Business to Business Exchanges and
Supply Chain Contracting

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Extended Abstract

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1 Motivation

In recent years, Business to Business (B2B) e-commerce has been showing tremendous promise. It is widely estimated that B2B e-commerce trading volume will exceed four trillion dollars by the year 2004 (compared to 11.5 trillion dollars estimated for the United States GNP for the same year). More than a thousand B2B exchanges opened on the Internet in the last two years, and Forrester Research estimates that by 2004, more than 50% of supply chain transactions will be carried out via on-line B2B exchanges.\footnote{One of our objectives is to scrutinize forecasts like this one.} One question that immediately comes to mind is whether and to what extent trading on these exchanges will make traditional supply chain contracting obsolete. Other questions that arise are: Under what conditions do these exchanges create value and for which partners? When the exchanges do create value, what are the factors that affect that value? How is the value shared by the supply chain partners, and what are the factors that affect this allocation? We introduce a framework for analyzing the effects of B2B exchanges on the supply chain, and the strategic behavior of participants who are potential users of the exchange.

To study the role and performance of B2B exchanges, we construct a model of a two-level supply chain with multiple manufacturers and suppliers. There are three periods in our model:

(i) In the first period, contracting between the suppliers and the manufacturers takes place. The contract determines the quantity of the intermediate product that will be supplied to the manufacturers and the unit price that will be paid by the manufacturers to the suppliers.

(ii) In the second period, both the manufacturers and the suppliers receive private information. If there is a B2B exchange for the intermediate good, the participants can trade on the exchange, based on their updated information.

(iii) In the third period, the consumer market clears with the quantity produced by the manufacturers, who engage in Cournot-style oligopolistic competition. Thus, the manufacturers can purchase the intermediate good from the suppliers either via contracting (in period 1) or on the B2B exchange (in period 2).\footnote{In fact, manufacturers can also sell the product in the B2B exchange, as described below.}
**Contracting**

In the contracting stage, there is no informational asymmetry between the partners. We assume that the contract price and quantities agreed to be supplied satisfy the following properties:

(i) Given the contract price, each manufacturer’s order quantity has to maximize her expected profits.

(ii) Given the other suppliers’ contracted quantities, each supplier’s contracted quantity maximizes her expected profits.

(iii) Total contracted demand equals total contracted supply.

As an alternative characterization of the same set of equilibrium conditions, the manufacturers can be thought of as submitting demand schedules to the suppliers who then compete as a Cournot oligopoly on the aggregate demand curve.

**B2B Exchange**

At the beginning of the second period, each manufacturer obtains a noisy signal about the realization of demand and each supplier obtains a noisy signal about the realization of the unit production cost of the intermediate product. The B2B exchange (when it exists) enables the participants to trade the intermediate good based on their updated information. When trading on the exchange, both the suppliers and the manufacturers can be either a seller or a buyer. (For instance, if a manufacturer gets a signal indicating a lower than expected consumer demand and she already has committed to a large contracted purchase level, she may be better off selling some of that quantity in the exchange).

**Equilibrium**

We examine the equilibrium in the exchange using the concept of noisy Rational Expectations Equilibrium with private information. One implication of this equilibrium concept is that the traders incorporate the information that is contained in the equilibrium price in their decision making process. This reflects the aggregation and transparency of information in a medium with very little or no friction such as an

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We find necessary and sufficient conditions for the existence of an equilibrium in the exchange given the contractually-committed quantities in the first stage for each of the manufacturers. We derive the functional forms of the manufacturers’ and the suppliers’ ex-ante expected profits as a function of the vector of contracted quantities. We then calculate the first period equilibrium, dynamically incorporating the second-period expected profits into the objective functions of the participants.

2 Model

There are $M$ suppliers and $N$ manufacturers in a two-level supply chain which ultimately serves a consumer market in one end. There are three time periods in the model indexed $t = 1, 2$ and $3$. At $t = 1$, taking into account all participants’ actions in periods 2 and 3, if both sides profit from it, contracting takes place between the suppliers and the manufacturers. The contracts specify the quantity of the intermediate product that will be delivered from each supplier to each manufacturer at time $t = 3$, and the unit price that each manufacturer will pay each supplier per unit. At time $t = 2$, the suppliers get private information about the actual production costs that they will incur for producing one unit of the intermediate good, and the manufacturers get information about the demand curve in the consumer market. Without a B2B exchange, the parties are unable to readjust their contractually-committed positions according to their updated information. However, if there is an exchange, then the participants can trade on the intermediate good on the exchange based on their information, and readjust their positions. At $t = 3$, the consumer market clears with the quantities produced by the manufacturers, who compete as a Cournot oligopoly. Figure 1 summarizes the timeline.

The consumer demand curve at $t = 3$ is assumed to be linear with slope normalized to 1. That is the demand in the consumer market is given by

$$p_d = K + \tilde{d} - \sum_{i=1}^{N} Q_i,$$  \hspace{1cm} (2.1)

where $p_d$ is the clearing price at the consumer market, and $Q_i$ is the quantity of the
end product that is produced for the consumer market by manufacturer $i$. In (2.1) $K$ represents the known component of the demand intercept while $\tilde{d}$ represents the uncertain component. We assume that $\tilde{d}$ is normally distributed with mean 0 and variance $\sigma^2_d$.

The suppliers’ unit production cost, denoted by $c$, is normally distributed with mean $c_0$ and variance $\sigma^2_h$. That is $c = c_0 + \tilde{h}$ where $\tilde{h}$ is normally distributed with mean 0 and variance $\sigma^2_h$.

The rest of this section describes the equilibrium in both stages of supply chain purchasing (contracting and the B2B exchange) and derives the equilibrium outcomes. We start with the equilibrium in the B2B exchange.

2.1 The B2B Exchange

Denote the contracted quantity for manufacturer $i$ at $t = 1$ by $w_i$ for $i = 1, \ldots, N$ and define $\mathbf{w} = (w_1, \ldots, w_N)$. At time $t = 2$, manufacturer $i$ receives the signal $s^d_i = d + \varepsilon_i$, where $\varepsilon_i$ are i.i.d. normal random variables with mean 0 and variance $\sigma^2_\varepsilon$. Similarly, supplier $j$ receives the signal $s^c_j = h + u_j$, where $u_j$ are i.i.d. normal random variables with mean 0 and variance $\sigma^2_u$. We also assume that the noise terms are independent of $d$ and $h$. We denote the beginning of period 2 expected total profits excluding the
contracted transactions for a manufacturer as $\pi^m$ and for a supplier as $\pi^s$.

2.1.1 Definition of Equilibrium

We examine the equilibrium in a linear noisy rational expectations framework. An equilibrium satisfies the following properties:

(i) Given her signal, the equilibrium price and the vector of contracted quantities, each manufacturer maximizes her expected profit:

$$E[\pi^m(x_i)|s^d_i, p_e, w] \geq E[\pi^m(x)|s^d_i, p_e, w] \text{ for all } x \in \mathbb{R}, \ 1 \leq i \leq N. \quad (2.2)$$

(ii) Given her signal, the equilibrium price and the vector of contracted quantities, each supplier maximizes her expected profit:

$$E[\pi^s(y_j)|s^c_j, p_e, w] \geq E[\pi^s(y)|s^c_j, p_e, w] \text{ for all } y \in \mathbb{R}, \ 1 \leq j \leq M. \quad (2.3)$$

(iii) The market clears:

$$\sum_{i=1}^{N} x_i - \sum_{j=1}^{M} y_j = 0. \quad (2.4)$$

In the linear equilibrium, the quantity traded by manufacturer $i$ in the market is denoted by $x_i$ and has the form

$$x_i = \alpha_0 + \alpha_s s^d_i + \alpha_p p_e, \quad (2.5)$$

where $p_e$ is the market clearing price at the exchange. Similarly, the quantity traded by supplier $j$, denoted by $y_j$, will have the form

$$y_j = \beta_0 + \beta_s s^c_j + \beta_p p_e. \quad (2.6)$$

In addition to these conditions, in equilibrium the demand curves slope down, i.e. $\alpha_p < 0$, $\beta_p > 0$, and the agents react to their signals rationally, i.e., $\alpha_s > 0$ and $\beta_s < 0$. 

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2.1.2 The Equilibrium Outcome

For given \( \alpha_s, b_s \), define

\[
\eta_1(\alpha_s, \beta_s) = \frac{\sigma_d^2(\alpha_s(N - 1)\sigma_d^2 + M(M\sigma_d^2 + \sigma_u^2)\beta_s)}{(N - 1)(N\sigma_d^2 + \sigma_e^2)\alpha_s^2 + M(M\sigma_d^2 + \sigma_u^2)(\sigma_d^2 + \sigma_e^2)\beta_s^2},
\]

(2.7)

\[
\eta_2(\alpha_s, \beta_s) = \frac{\sigma_d^2\beta_s^2(M - 1)\sigma_u^2 + N(M\sigma_d^2 + \sigma_u^2)\alpha_s^2}{(N - 1)(N\sigma_d^2 + \sigma_e^2)\alpha_s^2 + M(M\sigma_d^2 + \sigma_u^2)(\sigma_d^2 + \sigma_e^2)\beta_s^2},
\]

(2.8)

\[
\nu_1(\alpha_s, \beta_s) = \frac{\sigma_h^2(\beta_s^2(N - 1)\sigma_d^2 + \sigma_u^2)\alpha_s^2 + M(M\sigma_d^2 + \sigma_u^2)(\sigma_d^2 + \sigma_e^2)\beta_s^2}{(N - 1)(N\sigma_d^2 + \sigma_e^2)\alpha_s^2 + M(M\sigma_d^2 + \sigma_u^2)(\sigma_d^2 + \sigma_e^2)\beta_s^2},
\]

(2.9)

\[
\nu_2(\alpha_s, \beta_s) = \frac{(M - 1)\sigma_h^2\beta_s}{(N - 1)(N\sigma_d^2 + \sigma_e^2)\alpha_s^2 + M(M\sigma_d^2 + \sigma_u^2)(\sigma_d^2 + \sigma_e^2)\beta_s^2},
\]

(2.10)

and

\[
\rho(\alpha_s, \beta_s) = \frac{M(M\sigma_d^2 + \sigma_u^2)\beta_s^2}{(N - 1)(N\sigma_d^2 + \sigma_e^2)\alpha_s^2 + M(M\sigma_d^2 + \sigma_u^2)(\sigma_d^2 + \sigma_e^2)\beta_s^2}.
\]

(2.11)

The next proposition summarizes the necessary and sufficient conditions for the equilibrium.\(^4\)

**Proposition 2.1** Define

\[
\alpha_p(\alpha_s, \beta_s) = \frac{\beta_s}{N\nu_1 - \nu_1\nu_2 - M + 2},
\]

(2.12)

\[
\beta_p(\alpha_s, \beta_s) = \frac{(N\alpha_s\nu_2 - 1)\beta_s}{\nu_1 + (M - 1)\nu_2\beta_s},
\]

(2.13)

and let

\[
\lambda_1 = (M\beta_p - (N - 1)\alpha_p)^{-1}.
\]

(2.14)

Then the necessary and sufficient conditions for \( \alpha_s \) and \( \beta_s \) to be equilibrium signal coefficients are given by:

(i) \( \alpha_s \) and \( \beta_s \) solve the system

\[
\alpha_p(\alpha_s, \beta_s) = \frac{M\beta_p(\alpha_s, \beta_s)(\eta_2 + (\rho - 1)) - 1}{N(\eta_2 + \rho) + 1 + \lambda_1}
\]

(2.15)

\[
\alpha_s = \frac{\eta_1}{\eta_2 + \rho + 1 + \lambda_1}.
\]

(2.16)

\(^4\)The arguments \( \alpha_s \) and \( \beta_s \) are dropped henceforth wherever convenient for notational simplicity.
(ii) $\alpha_s$ and $\beta_s$ satisfy
\[
\frac{(M - 2)N(N\sigma_d^2 + \sigma_e^2)}{M(M - 1)\sigma_u^2} < \frac{\beta_s^2}{\alpha_s^2} < \frac{N(N\sigma_d^2 + \sigma_e^2)(1 + \sqrt{1 + \frac{4M}{(M-1)^2}})}{2M\sigma_u^2}.
\] (2.17)

In equilibrium, $\alpha_p$ and $\beta_p$ satisfy (2.12) and (2.13).

The next proposition summarizes the equilibrium outcome characteristics for a given contracted quantity vector $w \in \mathbb{R}^N$.

**Proposition 2.2**

(i) Define
\[
\Lambda_1(\alpha_s, \beta_s) = (N\alpha_p(\alpha_s, \beta_s) - (M - 1)\beta_p(\alpha_s, \beta_s))^{-1}
\] (2.18)
along with
\[
\gamma_0 = \frac{(\Lambda_1 + M\nu_2)NK + MNc_0(\eta_2 + \rho - 1)}{(\Lambda_1 + M\nu_2)((\eta_2 + (\rho + 1) + \lambda_1)) + M\nu_2(N - \eta_2 - \rho)}
\] (2.19)
\[
\gamma_1 = -\frac{(\Lambda_1 + M\nu_2)(N + 1)}{(\Lambda_1 + M\nu_2)((\eta_2 + (\rho + 1) + \lambda_1)) + M\nu_2(N - \eta_2 - \rho)}
\] (2.20)
where $\alpha_p$, $\beta_p$ and $\lambda_1$ are as given in Proposition 2.1.

Suppose $w = (w_1, ..., w_N) \in \mathbb{R}^N$ is given. Then in equilibrium
\[
\alpha_0(\alpha_s, \beta_s, w) = \frac{K(\Lambda_1 + M\nu_2) + M(\eta_2 + \rho - 1)(c_0 + \nu_2\gamma_0)}{(1 + \lambda_1)(\Lambda_1 + M\nu_2)} + \frac{M(\eta_2 + \rho - 1)\nu_2\gamma_1 - \Lambda_1 - M\nu_2}{(1 + \lambda_1)(\Lambda_1 + M\nu_2)} \sum_{k \neq i} w_k
\]
\[+ \frac{M(\eta_2 + \rho - 1)\nu_2\gamma_1 - 2(\Lambda_1 + M\nu_2)}{(1 + \lambda_1)(\Lambda_1 + M\nu_2)} w_i
\] (2.21)
and
\[
\beta_0(\alpha_s, \beta_s, w) = \frac{c_0 + \nu_2\gamma_0}{\Lambda_1 + M\nu_2} + \frac{\nu_2\gamma_1}{\Lambda_1 + M\nu_2} \sum_{i=1}^{N} w_i.
\] (2.22)

(ii) For $1 \leq j \leq M$, $\pi_j$ is a quadratic function of $\sum_{i=1}^{N} w_i$ and for $1 \leq i \leq N$, $\pi_i^m$ is a quadratic function of $\sum_{k \neq i} w_k$ and $w_i$. 

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For future reference, define
\[ \pi^s_j\left(\sum_{i=1}^N w_i\right) = \pi^s_0 + \pi^s_1 \sum_{i=1}^N w_i + \pi^s_2 \left(\sum_{i=1}^N w_i\right)^2 \]  
(2.23)
and
\[ \pi^m_i\left(\sum_{k \neq i} w_k, w_i\right) = \pi^m_0 + \pi^m_0 w_i + \pi^m_1 \sum_{k \neq i} w_k + \pi^m_2 \left(\sum_{k \neq i} w_k\right) w_i + \pi^m_3 \left(\sum_{k \neq i} w_k\right)^2 + \pi^m_4 \left(\sum_{k \neq i} w_k\right)^2 \]  
(2.24)
for all \(1 \leq j \leq M\) and \(1 \leq i \leq N\).

2.2 The Contracting Stage

Taking into account the outcome (and existence) of the exchange, we next examine the contracting decisions for the supply chain participants and the resulting equilibrium.

2.2.1 Definition of the Equilibrium

The manufacturers submit demand schedules stating the quantity that they would order for each given contracting price \((p_c)\). Taking the aggregate demand schedule, the suppliers engage in quantity competition maximizing their expected total profits. More formally, the equilibrium satisfies the following conditions:

(i) The quantity that each manufacturer orders \((w_i\) for manufacturer \(i\)) maximizes her total expected profits given \(p_c\), the other manufacturers’ demanded quantities and her information at the contracting stage:
\[ \pi^m_i\left(\sum_{k \neq i} w_k, w_i\right) - p_c w_i \geq \pi^m_i\left(\sum_{k \neq i} w_k, w_{mi}\right) - p_c w \text{ for all } w \geq 0 \text{ and } 1 \leq i \leq N. \]  
(2.25)

(ii) Each supplier’s chosen production quantity \((q_j\)) maximizes her total expected profits given the aggregate demand schedule of the suppliers, the total quantity produced by the other suppliers and her information at the contracting stage:
\[ (p_c\left(\sum_{i=1}^N w_i(q_j)\right) - c_0)q_j + \pi^s_j\left(\sum_{i=1}^N w_i(q)\right) \geq (p_c\left(\sum_{i=1}^N w_i(q)\right) - c_0)q + \pi^s_j\left(\sum_{i=1}^N w_i(q)\right) \]  
(2.26)
for all $q \geq 0$ and $1 \leq j \leq M$, where $p_c(\cdot)$ is the inverse aggregate demand function, i.e. the transaction price corresponding to a certain total quantity contracted by the suppliers.

(iii) The total demand contracted is equal to the total supply contracted:

$$\sum_{i=1}^{N} w_i = \sum_{j=1}^{M} q_j \quad (2.27)$$

Alternatively, we can formulate the equilibrium outcome as the solution to the following optimization problem for the suppliers. For each supplier $j$:

$$\max_{p_c,q_j} \quad (p_c - c_0) q_j + \pi^s_j \left( \sum_{i=1}^{N} w_i \right)$$

s.t.

$$w_i = \arg \max_{w \geq 0} \pi^m_i \left( \sum_{k \neq i} w_j, w \right) - p_c w \quad i = 1, 2, ..., N$$

$$\sum_{i=1}^{N} w_i = \sum_{k \neq j} q_k + q_j$$

In addition, in this characterization, the equilibrium price has to be the optimal price for all suppliers.

### 2.2.2 Equilibrium Outcome with Contracting

When there is no exchange, it can be shown that the equilibrium contract price satisfies $p_c = (K + M c_0)/(M + 1)$. In this case, the contractually committed quantity for each supplier will be $q_j = (N(K - c_0))/((N + 1)(M + 1))$, $j = 1, ..., M$, and the ordered quantity for each manufacturer will be $w_i = (M(K - c_0))/((N + 1)(M + 1))$, $i = 1, ..., N$. The following proposition summarizes the outcome at $t = 1$ with the exchange:

**Proposition 2.3** Given an equilibrium exists in the exchange, the necessary and sufficient conditions for the existence of an equilibrium in the first stage are $\pi^m_{ww} < 0$ and $N \pi^s_2 + (N - 1) \pi^m_{sw} + 2 \pi^m_{ww} < 0$. If an equilibrium exists, then in equilibrium:

(i) If $c_0 - \pi^m_{0w} - \pi^s_1 < 0$, then

$$q_j = \frac{N(c_0 - \pi^m_{0w} - \pi^s_1)}{(M + 1)(N \pi^s_2 + (N - 1) \pi^m_{sw} + 2 \pi^m_{ww})} \quad j = 1, ..., M \, , \quad (2.28)$$
\[ w_i = \frac{M(c_0 - \pi_{0w}^m - \pi_1^s)}{(M + 1)(N\pi_2^m + (N - 1)\pi_{sw}^m + 2\pi_{w}^m)} \quad i = 1, \ldots, N, \quad (2.29) \]

and
\[ p_c = \pi_{0w}^m + \frac{((N - 1)\pi_{sw}^m + 2\pi_{w}^m)(c_0 - \pi_{0w}^m - \pi_1^s)}{(M + 1)(N\pi_2^m + (N - 1)\pi_{sw}^m + 2\pi_{w}^m)} . \quad (2.30) \]

(ii) If \( c_0 - \pi_{0w}^m - \pi_1^s \geq 0 \) then \( q_j = w_i = 0 \) for all \( 1 \leq j \leq M \) and \( 1 \leq i \leq N \), i.e., no quantities are committed in the contracting stage.

2.3 Monopolistic Cases

2.3.1 Monopolistic Supplier

In this section we present and analyze the case with a monopolistic supplier (i.e., \( M = 1 \)). The next proposition gives the equilibrium outcome in the B2B exchange for the monopolistic supplier case.

**Proposition 2.4** When \( M = 1 \) a unique equilibrium exists in the exchange. Define
\[ B = \eta_2\left( \frac{N}{N - 1} + \frac{(N - 1)\sigma_d^2 + \sigma^2}{\sigma^2} + \frac{1}{2} - \frac{2N - 1}{N - 1} \right) . \quad (2.31) \]

In equilibrium,
\[ \rho = \frac{-B + \sqrt{B^2 + \frac{42N_{-1}^2}{N - 1} (\frac{N}{N - 1} \eta_2 + \frac{1}{2})}}{2(2N - 1)}, \quad (2.32) \]

where \( \eta_2 \) solves
\[ \eta_2 = \frac{(\sigma_d^2)\sigma_e^2(2N - 1)^2(2N(\eta_2 + \rho) - (N - 1))^2(\rho + \frac{1}{2} + \frac{N(\eta_2 + \rho)}{N - 1})(\sigma_h^2 + \sigma_u^2)}{4N^2(\sigma_h^2)^2(N - 1)((N\sigma_d^2 + \sigma_e^2)\eta_2 + (\sigma_d^2 + \sigma_e^2)(\rho + \frac{1}{2} + \frac{N(\eta_2 + \rho)}{N - 1}))^2} . \quad (2.33) \]

The equilibrium strategies satisfy
\[ \alpha_p = \frac{1}{2N - 1} - 1 \quad (2.34) \]
\[ \beta_p = -N\alpha_p \quad (2.35) \]
\[ \alpha_s = \frac{N^2(\sigma_d^2)^2\alpha^2\eta_2}{(N - 1)(\sigma_d^2 + \sigma_e^2)\sigma_e^2(\rho + 1 - \frac{1}{(2N - 1)\alpha_p})} \quad (2.36) \]
and
\begin{equation}
\beta_s = \frac{N\alpha_p\sigma^2_h}{\sigma^2_h + \sigma^2_u}. \tag{2.38}
\end{equation}

As the number of manufacturers becomes large the entire purchasing between suppliers and manufacturers shift to the exchange. The following proposition summarizes this result.\textsuperscript{5}

**Proposition 2.5** When $M = 1$, for any given $(\sigma_d^2, \sigma_h^2, \sigma_e^2, \sigma_u^2, K, c_0)$ there exists a $N^* \geq 2$ such that if $N > N^*$ no contracting takes place between the supplier and manufacturers in the first stage.

The following proposition summarizes the equilibrium manufacturer response to signals and the price and market liquidity as the number of manufacturers gets large.

**Proposition 2.6** For $M = 1$:

(i)
\begin{equation}
\lim_{N \to \infty} \alpha_p = 0^- \tag{2.39}
\end{equation}
\begin{equation}
\lim_{N \to \infty} \alpha_s = 0^+ \tag{2.40}
\end{equation}

(ii)
\begin{equation}
\lim_{N \to \infty} \lambda_1 = \frac{1}{2} \tag{2.41}
\end{equation}
\begin{equation}
\lim_{N \to \infty} \Lambda_1 = -1 \tag{2.42}
\end{equation}

### 2.3.2 Monopolistic Manufacturer

In this section we give the equilibrium outcome for the monopolistic manufacturer case, i.e. the case that $N = 1$. The following proposition summarizes the equilibrium outcome.

\textsuperscript{5}More discussion on this issue is given in Section 3.
Proposition 2.7 Define

\[ \Lambda_1(\beta_p) = \frac{-1}{2M\beta_p + 1 + (M - 1)\beta_p} \]  

(2.43)

and

\[ \eta_1 = \frac{\sigma_d^2}{\sigma_d^2 + \sigma_\varepsilon^2}. \]  

(2.44)

When \( N = 1 \), in equilibrium \( \beta_p \) solves:

\[
\sigma_h^2(M - 1)\sigma_u^2\Lambda_1(2M\beta_p + 1)^2 + ((\sigma_h^2 + \sigma_u^2)\Lambda_1 = (\Lambda_1 + 1)(\sigma_h^2 M + \sigma_u^2))\eta_1^2 M^2 \beta_p^2 (\Lambda_1 \beta_p + 1)\sigma_h^2.
\]  

(2.45)

Equilibrium parameters satisfy

\[ \alpha_p = -\frac{M\beta_p}{2M\beta_p + 1} \]  

(2.46)

\[ \alpha_s = \frac{\eta_1}{2 + \frac{1}{M\beta_p}} \]  

(2.47)

and

\[ \beta_s = \sqrt{\frac{-(\Lambda_1 \beta_p + 1)\alpha_s^2 \sigma_h^2}{(M - 1)\sigma_u^2\Lambda_1}} \]  

(2.48)

with

\[ \alpha_0 = \frac{K - 2w}{2 + \frac{1}{M\beta_p}} \]  

(2.49)

\[ \beta_0 = \frac{c_0 + \nu_2 \frac{K - 2w}{2 + \frac{1}{M\beta_p}}}{\Lambda_1 + M\nu_2}. \]  

(2.50)

3 Equilibrium Results

We next analyze the equilibrium outcome for the monopolistic supplier case for varying parameter values. We first consider the effects of manufacturers’ information quality. Figure 2 shows the outcome for \( N = 3 \). Panel (a) shows that as the manufacturers’ information quality decreases (i.e. \( \sigma_\varepsilon^2 \) increases), the contracted quality increases and the expected quantity traded on the B2B exchange decreases. This is because as their information quality increases, it is more profitable for the manufacturers to trade on
the exchange with their information. As a result, they decrease the amount that they contract for and shift their purchases to the exchange. Consumer surplus is increasing with the quality of the information (Panel (b)). Panel (c) shows that in this case, the expected manufacturer profits are higher with the exchange than they would be without the exchange, and they are increasing with the quality of the manufacturers’ information. Panel (d) shows that the supplier’s profit is lower with the exchange than without it, and decreases with the quality of the manufacturers’ information.

There are two major factors that determine the effect of increased information quality on manufacturer profits. On one hand, better quality of information means better decision-making and increased profits for the manufacturers. On the other hand, increased information quality also means that the manufacturers respond to their signals as well as the information they extract from the exchange more strongly, making the traded quantities more variable which reduces the manufacturer profits. This effect is stronger when the number of manufacturers is large, as can be seen in Figure 3 which shows the equilibrium outcome when $N = 10$. As can be seen in Panel (c) of this figure, the manufacturers’ profits are decreasing in the quality of their information. This shows that the reduction in expected profits caused by the uncertainty in traded quantities exceeds the increase in profits caused by the higher quality of the information. In fact, when the quality of information is very high, the expected manufacturer profits with the exchange fall below the expected profit level without the exchange. When we look at the expected supplier profits (Panel (d)) the profit reduction effect of increased uncertainty about the traded quantities is strong, and increased manufacturer profits cause a reduction in the expected supplier profits.

Another important thing to notice from Figure 3 is that with $N = 10$ the manufacturers choose not to engage in the contract at $t = 1$ at all and do all their purchases on the exchange (Panel (a)). This is the result of the powerful effect of information aggregation in the exchange. When the number of manufacturers is high, the aggregated information reflected in the clearing price is very strong. This means that by trading on the exchange, the manufacturers can benefit from improved decision making at such a level that ordering any positive level in contracting stage is suboptimal.

The effect of varying supplier information quality on the equilibrium is shown in Figure 4. The main effect of increased supplier information quality on the manufac-
Figure 2: Equilibrium outcome with three manufacturers and varying $\sigma^2_\varepsilon$. The parameters are $\sigma^2_d = 1$, $\sigma^2_h = 1$, $\sigma^2_u = 0.5$, $K = 20$ and $c_0 = 5$. 
Figure 3: Equilibrium outcome with 10 manufacturers and varying $\sigma^2_e$. The parameters are $\sigma^2_d = 1$, $\sigma^2_h = 1$, $\sigma^2_u = 0.5$, $K = 20$ and $c_0 = 5$. 
turers is to reduce the noise in the clearing price at the exchange. This means better
information for the manufacturers hence better decision making. As seen in Panel (a)
this decreases the manufacturers’ order quantities in the contracting stage and shifts
their purchases to the exchange. In fact, the expected overall quantities purchased by
the manufacturers and sold to consumers with exchange is lower than without it.

As explained above, increased information availability on the exchange also has
a downside. That is, it causes the manufacturers to react the available information
more strongly. This increases the variability of the traded quantities and reduces the
expected manufacturer profits. Again, this effect is more powerful when the number
of manufacturers is high, as can be seen in Panel (c) of Figure 5. When $N = 10$,
the manufacturers’ profits improve with the exchange but decrease as the supplier’s
information quality improves.

The effect of changing number of manufacturers on the equilibrium can better be
seen in Figure 6. In Panel (a) we can see that when $N$ is low, the manufacturers
choose to use both venues (i.e., contracting and the exchange) for purchasing. On the
other hand, as $N$ increases, information aggregation in the exchange makes trading
there more attractive, and in equilibrium the manufacturers choose not to order in
the contracting stage at all. Manufacturer profits and the expected quantities that
each manufacturer produces decline as the number of manufacturers increases because
of increased competition (Panel (c)). In Panel (d) we can see that supplier profits
with pure contracting is monotonically increasing with the number of manufacturers,
since with a large number of manufacturers and increased competition among them,
the supplier is able to better exploit her position. On the other hand the supplier’s
profit is non-monotonic in the number of manufacturers with the exchange. For low
$N$, as the number of manufacturers increases, increased competition among them helps
the supplier increase her profits. However, when the number of manufacturers exceeds
a critical value, the increased uncertainty starts reducing the supplier’s profits.

4 Strategic Value of the Exchange

The results of Section 3 show that the supplier is generally worse off with the exchange
while the manufacturers are in most cases better off with it. This is because, as
Figure 4: Equilibrium outcome with three manufacturers and varying $\sigma_u^2$. The parameters are $\sigma_d^2 = 1$, $\sigma_h^2 = 1$, $\sigma_e^2 = 0.5$, $K = 20$ and $c_0 = 5$. 
Figure 5: Equilibrium outcome with 10 manufacturers and varying $\sigma_u^2$. The parameters are $\sigma_d^2 = 1$, $\sigma_h^2 = 1$, $\sigma_\epsilon^2 = 0.5$, $K = 20$ and $c_0 = 5$. 
Figure 6: Equilibrium outcome with varying number of manufacturers. The parameters are $\sigma_d^2 = 1$, $\sigma_h^2 = 1$, $\sigma_e^2 = 0.5$, $\sigma_u^2 = 0.5$, $K = 20$ and $c_0 = 5$. 
discussed before, the exchange not only enables the manufacturers to act on their information, but it also serves as a mechanism for them to share their information by aggregating the participants’ information in the transaction price. On the other hand, the exchange creates uncertainty about the trading quantity and therefore reduces the supplier’s expected profits. Since the supplier is worse off with the exchange, can she just block it by not participating in it?

The answer turns out to be negative. The reason is in this case the manufacturers can form the exchange to trade among themselves and with it the supplier is induced to participate as well. Further, if there are at least three manufacturers and the exchange exists, there is an equilibrium in the exchange even when the supplier chooses not to participate.

Without the supplier the equilibrium condition (iii) given in Section 2.1.1 will be replaced by the new market clearing condition

\[ \sum_{i=1}^{N} x_i = 0. \]

The following proposition summarizes the equilibrium outcome in the exchange without the participation of the supplier.

**Proposition 4.1** If \( N > 2 \), an equilibrium in an exchange with no supplier exists. The equilibrium strategies satisfy

\[
\alpha_{0i} = \frac{K - \sum_{k \neq i} w_k - 2w_i}{2(N-1) + \frac{2N-1}{N-1} + 1} \quad \text{for} \quad 1 \leq i \leq N, \tag{4.2}
\]

\[
\alpha_s = \frac{N\sigma_d^2}{2(N-1)(N\sigma_d^2 + \sigma_e^2)}, \tag{4.3}
\]

and

\[
\alpha_p = \frac{2 - N}{(2N - 1)(N - 1)}. \tag{4.4}
\]

Now given that the exchange exists, it is suboptimal at \( t = 2 \) for the supplier not to participate in it using her updated information. Therefore given the manufacturer’s strategies, the supplier is forced to participate in the exchange even though this reduces her overall profits when compared to the case without the exchange. We conclude
that setting up an exchange among themselves is a profitable strategic move for the manufacturers to take back some of the profits that the monopolistic supplier extracts from them using her positional advantage.

5 Summary of the Results; Current and Future Work

We presented a framework for analyzing the role and value of Business to Business electronic exchanges and their interaction with supply chain contracting. We derive the equilibrium outcome with multiple suppliers and manufacturers, find the functional form of the value functions and derive the dynamic equilibrium for the entire game.

For the monopolistic supplier case, our results show that the exchange may make the supplier worse off, since the quantity variability in the exchange adds uncertainty and reduces her expected profits as compared to the case with pure contracting. We find that in general, the supplier benefits from increased accuracy of her own signal.

The manufacturers, on the other hand, benefit considerably from the existence of the exchange as long as their number is not too high. This is because the market enables them to aggregate their information in a competitive way, and hence improves their decision-making. If the supplier’s information quality improves, the exchange reveals more information to the manufacturers, increasing their profits. Hence the manufacturers become better off purchasing in the exchange, and the contracted quantity levels in the first period decrease.

When the number of manufacturers is large, some of the results are different. The increased number of manufacturers in the market has two effects. First, a large number of manufacturers increases the aggregation of information, making manufacturers better off not engaging in contracting at all and carrying out all their purchasing through the exchange. Second, as the number of manufacturers increases, the information content of the exchange is improved, causing the manufacturers to react more strongly. This results in increased variability of the quantity traded in the exchange. When the number of manufacturers is large enough, this effect reduces the expected manufacturer profits.
When the quality of the manufacturers’ information gets better, the quantity contracted in the first period comes down and purchasing shifts to the exchange. If the number of manufacturers is small, their expected profits increase with the quality of their information. However, as the number of manufacturers increases, the increased variability of the quantity traded in the exchange starts reducing the expected manufacturer profits when the quality of their information increases. The supplier’s profit is affected by two opposite factors. Increased manufacturer information quality results in an increase in the expected quantity demanded by the manufacturers, thus increasing supplier profits. On the other hand, increased variability in the quantity exchanged results in lower expected profits. With a large number of manufacturers, the second factor becomes dominant, and the supplier’s profits decrease as the quality of the manufacturer information increases.

Our results suggest that the strategic effects can reduce the potential benefits of the exchange to a level where all participants would be better off without it. This can hamper the development of B2B exchanges.

The results reported here address the case of a monopolistic supplier. An analysis of the implications of the equilibria derived here for the multiple supplier case is currently underway. Further, the framework presented here can be extended to study a number of related issues. For instance, how does the purchasing structure change when the demand and cost across the layers of the supply chain are correlated? What happens when the competitors in a given layer are asymmetric? What are the implications of the existence of the exchange in a repeated trading environment? The answers to these questions, which are the subject of current and future research, can help understand and improve the performance of Business to Business exchanges and their role in the supply chain.