Economists often use total surplus - the area between the demand curve and the marginal cost curve - as a measure of "welfare". Professor Sweeney showed in class that there is no difference in welfare, as defined in this way, between perfectly competitive markets and those with a perfectly price-discriminating monopolist. The issue is simply one of who gets the welfare, the consumers or the producers. One justification for the total surplus metric is that it maximizes the total size of the pie and that it is up to governments to distribute the pie equitably; there is a clear distinction between equity and efficiency. Does total surplus measure of welfare seem like a reasonable basis for policymaking? Why? (at least 100 words. We just want you to think about it.)

5. Assume that a firm with market power faces an advertising-responsive demand. Let the demand function be

\[
Q = A^{0.5} (B - P),
\]

where \( Q \) is the demand, \( A \) is the number of advertising hours on local
television, $B$ is a constant (the choke price), and $P$ is the price. Assume that the production cost of the product is proportional to output, with a marginal and average cost of $K$. Assume that the firm must pay a price of $V$ per unit of advertising.

a) Show that the profit-maximizing price of the product will be independent of the price of advertising and of the amount of advertising chosen by the firm.

b) Show that the profit-maximizing level of sales is inversely proportional to $V$.

c) Show that the profit-maximizing level of sales will be proportional to the cubic power of the difference between the choke price and the production costs.

6. Suppose that you are a regulator working for the US Department of Justice. A consumer group comes to you to register a complaint. They are facing a monopolist and are unhappy, claiming that they would have much greater consumer surplus if the monopolist were forced to charge competitive prices. Although they can tell you nothing about the cost or technology of the monopolist, they can tell you whatever you want to know about demand. In particular, they tell you that the demand function is $Q = 10p^{-20}$. You know that regulating the monopolist will be costly, so you would only be interested in doing so, if consumer’s welfare would in fact be “significantly” better off.

(a) As a first approximation to the loss in consumer surplus, what is the current distortion in prices away from the monopolist’s marginal cost?

(b) Assuming that the monopolist’s marginal cost function is nondecreasing and nonconcave, find an upper bound on the loss of consumer surplus due to monopoly pricing.

7. The manufacture of rhillip, a nonnarcotic nutrient, takes two stages. In the first stage, legume and kapitose are used to produce rhipigume. In the second stage, rhipigume and further amounts of kapitose are used to make rhillip. Rhipigume has no uses other than for the manufacture of rhillip; as you know full well by now, legume and kapitose are the bases for many specialty chemicals.

For purposes of this problem, suppose that legume and kapitose are traded in perfectly competitive markets. The price of kapitose is $1$, and the price of legume is $4$ (for the time being).
Rhipigume is produced in a perfectly competitive industry that is filled with many identical firms, each of which has production function

\[ r = k^{1/4}l^{1/4}, \]

where \( r \) is the amount of rhipigume produced from \( k \) units of kapitose and \( l \) units of legume. Each firm in this industry faces fixed costs of $16 (per firm) as well. There is completely free entry and exit to and from this industry.

Rhllipp is produced in another perfectly competitive industry. (For legal reasons, no firm can produce both rhllipp and rhipigume.) There are many firms in this industry as well, each of whom has production function

\[ R = k^{1/4}r^{1/4}, \]

where \( R \) is the amount of rhllipp produced from \( k \) units of kapitose and \( r \) units of rhipigume. Each firm in this industry has fixed costs of $72 (per firm). There is completely free entry and exit to and from this industry. Industry demand for Rhllipp is given by the equation

\[ P = 96 - R/10, \]

where \( P \) is the price of rhllipp and \( R \) is the total quantity of rhllipp brought to market.

(a) What is the long-run equilibrium in these two industries? I want to know prices for the two goods, numbers of firms in each industry, output supply and factor demands of each firm, profits per firm, and total industry supply and factor demands.

In the short run, firms in both industries can vary the amounts of kapitose they use in production, but not the amounts of legume or rhipigume. In the intermediate run, firms can vary the amounts of their factor inputs, but entry to or exit from either industry is not possible. In the long run, entry and exit becomes possible.

(b) Beginning from the position you computed in part (a), the price of legume suddenly falls to $.25. What will happen in these two industries in the short run? In the intermediate run? And in the long run? (For the long run, don't worry about fractional numbers of firms.)