Arthur F. Veinott, Jr.

Course Outline

1 Introduction to Supply-Chain Optimization. ($\frac{1}{2}$ week)

Optimization of and multi-firm cooperation/competition in supply chains. Significance of inventories in economy, types of inventories, motives for carrying inventories. Types of decisions. Network-flow and Leontief-substitution models. Single/multi-product and single/multi-facility systems. Roles of scale economies/diseconomies, stationarity/nonstationarity, horizon length, information certainty/uncertainty. Tools of analysis: dynamic, convex and lattice programming; convex-cost network flows—substitutes/complements/ripples and invariance theorem; concave-cost network flows; cooperative and noncooperative games; stochastic order, total positivity.

2 Lattice Programming and Supply Chains: Comparison of Optima. (1 week)

• Sublattices of $\Re^n$. Simple sublattices: i-decreasing j-increasing sets. Constructing sublattices by products, intersections, sublattice-preserving transformations, projections, and sections of sublattices. Polyhedral sublattices and duals of weighted distribution problems.

• Additive, Subadditive and Superadditive Functions. Representation of additive functions. Characterization of subadditive (resp., superadditive) functions by pairwise subadditivity (resp., superadditivity).

• Ascending Multi-Functions. Increasing selections from ascending multi-functions.

• Minimizing Additive and Subadditive Functions on Sublattices. Monotonicity of optimal solution in problem parameters. Additivity and subadditivity of minimum cost in problem parameters. Applications to show monotonicity of optimal cumulative production in cumulative sales forecasts in supply chains.

• Multiple Assignment Problem with Subadditive Costs. Optimality of identity permutation. Applications to order of issuing items with age-dependent value, order of selling securities to minimize taxes and order of introducing energy technologies.

3 Noncooperative and Cooperative Games: Competition and Cooperation in Supply Chains. (1 week)

• Nash Equilibria of Noncooperative Superadditive Games and Supply Chains. Existence and comparison of fixed points of monotone functions and of Nash Equilibria of Superadditive Games. Application to multi-firm competitive price setting in supply chains.

• CORE of a Cooperative Game and Supply Chain Alliances. CORE of a game. CORE of a linear-programming game. Application to sharing benefits of supply-chain alliances.

4 Convex-Cost Network Flows and Supply Chains: Substitutes/Complements/Ripples. ($1\frac{1}{2}$ weeks)

• Introduction to Graphs. Connectivity, biconnectivity, triconnectivity. Components. Planar and series-parallel graphs. Simple paths, chains, cycles, circuits.

• Ripple Theorem. Less-biconnected-to relation. Decomposition of circulations as sums of simple conformal circulations. Changing the flow in one arc causes a second arc's optimal flow to change by an amount that diminishes the less-biconnected the second arc is to the first.

• Substitutes and Complements. Network flows in which each arc's flow cost is subadditive in the arc's flow and a parameter (e.g., an upper/lower bound on the arc's flow or a parameter of its flow cost) associated therewith. Conformality of arcs in general, planar and series-parallel graphs. Directions and rates of change of an optimal arc flow resulting from changes in a conformal arc's parameter.

• Parametric Algorithms for Finding Optimal Flows.

- Network-Flow Models of Supply Chains. Applications of substitutes/complements/ripples to show the monotonicity of optimal single/multi-plant single/multi-product procurement/production, storage, distribution and sales decisions in wage rates, wage guarantees, market prices, sales levels, and timing of sales. Parametric algorithms.

- Quadratic Costs and Linear Inventory Decision Rules. Closed-form solution of dynamic inventory problems with convex quadratic costs. Linearity of optimal production levels in forecast demands.

5 Convex-Cost Network Flows and Supply Chains: Invariance. (1 week)

- Conjugate Functions. d-additive convex functions and their conjugates.

- Invariant Network Flows and Supply Chains. Existence of an invariant optimal flow, i.e., a flow simultaneously minimizing every d-additive convex function of the flows emanating from a single node of a network. Invariance of order of optimal dual variables. Specialization to taut-string solution. Application to find optimal dynamic supply-chain production levels and price-speculation strategies graphically.

6 Concave-Cost Network Flows and Supply Chains. (2 weeks)

- Minimum-Additive-Concave-Cost Networks Flows. An $n3^d + n^2 2^d$ dynamic-programming (send-and-split) method for n-node (d+1)-demand-node uncapacitated network flows. Extension to capacitated flows. Strong connectivity. Applications to plant location, supply chains, and network design (minimum-cost chains, Steiner trees and arborescences, road/rail/sewage networks) with economies of scale.

- Minimum-Concave-Cost Flows in Planar Networks. An $nd^3 + n^2 d^2$ refinement of the algorithm for 1-planar and nearly 1-planar networks. Application to finding optimal production schedules for finite-horizon/cyclic, single-product, single/serial-facility supply-chains with/without backorders and with economies of scale.

- 94%-Effective Lot-Sizing in One-Warehouse N-Retailer Supply Chains. An $O(N \log N)$ time approximation algorithm for the stationary infinite-horizon continuous-time problem with set-up ordering costs, linear storage-cost rates, and no backorders.

7 Stochastic Order, Subadditivity Preservation, Total Positivity and Supply Chains. ($\frac{1}{2}$ week)

Stochastic order, a partial ordering of the distributions by location. Equivalence of stochastic and pointwise order. Total positivity and its sign-variation-diminishing properties. Subadditivity and convexity preservation. Application to showing monotonicity of optimal dynamic production and inventory levels as a function of demand distributions, costs, prices, advertising, etc.

8 Dynamic Supply Policy with Stochastic Demands. (2 weeks)

Use of dynamic-programming recursions—and often lattice programming—to characterize the form of optimal policies and minimum-cost functions for supply-chains with convex and set-up ordering costs. Optimality of (s, S) policies using (K, S)-quasiconvexity and K-convexity. Backorders and lead times. Multiple products. Complex demand processes: Markovian, Bayesian, percent-done estimating, bookings. Production smoothing. Decomposition of serial supply chains by showing additivity of minimum-cost function.

9 Myopic Supply Policy with Stochastic Demands. ($\frac{1}{2}$ week)

Conditions assuring single-period optimality implies multi-period optimality for single- and multi-item supply chains. Use of lattice programming and the theory of substitutes and complements in network flows to satisfy these conditions. Linear ordering costs, substitute products, batch ordering.