Final Exam
6 problems / 200 points
Open only Green Book, Open Notes

Ex 1. _____ /40 points
Ex 2. _____ /40 points
Ex 3. _____ /25 points
Ex 4. _____ /35 points
Ex 5. _____ /30 points
Ex 6. _____ /30 points
Total _____ /200 points

Please print your full name.

Last Name:

First Name:

Please read the Honor Code and sign.

The Stanford University Honor Code

1. The Honor Code is an undertaking of the students, individually and collectively:

   (a) that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;

   (b) that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.

2. The faculty on its part manifests its confidence in the honor of its students by refraining from proctoring examinations and from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above. The faculty will also avoid, as far as practicable, academic procedures that create temptations to violate the Honor Code.

3. While the faculty alone has the right and obligation to set academic requirements, the students and faculty will work together to establish optimal conditions for honorable academic work.

I acknowledge and accept the Honor Code. (Signed) _____

1. (SLIM computer) The SLIM computer company purchases DRAM from two suppliers. Both of these suppliers limit the amount of DRAM they will commit to SLIM and purchases are governed by on-going contracts. SLIM requires 10 units of DRAM each quarter for the next year. (A unit of DRAM is 1 million chips).
Supplier A will provide up to 10 units each quarter and the price will be set equal to the market price of DRAM.
Supplier B is a smaller company and is anxious to obtain business from SLIM, however, it must limit its supply commitments. Currently, two possible contract arrangements are being considered.

(1) SLIM will be provided up to 5 units of DRAM each quarter at a price that is 20 cents (per chip) below the market price.

(2) SLIM will be provided supply up to 4 units of DRAM each quarter and the price will be the market price as long as that price is below \$7.50 per chip; otherwise it will be \$7.50.

Now, supplier A got to know that SLIM will buy only 5 units from it due to the attractive contracts of B described above. However, supplier A is so greedy that it has just decided to offer SLIM a new contract below to beat both of supplier B's contract arrangements.

(3) SLIM will be provided according to the contract (1) above. Additionally, SLIM can choose once to change the contract to (2) at any time during the contract. Once SLIM changes the contracts, it will only be provided up to 4 units according to (2). (Note that the amount left still has to be purchased at the market price.)

An investment science expert has deduced (from historical data and from concepts taught in a more advanced course) that the risk-neutral price of DRAM can be represented by a lattice which begins at \$7.00. At each quarter, the lattice increases by \$0.50 or decreases by \$0.50. The "up" probability is $\frac{1}{3}$. The risk-free rate is 0%. Note that holding DRAM is strictly less desirable than holding cash; this is the case because DRAM is purchased as a necessity and not as an investment.

(a) Make a lattice of DRAM market prices spanning 3 quarters. We are assuming that the current quarter is immediate and hence 10 units will be purchased now under the terms chosen. The contract will last three more quarters so you should reach a possible price of \$8.50.
(b) what are the total values to SLIM of arrangements (1) and (2)?
(c) What is the total value to SLIM of arrangements (3)?

1. (Continued)

2. (Wagers) A game of chance based on a spinning wheel is available that pays n times the money bet in case of a win, or nothing in case of a loss. A gambler can buy a device by which he can secretly exercise some control of the wheel, so that the chance of winning is $\frac{\alpha}{n}$ where $\alpha > 1$.
(a) If the gambler has Y dollars, how much should he bet if he wishes to be log-optimal?
(b) What is the expected value of the logarithm of wealth after a single play?
(c) Suppose that n is very large, and the gambler uses his strategy n times. What is the expected value of the logarithm of his final wealth (as a function of n as $n \to \infty$). [Ignore all terms that are $o(1)$ or smaller.]

3. (Discount factor structure) The short discount factor is defined as $d = \frac{1}{1+r}$, where r is the short rate. Suppose that the short discount factor at time 0 is d_0. Each year, there is a (risk neutral) probability of 0.5 that this discount factor will increase (i.e. be multiplied by a factor of $(1+\alpha)$) and a (risk neutral) probability of 0.5 that this discount factor will decrease (i.e. be multiplied by a factor of $(1-\alpha)$). [Note: We assume here that $\alpha > 0$ and $(1+\alpha)^k d_0 < 1$ for a foreseeable future.]

Let $d_{0,k}$ be the factor by which cash at time k is discounted to obtain its present value (at time 0). For example, $d_{0,1}$ is just d_0.

1. (10 points) Find an expression for $d_{0,3}$ in terms of α and d_0.

2. (10 points) Find the current value of the forward rate $f_{2,3}$ in terms of α and d_0.

4. (Risky forward) We learned how to price a forward contract when there is no risk that either party will default on their obligation. Using option theory, we can now model more realistic problems.

1. (5 points) The RiskBank commits to buy from the UncertBank, in six months, one share of HiTech Corporation that trades currrently at $100. Assume that the 6-month Treasury note currently yields 8% and can be considered risk free. If HiTech stock doesn't pay any dividends and if the two banks were default free, what would the forward price be?

2. (10 points) Better than arrange a formal forward contact, the two parties agree to duplicate the forward contact by issuing between them combination of calls and puts on the underlying, i.e. HiTech stock. Determine the combination of puts and calls that the two banks should issue to each other to replicate a forward contract.

3. (15 points) In reality, the two banks have poor capitalization. There is a non-zero probability that either one of them may go bankrupt within the next 6 months and fail to honor its obligations. Assuming that the default risk is priced in the price of the financial securities that the banks issue, what should the cask transfer between the two banks be at the signing of the forward contract with the forward price from (a) in order to compensate their different credit risks. You might find the following date useful:

The prices of selected securities underwritten by RiskBank are:

- European Call Option on HiTech, expiring in 6 months, with the strike price equal to the forward price from (a): $4.95
- European Put Option on HiTech, expiring in 6 months, with the strike price equal to the forward price from (a): $4.95
- Zero-coupon bond, expiring in 6 months, with face value of $100: $94.80

The prices of selected securities underwritten by UncertBank are:

- European Call Option on HiTech, expiring in 6 months, with the strike price equal to the forward price from (a): $4.90
- European Put Option on HiTech, expiring in 6 months, with the strike price equal to the forward price from (a): $4.90
- Zero-coupon bond, expiring in 6 months, with face value of $100: $93.56

Assume that the performance of HiTech stock does not influence the two banks' creditworthiness and that all their debts (bonds, notes, forward contracts, etc.) issued by a given bank will be treated the same by the bankruptcy court.

4. (Continued)

5. (Foreign Currency Swap) Suppose that the term structure of interest rates is flat in both the US and Japan. The interest rates in the US are 9% per annum and in Japan are 4% per annum, both with continuous compounding. A financial institution has entered into a currency swap where it receives 5% per annum in yen and pays 8% per annum in dollars once a year. The principals in the two currencies are $10 million and 1200 million yen. The swap will last for another 3 years and the exchange rate between the two currencies is 100 yen = $1. What is the value of the swap contract in US$.

6. (Arbitrage or not?) An option to exchange a stock of company A for a stock of company B at expiration T has a payoff at T of $(S_T^B - S_T^A)^+$. Such an option is called a Magrabe Option. One day you call your broker and he gives you the following quotes. Assuming that you could short as well as long your position, does there exist an arbitrage opportunity? If no, then why not? If yes, then give a clear arbitrage argument.
All quotes are for expiration time T.

- Call option on stock of A with strike equal to \$25: \$5.25
- Put option on stock of A with strike equal to \$25: \$4.15
- Call option on stock of B with strike equal to \$25: \$7.85
- Put option on stock of B with strike equal to \$25: \$5.15
- Option to exchange stock of A for a stock of B: \$1.45