Midterm Exam
4 problems / 100 points
Open only Green Book, Open Notes

Ex 1. _____ /25 points
Ex 2. _____ /25 points
Ex 3. _____ /25 points
Ex 4. _____ /25 points
Total _____ /100 points

Please print your full name.

Last Name: ________________________________
First Name: ________________________________

Please read the Honor Code and sign.

The Stanford University Honor Code

1. The Honor Code is an undertaking of the students, individually and collectively:
   (a) that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;
   (b) that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.

2. The faculty on its part manifests its confidence in the honor of its students by refraining from proctoring examinations and from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above. The faculty will also avoid, as far as practicable, academic procedures that create temptations to violate the Honor Code.

3. While the faculty alone has the right and obligation to set academic requirements, the students and faculty will work together to establish optimal conditions for honorable academic work.

I acknowledge and accept the Honor Code. (Signed) ________________________________
1. **(Minimum Variance Hedge)** A farmer produces apples. He has a contract to supply $w_1$ units of apples 1 year from now. He would like to hedge his position. Currently there exist no futures in the market for Apples, but there are futures for Oranges and Mangoes. The prices of Oranges and Apples have a correlation of $\rho_{OA}$, Oranges and Mangoes have correlation of $\rho_{OM}$ and Apples and Mangoes have a correlation of $\rho_{AM}$. The variance of the prices of Apples, Mangoes, Oranges are $\sigma_A^2$, $\sigma_M^2$, and $\sigma_O^2$ respectively.

(a) As a financial consultant, what portfolio would you advice the farmer to hold for a minimum variance hedge?

(b) What would be the resulting variance of the final cash flow?
2. (Ito’s Formula) Let $Z_t$ be a standard Wiener process and define $M_t = \exp(\sigma Z_t - \frac{1}{2} \sigma^2 t)$. Determine the process followed by $M_t$. 
3. **(Futures Options)** Options on futures are exchange-traded instruments that give the holder the right to enter in a futures contract at a specified futures price. Options on futures are typically American style as you have seen in the class. However, assume that the options are European instead of American, i.e. the holder cannot exercise the option before the expiration date. Put and call options on futures have a relation of the form

\[ C_F - P_F + \gamma \cdot K + \delta \cdot F_0 = 0, \]

where \( C_F \) and \( P_F \) are the price at the current time, \( t = 0 \), of a European call and a European put option on futures with a strike price of \( K \) and time to expiration \( T_1 \). The options are defined on the same futures contract that has a future price of \( F_0 \) at time 0. The time to expiration of the futures contract is \( T_2 \). Note that \( T_2 \) is typically larger than \( T_1 \). Let \( d(t_1, t_2) \) be the price at \( t_1 \) of zero coupon bond paying $1 at \( t_2 \). Interest rates are assumed to follow expectations dynamics. Find the constants \( \gamma, \delta \).

(Hint: Payoffs of a call and a put option on futures at expiration are \( \max(0, F_{T_1} - K) \) and \( \max(0, K - F_{T_1}) \).)
4. (Linear pricing) In Oiland’s stock exchange, put and call options based on the stock price of Oilco with various strike prices are traded. The stock exchange has decided to introduce a new European certificate whose pay-off function at maturity is

\[ P = \begin{cases} 
4,000 & \text{if } G \leq 18, \\
(2 + \frac{G - 18}{5}) \cdot 2000 & \text{if } 18 < G < 23, \\
6,000 & \text{if } G \geq 23,
\end{cases} \]

where \( G \) is the price of stock of Oilco at maturity. This certificate has an expiration date 1 year from now. The current stock price of Oilco is $20. The risk-free interest rate is 50% annually. We know that the stock price of Oilco after 1 year will be either $40 or $10.

(a) Decompose the new certificate into bonds without coupon payments and options based on the stock price of Oilco with 1 year to expiration.

(b) Find the price of the new certificate.