1. Show that the following assertions follow from the theory developed in class. Write down your assumptions explicitly. For the first five parts, assume that the economy is below full employment level of GDP.

   (a) an increase in government spending financed by issuing bonds will increase both the interest rate and the GDP.

   (b) an increase in government spending financed by issuing money will increase the GDP but could either increase or decrease the interest rate.

   (c) If the federal reserve system issues money and uses it to buy bonds on the open market, the GDP will increase and the interest rate will decline.

   (d) a reduction in the budget deficit (assume there is one) could lead to a reduction in the trade deficit.

   (e) actions which increase the U.S. exports will increase GDP. However, the precise impact on GDP and on the interest rate will depend on how people in other countries allocate the necessary change in their holding of U.S. currency: do they mostly hold fewer U.S. dollars or do they invest less in U.S. securities?

   (f) if the economy is at full employment, an increase in government spending, financed by an increase in the money stock, will lead to inflation but no increase in real GDP.

2. (Balanced budget multiplier) Consider the following model which ignores the money and bond markets:

   \[ Y = C + I + G, \text{ and } C = C(Y - T), \]  
   \[ Y = C(Y - T) + I + G, \text{ where } C \text{ is the consumption function, } T \text{ are total taxes, } G \text{ is government spending, and } I \text{ is national investment.} \]

   Suppose \( G \) and \( I \) are exogenous variables, and that taxes are independent of income.

   (a) Find the tax multiplier.

   (b) Find the government spending multiplier.

   (c) Suppose government spending \( G \) and taxes \( T \) increase by the same amount. For this change the budget deficit \( D = G - T \) does not change. Show the multiplier is 1. Explain.
3. Suppose that the production function is \( f(k, L) = k^\alpha L^{1-\alpha} \).

a) Find expressions for \( k^* \), \( y^* \), and \( c^* \) as functions of the parameters of the model, \( s \), \( n \), \( g \), \( \delta \) and \( \alpha \).

b) What is the golden-rule value of \( k \)?

c) What is the savings rate needed to yield the golden-rule capital stock?

4: An increase in the labor force. Shocks to an economy, such as wars, famines, or the unification of two economies, often generate large one-time flow of workers across borders. What are the short-run and long-run effects on an economy of a one-time permanent increase in the stock of labor? Examine this question in the context of the Solow model with \( g = \frac{\dot{A}}{A} = 0 \) and \( n = \frac{\dot{L}}{L} = 0 \).

5: A decrease in the Investment rate. Suppose the US congress enacts legislation that discourages saving and investment, such as the elimination of the investment tax credit that occurred in 1990. As a result, suppose the investment rate falls permanently from \( s' \) to \( s'' \). Examine this policy change in the Solow model with technological progress - that is, \( g = \frac{\dot{A}}{A} > 0 \) - assuming that the economy begins in steady state. Sketch a graph of how (the natural log of) output per worker evolves over time with and without the policy change. Make a similar graph for the growth rate of output per worker. Does the policy change permanently reduce the level or the growth rate of output per worker?