Solving Stochastic Problems
(and the DECIS System)

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Synopsis

- Uncertainty plays a key role in many decisions
  - Uncertain prices, demands, and availability of resources...
- Optimization under uncertainty has become practical
  - Commercial Software (DECIS, OSL-SE, Lindo, Aimms)
  - Specialized Solutions
  - Still very early in its development
- Multistage problems are still challenging
Approaches to stochastic programming

• Deterministic methods
  – Solving deterministic equivalent problem exploiting structure
    • Decomposition, interior point methods (Lustig et al., 1991), parallel processors, etc.

• Bounding techniques
  – Deterministic lower and upper bounds and refinement through partitioning
    • Jensen 1906, Edmundson, 1956 and Madansky, 1959; Ben Tal and Hochmann, 1972; Kall, 1974; Huang Ziemba and Ben Tal, 1977; Frauendorfer, 1992, Kuhn 2005, etc.

• Sampling techniques - probabilistic bounds
  – Pre-sampling (SAA)
    • Epi Convergence, King and Wets, 1989, SAA, Shapiro, 1991; Shapiro and Homem-de-Mello, 1998; Bayraksan and Morton, 2007
  – Sampling within the decomposition

• Others
• Multi-stage problems with many stages still difficult to solve
Classification of multi-stage stochastic linear programs

• Type of stochastic process
  – General
  – Autoregressive
  – Serial independence

• Type of stochastic parameters
  – Anywhere
  – Transition matrix and right-hand side only
  – Right-hand side only
The multi-stage SLP

\[
\begin{align*}
\min \ z = & \quad c_1 x_1 + E(c_2 x_2^{\omega_2}) + \cdots + E(c_{T-1} x_{T-1}^{\omega_{T-1}, \omega_2}) + E(c_T x_T^{\omega_T, \omega_2}) \\
\text{s.t} \quad & A_1 x_1 + B_1^{\omega_2} x_1 + A_2 x_2^{\omega_2} = b_1 \\
& \quad \vdots \\
& -B_{T-1}^{\omega_T} x_{T-1}^{\omega_{T-1}, \omega_2} + A_T x_T^{\omega_T, \omega_2} = b_T^{\omega_T} \\
& \quad x_1, x_2^{\omega_2}, \ldots, x_{T-1}^{\omega_{T-1}, \omega_2}, x_T^{\omega_T, \omega_2} \geq 0 \quad (2.1) \\
& \quad \omega_t \in \Omega_t, \ t = 2, \ldots, T.
\end{align*}
\]
Using dual decomposition

The Stage $t$, $t = 2, \ldots, T - 1$, Problems:

$$\min_{s/t} z_t^{\omega_t} = c_t \bar{x}_t^{\omega_t} + \theta_t^{\omega_t}$$

$$\pi_t^{\omega_t} : A_t \bar{x}_t^{\omega_t} = b_t^{\omega_t} + B_t^{\omega_t} \hat{x}_{t-1}$$

$$\rho_t^{l_t, \omega_t} : -G_t^{l_t, \omega_t} \bar{x}_t^{\omega_t} + \alpha_t^{l_t} \theta_t^{\omega_t} \geq g_t^{l_t}, \quad l_t = 1, \ldots, L_t \geq 0.$$
Solution

- Nested Benders decomposition
- Tree traversing strategies
- Cut sharing, Infanger and Morton, 1996
Cut sharing: dual feasibility

\[ z_t^{\omega_t}(\hat{x}_{t-1}) = \max_{s/t} (\pi_t^{\omega_t}(b_t^{\omega_t} + B_t^{\omega_t}\hat{x}_{t-1}) + \rho_t^{\omega_t}g_t) \]

\[ \pi_t^{\omega_t} A_t \]

\[ -\rho_t^{\omega_t} G_t \leq c_t \]

\[ \rho_t^{\omega_t} 1 = 1 \]

\[ \rho_t^{\omega_t} \geq 0, \omega_t \in \Omega_t, \]
Cut sharing

• Inter-stage independence
  – Cuts can be shared between different scenarios in each stage

• Inter-stage dependency
  – For the general case of autoregressive dependency, establishing dual feasibility for all scenarios in a stage and any history is at best difficult
  – Cuts cannot be shared
Cut sharing (cont.)

- We can handle
  - For autoregressive dependency of the right-hand sides and inter-stage independence of the transition matrices, cuts in any stage can be adjusted to be valid for any scenario
  - For autoregressive dependency up to three stages (and inter-stage independence thereafter) cuts can be adjusted to be valid for any scenario in the second stage
Sampling-based approaches

• Multi-stage decomposition and sampling with the decomposition
  – Multi-stage models with serial independence or autoregressive dependency in the right-hand side
  – Multi-stage stage models with autoregressive dependency in the transition and recourse matrix up to stage three

• Pre-sampling, Multi-Stage SAA
  – Multi-stage models with general dependency or autoregressive dependency
The DECIS solver

- Two-stage code released
  - Two-stage decomposition, Monte Carlo sampling with variance reduction techniques
  - Useful as deterministic method for general class of multi-stage problems

- Multi-stage code experimental
  - Multi-stage decomposition, Monte Carlo sampling with variance reduction techniques for autoregressive dependency in right-hand sides and serial independence elsewhere
Research in applications

• Finance
  – Risk Management and Portfolio Optimization (Mark Prindiville, Ph.D. thesis; Gerd Infanger, project with Freddie Mac)
  – Pricing of options in the presence of transaction costs (Ph. D thesis Kazimoro, with advisor John Weyant)
  – Dynamic Asset Allocation under serially correlated asset returns, Alexis Collomb, Ph.D. thesis

• Energy
  – Expansion and Operations Planning of Electric Power Systems
  – Gas Portfolios (PG&E)
  – Bidding on energy markets (Electrabel), energy markets (EPRI)
Research in applications (cont.)

- Transportation
  - Vehicle allocation (Bert Hackney, Ph.D. thesis)

- Supply chain
  - Phasing out products at HP (Marty O’Brian, Ph.D. thesis)
  - General supply chain planning (Rene Schaub, Ph.D. thesis)
Parallel processing experience

- Stochastic programs naturally lend themselves to parallel processing
- Significant improvements in solution time have been achieved
- Efficiency improves with problem size
- Efficiency improves with sample size
Parallel Decis M (based on elapsed solution time)

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DECIS Architecture

- Decomposes problem in master and sub
- Stores only one instance of the problem and generates scenario sub-problems as needed
- Optimizers used for solving sub-problems (integrated as a subroutine):
  - CPLEX
  - MINOS
- Provides various solution strategies
  - Decomposition, universe problem, sampling (importance sampling, control variates), pre-sampling, regularized decomposition
DECIS Interfaces

• DECIS API
  – Subroutine library

• (S (stochastic) MPS interface
  – Core file, time file, stoch file
  – Program control (options) file
  – Standard MPS output, expected cost, confidence intervals)

• GAMS-DECIS
  – integrated into and callable directly from Gams
  – Improvements on the Gams language to model stochastic programs and on the Gams-Decis interface are currently under way
Conclusion

- Solving stochastic programs has become practical, but there is still a long way to go
- General multi-stage problems still challenging
- DECIS has been used successfully for the solution of a variety of very large problems
- Using stochastic optimization promises profits and competitive advantage in different areas of application, especially in finance.