1) Assume there are two people (A and B) and two goods (x₁ and x₂) in a pure exchange economy. They have different initial endowments of the two goods and different preferences. Initial endowments are:

\[(w_1^A, w_2^A) = (1,2)\]
\[(w_1^B, w_2^B) = (4,3)\]

For each of the following pairs of utility functions, do the following 3 things:

i) Find a mathematical expression for all Pareto Optimal allocations of the two goods.

ii) Find the competitive equilibrium.

iii) Diagram and label your answers to parts i) and ii) on the same diagram.

a) \[u(x_1^A, x_2^A) = 0.5\ln(x_1^A) + 0.5\ln(x_2^A)\]
\[u(x_1^B, x_2^B) = 0.25\ln(x_1^B) + 0.75\ln(x_2^B)\]

b) \[u(x_1^A, x_2^A) = \min\{x_1^A, x_2^A\}\]
\[u(x_1^B, x_2^B) = 0.25\ln(x_1^B) + 0.75\ln(x_2^B)\]

c) \[u(x_1^A, x_2^A) = \max\{x_1^A, x_2^A\}\]
\[u(x_1^B, x_2^B) = 0.25\ln(x_1^B) + 0.75\ln(x_2^B)\]

d) \[u(x_1^A, x_2^A) = x_1^A + x_2^A\]
\[u(x_1^B, x_2^B) = x_1^B + 3x_2^B\]

2) Consider an economy with 15 consumers and 2 goods. Consumer 3 has Cobb-Douglas utility function \[u(x_1^3, x_2^3) = x_1^3 x_2^3\]. At a certain Pareto efficient allocation \(x^*\), consumer 3 holds (10, 5). What are the competitive prices that support allocation \(x^*\).

3) Consider the following two-person economy. Individual A earns income \(W_0\) in period zero and nothing in period 1. Individual B earns \(W_1\) in period 1 and nothing in period zero. Let \(r\) be the equilibrium interest rate. The two people can borrow or lend from one
another, and a person can consume up to the amount of money he or she has. They can
save money from period 0 to be consumed in period 1. Assume both A and B have utility
functions of the following form: \( U(c_0, c_1) = \ln(c_0) + b \ln(c_1) \), for \( 0 < b < 1 \), and where \( c_0 \)
and \( c_1 \) are consumption in period 0 and 1, respectively.

1. State all the constraints faced by individuals A and B.

2. Determine the equilibrium interest rate in this economy. Is it always greater than zero?
   (Note that this problem is very similar to Problem 2, Chapter 7 in Luenberger.)

4) Consider an economy in which there are two nonproduced factors of production, land
and labor, and two produced goods, apples and bandannas. Apples and bandannas are
produced with constant returns to scale. Bandannas are produced using labor only, while
apples are produced using labor and land. There are \( N \) identical people, each of whom
has an initial endowment of 15 units of labor and 10 units of land. They all have utility
functions of the form \( U(A, B) = c \ln A + (1-c) \ln B \) where \( 0 < c < 1 \) and where \( A \) and \( B \) are
a person’s consumption of apples and bandannas, respectively. Apples are produced with
a fixed-coefficients technology that uses one unit of labor and one unit of land for each
unit of apples produced. Bandannas are produced using labor only. One of labor is
required for each bandanna produced. Let labor be the numeraire for this economy.

a) Find competitive equilibrium prices and quantities for this economy.

b) For what values (if any) of the parameter \( c \) is it true that small changes in the
endowment of land will not change competitive equilibrium prices?

c) For what values (if any) of the parameter \( c \) is it true that small changes in the
endowment of land will not change competitive equilibrium consumption?

5) Consider a simple economy in which there are two factors of production, land (N) and
labor (L), in fixed quantities, N=1000 and L=3000. Two goods can be produced using
these two factors: Food (F) and Clothing (C). Production of each is constant returns to
scale and approximately Cobb-Douglas as follows:

\[
F = 10L_F^5 N_F^5 \\
C = 20L_C^2 N_C^8
\]

Let \( P_N \) and \( P_L \) be the prices of land and labor, respectively.

a) Using an Edgeworth box, diagram the locus of quantities of land and labor to be
allocated to the production of the two goods, food and clothing, which would be
consistent with a competitive equilibrium. In your diagram, be sure to label the axes,
indicating the lengths of the box sides. Be sure to show relevant isoquants and diagram
the competitive equilibrium allocations.
b) For the competitive equilibrium allocations of land and labor, derive simple mathematical expressions relating the ratio $L_F/N_F$ and the ration $L_C/N_C$ to the ratio of prices $P_N/P_L$.

c) Develop closed form expressions for $L_F$, $N_F$, $L_C$, and $N_C$ each as a function of the ratio of prices $P_N/P_L$.

d) For what range of the ratio $P_N/P_L$ will the competitive equilibrium production of food be equal to zero? For what range of the ratio $P_N/P_L$ will the competitive equilibrium production of clothing be equal to zero?

e) Develop closed form expressions for the competitive equilibrium prices of clothing and of food, as functions of $P_N$ and $P_L$. Show that the equilibrium prices of clothing and food are homogenous of degree 1 in $P_N$ and $P_L$. 