Optimal hedge ratios for credit versus equity trades

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Abstract—In this paper, we investigate whether relative valuation techniques for credit versus equity are relevant to CDS hedging and we discuss how to use these techniques for this purpose.

Credit and equity should exhibit correlation – as a company’s overall performance deteriorates, the prices of its equity and debt fall, because its future earnings potential falls and its probability of default rises. We use a structural model deriving from the CreditGrades framework to factor this correlation into the default probabilities of a given name. Structural models define credit and equity instruments as claims on a firm’s value. They imply no-arbitrage relationships between par credit spreads, equity prices and volatilities. We utilize these relationships to build hedge ratios at implied credit spread, implied volatility and implied equity price. We investigate the performances of the resulting hedging strategies in CDS hedging.

We compare this approach with a purely statistical approach where credit spread moves are learnt from equity moves.

Finally, we use these results to design optimal strategies for hedging CDSs with equity instruments.

I. INTRODUCTION

Credit default swaps (CDSs) may be considered as hedging instruments against the risk of default of a debt claim. However, as securities per se, CDSs may also need to be hedged against their specific risks. The value of a CDS – whose spread was fixed at inception – is quoted through its fair spread, or par spread, i.e. the spread that should set its present value to zero. There is no univocal way to convert a spread into a CDS value and there is recent evidence suggesting that fair spreads are not solely driven by default probabilities, at least for investment grade entities (see [1]). Therefore, daily changes of the marked–to–market value of a CDS contract should be decomposed along two drivers:

- spread risk: the spread risk is defined by the changes in the value of a CDS which are caused by changes in its fair credit spread. The Dodd-Frank act[1] requires that over–the–counter swaps, including CDSs, be cleared through exchanges and central clearing counterparties (CCPs). In this context, spread risk translates directly into margin requirements, so parties and CCPs need to agree on a methodology to convert fair spreads to CDS values[2]. Currently, the ISDA CDS Standard Model seems to be the norm for liquid contracts (e.g., CDSs on indices) in developed markets, although banks and hedge funds may be using proprietary valuations in addition to this model in order to get more accurate views on their eventual positions. For the sake of simplicity,

1Cf. title VII, “A bill to to improve the regulation of swap and security-based swap activities, and for other purposes”.

2The same applies if the trade is collateralized without a CCP.
we stick to the ISDA CDS Standard Model: all our CDS values are derived from par spreads through it. This paper focuses on spread risk. It aims at using models which capture some sources of spread variations and to incorporate them into hedge ratios for CDS contracts.

- **Jump-to-default risk:** the jump-to-default risk is the risk that a credit event occurs on a CDS. Acceptable credit events are defined in the terms of any CDS contract and should broadly match the notion of default on the reference obligation. Unlike estimated default probabilities, a jump to default is by nature discontinuous in time and is often associated with incomplete markets. In this paper, we do not consider jump-to-default risk, as all our entities remained solvent during the chosen period of time regardless of their credit ratings.

To predict spread moves, the approach of this paper is to use relative-value relationships within the capital structure (credit versus equity) of reference entities. Credit and equity should exhibit correlation – as a company’s overall performance deteriorates, the prices of its equity and debt fall, because its future earnings potential falls and its probability of default rises; and vice versa. Relative-value investors use theoretical and empirical models to think about both the “right” price of debt versus equity (and hence, when to put on a long equity, short credit trade or vice versa) and about appropriate hedge ratios for those trades.

One way to approach trading credit against equity is to model debt securities as short out-of-the-money put option positions on the company’s assets (a debt security yield a fixed premium, unless the company’s performance is very poor, in which case it results in a net loss) and equity securities as long in-the-money call option positions on assets (an equity security entitles to all the upside from increased asset value, with downside limited to the money which was put in). This structural modeling approach generates fair-value relationships and hedging relationships between credit, equity and volatility, and captures the underlying nonlinearities in credit-equity relationships (specifically, the increase in sensitivity of credit to equity as a company’s financial leverage increases). This approach is expected to perform well when those features of credit and equity as claims on a firm’s value are most obvious, i.e. for troubled assets.

An alternative is a more reduced-form statistical approach, in which the investor estimates mean-reverting statistical relationships between credit and equity. This approach can be placed under the reduced-form umbrella in the sense that sources of defaults are neither directly explained nor modeled, contrary to structural models where defaults happen when a threshold in the firm’s value is reached.

The main objective of this paper is to propose and check the robustness of “optimized” hedge ratios based on these different approaches.

In order to present the above-mentioned strategies, a limited number of firms (five) were culled from a larger sample based on their diversity in terms of industries and credit ratings and are singled out in this paper for concrete comparisons. Section II introduces them. Sections III and V describes the statistical approach as a base technique with which the structural model will be compared. Section V moves on to the CreditGrades structural model and sets forth the customizations that we applied to it. In section VI sensitivity definitions are detailed together with their application to delta and delta-vega hedging of CDS contracts. Section VII lays out what optimized hedging strategies we suggest to use. Section VIII concludes, while section IX gives details about our data sources and computations.

II. Selected Firms

In order to propound positive comparisons of the hedging techniques described in this paper, five firms were picked from a larger pool (fifty-seven names) based on their industries and credit ratings. A succinct presentation is given for each one of them below. As of June 2011, two had substantially speculative ratings, one was at the higher-end of the speculative spectrum and two were investment-
grade as is spelled out in the following list of the five names:
A. Kodak, formally known as The Eastman Kodak Company – extremely speculative,
B. Ryland Homes – non-investment grade speculative,
C. Ingersoll Rand – low medium grade,
D. Dell – upper medium grade,
E. Campbell’s, also known as Campbell Soup – upper medium grade.
All the time series for these firms run from January 3rd, 2005 to April 14th, 2011.

A. Kodak (The Eastman Kodak Company)
Kodak, the imaging product company, has been facing long-standing financial problems with the transition to digital imaging. It filed for bankruptcy protection in January 2012, though this is not reflected in the period of time considered in this paper. Its NYSE ticker is EK.
Rating & outlook: As of June 2011, S&P’s rating for EK was CCC with a negative outlook. As a concern mentioned by the rating agency, Kodak’s pace of cash consumption would have remained high over the near term.

B. Ryland Homes
Ryland is the ninth largest US developer and home builder. Based in Calabasas, California, Ryland Homes has built over 285,000 homes across
the United States since its inception, focusing primarily on first-time homebuyers as well as first- and second-time move-up buyers. It trades on the NYSE under the ticker RYL.
Rating & outlook: As of June 2011, S&P’s rating for RYL was BB- with a stable outlook. Ryland has been exposed to the continued poor performance of the housing market.

This static picture could be supplemented with rating histories.

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Fig. 1: Stock price and 5Y spread for Kodak since 2005

Fig. 2: Kodak’s leverage since 2005

Fig. 3: Stock price and 5Y spread for Ryland Homes since 2005

Fig. 4: Ryland Homes leverage since 2005
C. Ingersoll Rand

Ingersoll Rand is a conglomerate which defines itself as an international supplier to the transportation, manufacturing, construction, and agricultural industries. It produces everything from air compressors to golf cars, including door closers. The NYSE ticker for Ingersoll Rand is IR.

Rating & outlook: As of June 2011, S&P’s rating for IR was BBB+ with a stable outlook. Ingersoll Rand was included in the sample in particular because of potential “conglomerate effects” in its stock price and its credit rating.

Fig. 5: Stock price and 5Y spread for Ingersoll Rand since 2005

Fig. 6: Ingersoll Rand’s leverage since 2005

D. Dell

Dell is one of the largest technological corporations in the world. Based in Round Rock, Texas, Dell develops, sells and supports computers and related products and services. The NYSE ticker for Dell is DEll.

Rating & outlook: As of June 2011, S&P’s rating (which dates back to 2007) for DEll was A-, with a stable outlook.

E. Campbell’s (Campbell Soup)

Campbell Soup produces canned soups that are sold in 120 countries. Based in Camden, New Jersey, Campbell is made up of three divisions: Simple Meals (which consists largely in soups), Baked Snacks and Health Beverage. The NYSE ticker for Campbell Soup is CPB.

Rating & outlook: As of June 2011, S&P’s rating for CPB was A (published in 2007), with a negative outlook imposed on Feb 22, 2011.

The next section introduces the statistical approach to hedging CDS contracts with equity.

III. STATISTICAL APPROACH: MODEL

This section presents a simple statistical approach to hedging credit against equity. Linear relationships
between CDS values and equity prices are estimated. The resulting linear combinations are mean-reverting and can therefore be used to maintain stable P&Ls, by betting against the sustainability of large deviations from the long-term equilibrium. The analogy is made with pairs trading.

In more detail, the idea behind the statistical approach is that CDS spreads and market values are correlated with equity prices (cf. supra) and that it should be possible to estimate this correlation on a historical basis. Time series of CDS spreads and market values are non-stationary, just as time series of equity prices, at least because of their trends (even after taking a log transform). This is illustrated in fig. 11. This figure also makes it obvious that both series have similar trends when appropriately scaled. There could be some combination of the CDS market value and the equity price that would be stationary. Indeed, fig. 12 shows that it is possible to find a linear combination of the two time series which looks stationary. That combination was estimated through an error-correction model. This concept is briefly reviewed below, together with the underlying concept of cointegration. However, it is first explained how to make use of a mean-reverting relationship such as the one exhibited in fig. 12 in the context of CDS hedging. The focus is on time series of CDS values (not CDS spreads). As a reminder, CDS values are deduced from spreads through the ISDA CDS Standard Model.

A. Hedging by betting against large deviations

Exploiting the stability of a mean-reverting linear relationship is very similar to a pair trading strategy, a concept that readers may be familiar with. The idea behind a pair trading strategy is to trade on the oscillations around the equilibrium value of a signal \( w_t \) (in a moment, \( w_t \) will be a linear combination of a CDS market value and of an equity price. For the moment, it should be thought of as a combination of two stocks). The oscillations in the signal occur because it is mean reverting. When
Fig. 12: Cointegrated series for Ryland: \(-1.2 \log(\text{CDS value}) - 6.4 \log(\text{equity})\).

\(w_t\) deviates substantially from its mean, a trade is entered into. It is unwind when the \(w_t\) comes back close to equilibrium. In practice, how big the deviation needs to be in order for the trades to be profitable depends on several factors. Trading costs, marginal interest rates, and the bid–ask spreads of the two stocks are three factors to take into account. Mathematically, let \(\eta\) be the cost involved in carrying out a pair trading. Let \(\Delta\) be a target deviation of \(w_t\) from its mean \(\mu_w\) for entering into a trade. Then, conditioned on \(2\Delta > \eta\), a simple trading strategy is as follows:

- buy a share of stock 1 and short \(\gamma\) shares of stock 2 at time \(t\) if \(w_t = p_{1t} - \gamma p_{2t} = \mu_w - \Delta\),
- unwind the position at time \(t + i\) \((i > 0)\) if \(w_{t+i} = p_{1,t+i} - \gamma p_{2,t+i} = \mu_w + \Delta\).

The time point \(t + i\) will occur because the series is mean–reverting. Instead of taking two stocks for pair trading, we can consider making a pair out of a 5–year CDS spread and a stock.

B. Least–squares estimation

A quick test to verify that credit and equity are suitable for “pair trading” is to regress \(\log 5Y\) CDS market values on \(\log\) stock prices (see fig. [1]) and to check whether the correlation seems high. However, the regression could be spurious. A discussion on how to estimate linear combinations correctly follows in the next subsection.

C. Cointegration

The following treatment is inspired from [3] and [4]. A few definitions need to be introduced. Below, the multivariate time series \(y_t\) should be thought of as a vector whose components are the logarithm of a CDS market value and the logarithm of the corresponding equity price.

i) A multivariate time series \(y_t\) is said to be unit–root nonstationary if it is nonstationary but \(\Delta y_t\) is stationary (in the weak sense).

ii) A nonzero \(k \times 1\) vector \(b\) is called a cointegration vector of a unit–root nonstationary time series \(y_t\) if \(b' y_t\) is weakly stationary.

iii) A multivariate time series is said to be cointegrated if all its components are unit–root nonstationary and there exists a cointegration vector. If the linear space of cointegrating vectors (with \(0\) adjoined) has dimension \(r > 0\), then the time series is said to be cointegrated with order \(r\).

Unit–root nonstationary time series are challenging in the sense that regressing one component on another may be spurious. However, the regression is not spurious when there is a cointegrating vector (with nonzero components for the entries considered). Therefore, it is important to have tests for unit–root nonstationarity and cointegration. In the vast majority of the firms that we considered (and all the firms presented in this paper), the tests presented below concluded that the multivariate time series (\(\log(\text{CDS value}), \log(\text{equity price})\)) were unit–root nonstationary and cointegrated.

We focus here on the Vector Autoregressive (VAR) case. Consider a Gaussian VAR(\(p\)) model with a linear trend component:

\[ x_t = D(t) + \sum_{i=1}^{p} \Phi_i x_{t-i} + \varepsilon_t \]  \hspace{1cm} (1)

where \(D(t) = d_0 + d_1 t, d_i\) are constant vectors and \(\{\varepsilon_t\}\) is a sequence of i.i.d. Gaussian random vectors.
with mean zero and positive definite covariance matrix \( \text{Var}[\varepsilon_t] = \Sigma \). Define \( \Phi(z) = I - \sum_{i=1}^{p} \Phi_i z^i \). The \( \text{VAR}(p) \) model may be rewritten in another form:

\[
\Delta x_t = d_1 + \Pi x_{t-1} + \sum_{i=1}^{p-1} \Phi_i^* \Delta x_{t-i} + \varepsilon_t \tag{2}
\]

where \( \Pi = -\Phi(1) \), \( \Phi_i^* = -\sum_{j=i+1}^{p} \Phi_j \) for \( i = 1, \ldots, p-1 \) and \( \Delta x_t = x_t - x_{t-1} \). We assume that \( |\Phi(1)| = 0 \) (\(|\cdot|\) being the determinant) and that the other complex roots of \( |\Phi(z)| = 0 \) are outside the unit circle. In this case, \( x_t \) is unit-root nonstationary. Let \( m \) be the rank of \( \Pi \). We shall deal with the case where \( k > \text{Rank}(\Pi) = m > 0 \). In this case, model (2) is called an Error-Correction Model (ECM) and \( x_t \) has \( m \) cointegrating vectors and \( k - m \) unit roots; also, there exist \( k \times m \) full-rank matrices \( \alpha \) and \( \beta \) such that

\[
\Pi = \alpha \beta' \tag{3}
\]

The columns of \( \beta \) are the cointegrating vectors for \( x_t \).

**Estimation and Johansen’s test for cointegration relationships:** ECM models are estimated by maximum likelihood under the constraint that \( \text{rank}(\Pi) = m \) (reduced-rank regression). \( m \) is chosen through Johansen’s procedure: Johansen’s test for the hypothesis \( m \leq r \) against \( m > r \) is computed successively for increasing values of \( r \) until the test is accepted or \( r = k \).

**IV. Statistical approach: Hedging**

This section presents some results of the statistical approach to hedging CDS contracts with equity. The experience of the authors is that performances were highly unpredictable. Co-integration vectors are estimated using the first half of the data.\(^7\) Results are presented on fig. 13 for Ingersoll Rand and on fig. 14 for Ryland.

\(^6\)Indeed, if \( m = k \), then \( \Phi(1) \) would not be singular and if \( m = 0 \), \( \Delta y_t \) is a \( \text{VAR}(p-1) \) process.

\(^7\)Up to March 2008. The hedging P&Ls for the first half of the series should be considered cautiously: they also rely on co-integration relationships estimated on the first half of the data, which means that they use “days in the future”.

On the one hand, the hedging strategy performs reasonably well for Ingersoll Rand with the maximum P&L being around ca. 4% of the $1 notional. As mentioned above, Ingersoll Rand is a medium-grade firm. On the other hand, the hedging strategy was poor for low-rated Ryland. Significant P&L fluctuations are associated with a maximum at ca. 140% of the notional.

Hence, hedging using the statistical approach is unstable and may perform poorly in particular for...
low–rated names. This strategy will be used as a benchmark for the structural model presented next.

V. Structural approach: model

The structural approach used in this paper is based on the CreditGrades model and resorts to simple customizations. CreditGrades results from joint work between RiskMetrics Group, Goldman Sachs, JPMorgan and Deutsche Bank. It is laid out in the CreditGrades Technical Document [5], where it is introduced as a straightforward, “practical implementation of the standard structural model” set out by Merton [6] as well as Black and Scholes [7] (refinements include Black and Cox [8] and Leland [9]). The “standard structural model” defines equity and debt as claims on a stochastic process which may be thought of as the firm’s value. Default occurs if the firm’s value crosses a threshold, called the default barrier. To these foundations, CreditGrades adds continuous monitoring, a stochastic default barrier which is revealed upon default only and practical considerations (definition of equity, debt as well as calibration).

This section reviews the points in the CreditGrades framework that are essential to this paper (following and summarizing the CreditGrades Technical Document [5]) and puts forth the modifications that were brought to it.

A. The CreditGrades framework

The basic assumptions of the model are illustrated on figure [15] which was taken up from [5]. Define a stochastic process $V$ – the firm’s value. It is stipulated that default is the first time that $V$ crosses some threshold – the default barrier. The default barrier may be thought of as the recovery value

that debt holders receive after default, i.e. $L \cdot D$, where $L$ is the average recovery on the debt and $D$ is the firm’s debt. In the following, $V$ and $D$ are expressed on a per–share basis.

In CreditGrades, the firm’s value is a continuous, diffusive process which evolves as geometric Brownian motion under a risk–neutral probability measure:

$$
\frac{dV_t}{V_t} = \sigma dW_t + \mu_D dt
$$

(4)

where $W$ is standard Brownian motion, $\sigma$ is the asset volatility, and $\mu_D$ is the asset drift. For ease of exposition, assume $\mu_D = 0$ (as per [5]).

![Fig. 15: Sketch of the structural model (from [5])](image)

Recovery rates vary widely across default events. In addition, requiring that the firm’s value be continuous unrealistically decreases the default risk on the short–term ($V$ cannot jump below the barrier abruptly). For those reasons, CreditGrades makes a fundamental assumption according to which the average recovery $\bar{L}$ follows a lognormal distribution, with mean $\bar{L}$ and “percentage” standard deviation $\lambda$ in the equations below:

$$
\bar{L} = \ln \frac{1}{\lambda \sqrt{2}}
$$

$$
\lambda^2 = \text{Var} \left[ \log(L) \right]
$$

(5)

(6)

Here, $Z \sim N(0,1)$ is standard normal variable independent of $W$.

For an initial asset value $V_0$, default does not occur as long as

$$
V_t > L \cdot D
$$

(6)

$L$ is released upon default, when $V$ crosses $L \cdot D$. 

8Merton [6] uses the phrase “firm’s value”, whereas the CreditGrades Technical Document uses the phrase “firm’s assets”. The view according to which a firm defaults when its value or assets fall below a threshold is appealing. However, it is obviously a simplified slant on a more intricate reality: a firm defaults when it fails to meet some obligation on its debt and a credit event should be triggered on a CDS when the covenants say it should be. Therefore, the exact meaning of the stochastic process called the firm’s value or the firm’s assets is subject to interpretations. We use the term “firm’s value”, since “assets” may have an accounting connotation.
1) Survival probabilities: The survival probability of the company up to time \( t \) is given by the probability that the asset value \( q \) does not reach the barrier \( L \cdot D \) before time \( t \). Introducing a process \( X_t = \sigma W_t - \lambda Z - \sigma^2 t - \lambda^2 / 2 \), the survival condition can be rewritten as:

\[
X_t > \log \bar{L}D/V_0 - \lambda^2
\]

Notice that for \( t \geq 0 \), \( X_t \) is normally distributed with

\[
\mathbb{E}[X_t] = -\frac{\sigma^2}{2}(t + \frac{\lambda^2}{\sigma^2})
\]

\[
\text{Var}[X_t] = \sigma^2(t + \frac{\lambda^2}{\sigma^2})
\]

CreditGrades “trick” is to assimilate \( X \) to Brownian motion with matching moments. The corresponding Wiener process is assumed to start in the past to tally with the variance at time 0. Quoting [5]: “Intuitively, this approximation replaces the uncertainty in the default barrier with an uncertainty in the level of the firm’s value at time 0.” It also makes it possible to derive the survival probability up to time \( t \), \( P(t) \), in closed–form:

\[
P(t) = \Phi(-A_t^2 + \frac{\log d}{A_t}) - d \Phi(-A_t^2 - \frac{\log d}{A_t})
\]

where

\[
d = \frac{V_0 e^{\lambda^2 / 2}}{\bar{L}D}
\]

\[
A_t^2 = \sigma^2 t + \lambda^2
\]

Fig. 16 (from [5]) illustrates the impact of \( \lambda \) on the annualized probability of default \( p(t) = -\log (P(t))/t \). “As expected, higher uncertainty of debt recovery leads to higher short–term probability of default”. Fig. 16 also exhibits an inversion of the slope as the uncertainty on the debt recovery level \( \lambda \) increases.

2) Pricing CDSs and computing par spreads:

Starting from the survival probability function given by equation [7] above, usual derivations apply to get CDS values and fair credit spreads for a given CDS contract. In particular, the par spread is the spread that equates the premium leg and the default leg. Introducing the recovery \( R \) on the reference obligation (specific to this obligation and possibly distinct from the mean value across debt classes \( \bar{L} \)), the par spread for a CDS with maturity \( t \) may be expressed as

\[
c^* = r(1 - R) \times \ldots
1 - P(0) + e^{r\xi}(G(t + \xi) - G(\xi))
\]

\[
\frac{P(0) - P(t)e^{-rt}}{P(0) - P(t)e^{-rt} - e^{r\xi}(G(t + \xi) - G(\xi))}
\]

where \( \xi = \frac{\lambda^2}{\sigma^2} \), and the function \( G \) is given by Rubinstein and Reiner [10]:

\[
G(u) = d^{z+1/2} \Phi(-\frac{\log d}{\sigma \sqrt{u}} - z \sigma \sqrt{u})
\]

\[
+ d^{-z+1/2} \Phi(-\frac{\log d}{\sigma \sqrt{u}} + z \sigma \sqrt{u})
\]

with \( z = \sqrt{1/4 + 2r/\sigma^2} \). When the spread is \( c \) instead of \( c^* \), the CDS value is non–zero and given by:

\[
CDS = (1 - R)(1 - P(0))
\]

\[
- \frac{c}{r}(P(0) - P(t)e^{-rt})
\]

\[
+ \left(1 - R + \frac{c}{r}\right)e^{r\xi}(G(t + \xi) - G(\xi))
\]

(9)
Note: in formula (9) above, CDS is the marked-to-market value of the CDS (not its par spread, for example).

3) Relationship with market observables: (section 2.2) explains how to link the volatility of the firm’s value $\sigma$ to the volatility of its equity, denoted by $\sigma_S$, which is a more widely-spread concept of volatility in finance:

$$\sigma = \sigma_S \frac{S}{S + LD} \quad (10)$$

Also, for an initial value $V_0$ at time $t = 0$, we have

$$V_0 = S_0 + \bar{L}D \quad (11)$$

where $S_0$ is the current stock price.

4) Debt-per-share: The definition of the debt-per-share used in this paper follows exactly the one in [5], appendix B (see also section IX for details).

B. Customizations

Two customizations were applied to the CreditGrades model as described above.

1) Default barrier: In CreditGrades, there is no conditioning on the fact that the company has not yet defaulted. Hence, for a relatively leveraged company with a non-trivial default barrier, the model implies a significant very short-term (“overnight”) default risk, which may be larger than for longer horizons. For this reason, the standard deviation of the default barrier distribution $\lambda$ is not taken to be constant in this paper. Instead, it increases with leverage according to the formula:

$$\lambda = -0.4 \log(\text{leverage})$$

where leverage = debt/(equity + debt).

2) Volatility: In practice, CreditGrades starts from equity volatility measures and converts them to a volatility figure for the total firm’s value. Since CreditGrades relies on risk-neutral valuation, volatilities should be market-implied, not historical. Moreover, the volatility smile needs to be taken into account. In structural models, credit instrument are akin to deep out-of-the-money put options. The strike of the “option” would be even further out of the money than listed equity options, because typically in a default situation the stock is going to zero. Therefore, the approach in this paper is twofold.

- Take an estimate of the longest and deepest out-of-the-money put option from Bloomberg (e.g. 6-month, 1- or 2-year, 25-delta or 80% moneyness equity- implied volatilities). Data series were checked for obvious signs of illiquidity (not changing for several days in a row, or spikes).
- Construct a forward volatility assumption out to the duration of CDSs. We used data starting in 2010 on long-dated SPX implied volatilities and assumed that the slope of the forward term structure for the individual companies out to the duration of CDSs is the same as that observed for the S&P. To go back before the start of this data (in 2010), empirical relationships between observed less-than-two-year implied volatility and greater-than-two-year implied volatility in the sample was estimated to back-fill counterfactual values for greater-than-two-year implied volatility historically.

VI. STRUCTURAL APPROACH: HEDGING

A. Sensitivities in the structural model

The structural model defines equilibrium relationships between equity prices, credit spreads, debt levels and volatilities (which are translated into variables relating to the value process). Thus, the model produces:

- implied credit spreads as a function of equity price, debt and equity volatility. These implied credit spreads are exactly the fair credit spreads (or par spreads) described in the previous section,
- implied volatilities as a function of equity price, debt and credit spread. These implied volatilities may be used in the same way as implied equity volatilities, for example to compare pricing levels of similar options,
- implied equity prices as a function of debt, equity, volatility and credit spread.

In other words, these four variables – equity price, credit spread, debt and volatility – are not independent in the structural approach. Each of these
variables is determined, i.e. implied – by the three others. The term “credit–implied” will refer to quantities implied in the context of the structural model.

In practice, the structural model does not agree with market quotes: the fair credit spread implied by the structural model at the current equity price, debt level and equity–implied volatility differs from the par spread quoted on the market; similar discrepancies appear for credit–implied volatilities and credit–implied equity prices. Two alternatives may explain these observations.

1. The structural model could depart significantly from reality. The experience of practitioners seems to be that even refined structural models exhibit similar discrepancies.
2. If we do believe that the structural approach is correct, then the other explanation is that one market at least is mispriced. The problem is, in this case, that nothing in the theory gives any indication about which market is mispriced!

Therefore, in the structural model, computing the Greeks is not independent from a choice of an implied quantity.

- There is one Greek for each variable versus the others, e.g. credit delta (bps change in credit per $ change in stock price) or credit vega (bps change in credit per 1% change in volatility).
- Each Greek may be computed at credit–implied spread, credit–implied volatility or credit–implied equity price.

Details are given below in the context of CDSs. The CreditGrades Greeks are presented first, then the actual computation using both the structural model and the ISDA CDS Standard Model is made explicit. Figures are given for a USD 1,000 notional 5-year credit default swap on a firm with a debt–per–share of USD 30 and CDS recovery (called asset specific recovery in 5) R of 30 percent. Greeks are computed with finite difference approximations rather than analytical formulae.

1) Greeks in the CreditGrades model:

a) Delta: The CreditGrades delta represents the $ change of the value of a CDS per $ change in the firm’s equity price and, as usual, can be interpreted as the number of equity shares required to offset small price moves ($S$). Numerically, one may use a centered difference approximation with a second order error term:

$$\Delta(S) = \frac{CDS(S + \delta S) - CDS(S - \delta S)}{2\delta S} + O(\delta S)^2$$

(12)

Figure 17 shows the relationship between delta and the equity spot price in CreditGrades. The number of shares needed to hedge the CDS increases as the spot price drops. Note that other finite difference schemes are admissible. For example, the following difference approximation has a fourth order error term as may be proved by a Taylor expansion and would provides greater accuracy:

$$\Delta(S) = \frac{1}{12\delta S} \times \ldots$$

$$\left\{ \begin{array}{l} CDS(S - 2\delta S) - CDS(S + 2\delta S) \\ + 8 CDS(S + \delta S) - 8 CDS(S - \delta S) \end{array} \right\} + O(\delta S)^4$$

(13)

Favoring one scheme over the other (or choosing to compute every Greek analytically) has an impact – limited in most cases – on hedging “decisions” and on eventual P&Ls.

b) Gamma: The CreditGrades gamma is the second derivative of the CDS value with respect to the spot price. It may be approximated via:

$$\gamma(S) = \frac{1}{(\delta S)^2} \times \ldots$$

$$\left\{ \begin{array}{l} CDS(S + \delta S) - 2 CDS(S) \\ + CDS(S - \delta S) \end{array} \right\} + O(\delta S)^2$$

The order of the error term should be coherent with the one used for deltas. Figure 18 shows that gamma decreases with the stock price.

9This sped computations up and, at least at the level of this paper, provided similarly accurate results.

10Considering the equity price $S$ is quoted with two decimals, taking $\delta S \sim 10^{-6}$ should be adequate.
Fig. 17: Delta versus spot price for a 5–year CDS with $\bar{L} = 0.5$, $\lambda = 0.3$, $D = 30$ and $R = 0.3$

Fig. 18: Gamma versus spot price for a 5–year CDS with $\bar{L} = 0.5$, $\lambda = 0.3$, $D = 30$ and $R = 0.3$

c) Vega: The CreditGrades vega represents the $ change in the CDS value per 1% move in the volatility of the firm’s value. The asset vega can be converted to an equivalent equity vega using 10.

$$v(\sigma) = \frac{-CDS(\sigma + 2\delta\sigma) + 8CDS(\sigma + \delta\sigma)}{12\delta\sigma} - \frac{8CDS(\sigma - \delta\sigma) + CDS(\sigma - 2\delta\sigma)}{12\delta\sigma} \ldots + O(\delta\sigma)^4$$

Not surprisingly, all of the sensitivities shown in Figures 17 to 18 are quite similar to those of an equity put option.

Note – Error in the CreditGrades Technical Document: At this point, the authors of this paper would like to point out that they think there is a mistake in the original CreditGrades paper [5]. Equation 8 is correct, but was probably badly implemented. With the error, the graphs obtained are shown in figure 19 and 20 (reproduced from the CreditGrades Technical Document):

Fig. 19: CreditGrades delta versus spot price for a 5–year CDS with $\bar{L} = 0.5$, $\lambda = 0.3$, $D = 30$ and $R = 0.3$

2) Customizations to the Greeks – combining CreditGrades and the ISDA CDS Standard Model: in this paper, CDS values are computed using the ISDA CDS Standard Model exclusively. Therefore, in the structural approach of this paper, any sensitivity is decomposed multiplicatively into the sensitivity of the CDS value with respect to the spread and the residual sensitivity of the spread.

For example:

$$\frac{\Delta(CDS)}{\Delta(S)} = \frac{\Delta(CDS)}{\Delta(\text{Spread})} \times \frac{\Delta(\text{Spread})}{\Delta(S)}$$

B. Hedging: results

Delta hedging: Hedging strategies using deltas at implied spread and at implied equity are presented
below for a high grade firm (Campbell Soup – CPB) and for a low grade firm (Ryland Homes – RYL). Results for delta–hedging at implied volatility is not printed here as it did not yield comparatively fruitful results.

As we see in fig. 21 & Fig. 22 the hedging strategy (at both implied spread and implied equity) performed well for the high grade firm with maximum P&Ls at 3% and 6.5% of the notional. On the contrary, the strategy performs relatively badly when applied to the low grade firm with maximum P&Ls reaching 30% and 25% of the notional.

_Delta–plus–vega hedging:_ Delta–vega hedging is performed using stock and put options. We compute deltas and vegas at implied spread, implied equity and implied volatility.

Results are in fig. 25 and 26. This technique did not give any better hedge than delta hedging and performed poorly during the crisis scenario of 2008. P&Ls for other firms were worse and are not proffered here.

**VII. Optimizing Hedge Ratios**

This section puts forth two hedging strategies that incorporates and improves the strategies described previously:

1) _Optimal blend of strategies:_ delta hedging using a combination of the three implied parameters: implied spread, implied equity and implied volatility.

2) _Stability method:_ delta hedging using the most stable implied parameters: implied spread, implied equity and implied volatility.

The remainder of this sections gives details on these two strategies.
Fig. 23: P&L at implied spread using the structural model for low grade Ryland Homes (maximum P&L at ca. 30% of the notional).

Fig. 24: P&L at implied equity using the structural model for low grade Ryland Homes (maximum P&L at ca. 25% of the notional).

Fig. 25: P&L at implied equity using the structural model implied vega hedge for medium-grade Ingersoll Rand (maximum P&L at ca. 5% of the notional).

Fig. 26: P&L at implied equity using the structural model implied vega hedge for high-grade Dell (maximum P&L at ca. 5% of the notional).

Optimal blend of strategies: This strategy is based on the estimation of the best linear combination of the models on a rolling basis (past 100 days), blending delta hedge ratios at the three implied quantities (implied spread, implied equity and implied volatility). Let

- \( N_{OBS} \) be the number of stocks from the optimal blend of strategies (yet to be defined),
- \( N_{IS} \) be number of stocks from delta–hedging at implied spread,
- \( N_{IE} \) be number of stocks from delta–hedging at implied equity,
- \( N_{IV} \) be number of stocks from delta–hedging at implied volatility.

\( N_{OBS} \) is estimated through an OLS regression:

\[
N_{OBS} = \alpha N_{IS} + \beta N_{IE} + \gamma N_{IV} + \text{residual} \quad (15)
\]
The regression is run for each day of the series on the past 100 days and the parameters $\alpha, \beta, \gamma$ are re-estimated daily. Fig. 27 compares the P&L given by optimal blends with the P&L at implied equity for medium-grade Ingersoll Rand. We observe that during the crisis, the delta hedge using ratios at implied equity performed poorly compared to the delta hedge from our optimal blend of strategies (which is stable throughout).

Fig. 27: P&L at implied equity using the structural model and optimal blends of strategies for medium-grade Ingersoll Rand (IR). Maximum P&L (implied equity using the structural model) at ca. 17% of the notional and maximum P&L (optimal blend of strategies) at ca. 4% of the notional.

Stability method: This strategy aims at creating a delta hedge using the more stable implied parameter out of the implied spread, the implied equity price and the implied volatility. Indeed, for every firm, implied parameters are unstable throughout the period, with significant jumps during the 2008 crisis. Several solutions could be considered.

1) Take for the initial guess of the optimization routine (root finding) the implied parameter of the day before and add a penalty term for deviations from this guess (for example, the squared distance). It could be the case that very large penalties are necessary to regularize the time series of the implied parameters. This solution is still to be evaluated.

2) Use parametric forms for the implied parameters. Although this does make sense in the equity-implied volatility context, the authors lacked data for a systematic study of the smiles of credit-implied spread, equity price and volatility curves. Data may well be scarce for all market participants.

3) Use the most stable parameter. This strategy was chosen. For each trading day, the strategy consists in computing the variance of the three implied parameters – implied spread, implied equity price and implied volatility – over the past 100 days. The hedge ratio uses the quantity with the lowest variance.

Fig. 28 compares the P&L from the stability method with P&L using hedge ratios at implied equity. The analysis is performed on medium-grade firm Ingersoll Rand (IR). We observe that during the crisis the delta hedge using ratios at implied equity performed poorly when compared to the delta hedge from our stability method (which is stable throughout). At the same time, we are still assessing the pertinence of this strategy and are trying to improve its performance so that it may bring P&L improvements in all circumstances.

VIII. CONCLUSION & FUTURE WORK

This paper put forth two approaches to hedging CDS contracts with equity: one is based on the estimation of statistical relationships, the other on a structural model of debt and equity as claims on a firm’s value. There are significant differences between the performances of the resulting strategies: the statistical approach often performs poorly while the structural approach has some successes.

The success in hedging highly depends on credit ratings. Both the statistical and the structural approaches seem to work better for highly rated firms. For the statistical approach, this could be an effect resulting from stability away from default, making parameter estimation more reliable. For the structural approach, this is surprising, because the behavior that it assumes for credit and equity should be closer to reality when firms are near default.

11 This step was carried out in all optimizations.
Fig. 28: P&L at implied equity using the structural model and the stability method for medium-grade Ingersoll Rand (IR). Maximum P&L (ratios at implied equity using the structural model) at ca. 40% of the notional and Maximum P&L (stability method) at ca. 6% of the notional.

When considering low rated companies, the optimized strategies described in section VII seem to introduce some robustness compared to the statistical and structural approaches.

Future work could, of course, improve the statistical and the structural models. Delta–vega hedging needs to be rethought in the light of its poor performance. Optimized hedging strategies could be refined in order to allow hedges against jump–to–default risk. Finally, the performance and the robustness of the stability method should also be improved.

IX. DATA AND IMPLEMENTATION

This section details our data sources for various items.

1) **CDS spreads**: CDS spreads for 3–year, 5–year and 10–year contracts were obtained from Overland Advisors.

2) **Equity prices**: Equity prices are taken to be the closing prices from Bloomberg (PX_LAST). The data is needed to compute debt–per–shares and are inputs of the structural model. Further, it is also needed for hedging positions.

3) **Equity–implied volatilities**: Equity–implied volatilities are important inputs of the structural model. The approach was described in section V-B2. The Bloomberg fields that were used are, for example: 24MO_PUT_IMP_VOL, 2M_PUT_IMP_VOL_25DELTA_DFLT and 12MTH_IMP_VOL_80%MNY_DF.

4) **Financial debt**: The definition of debt–per–share is the one given by [5]: “The debt-per-share $D$ is based on financial data from consolidated statements.

   - We first calculate all liabilities that participate in the financial leverage of the firm. These include the principal value of all financial debts, short-term and long-term borrowings and convertible bonds. Additionally, we include quasi-financial debts such as capital leases, under-funded pension liabilities or preferred shares. Non-financial liabilities such as accounts payable, deferred taxes and reserves are not included.

   - The equivalent number of shares includes the common shares outstanding, as well as any shares necessary to account for other classes of shares and other contributors to the firm’s equity capital.

   Debt-per-share is then the ratio of the value of the liabilities to the equivalent number of shares.”

Financial debt data is based on consolidated statements items from Bloomberg. We followed the guidelines from appendix B of the CreditGrades Technical Document. Accordingly, debt–per–share calculation uses the following fields from Bloomberg: BS_ST_BORROW (short–term borrowings – interest–bearing financial obligations), BS_LT_BORROW (long–term borrowings – interest–bearing financial obligations), BS_OTHER_ST_LIAB (other short–term liabilities – non–interest bearing obligations), BS_OTHER_LT_LIAB (other long–term liabilities – non–interest bearing obligations), TRAIL_12M_MINORITY_INT (minority interests), CUR_MKT_CAP (current market
value of all classes of common equity) and BS_PFD_EQY (book value of the preferred shares). Financial debt is given by:

\[
\text{Financial Debt} = \text{ST_Borrow} + \text{LT_Borrow} + 0.5 \times \left( \text{Other_ST_Liabilities} + \text{Other_LT_Liabilities} \right)
\]

and debt per share is simply the ratio between financial debt and the number of common shares outstanding. Debt per share is an input of the structural model.

5) **Interest rates:** This paper focused on the US market. All interest rates were taken from the bootstrapped USD zero curves provided by Markit (see [11] for details on how the curves are constructed and where to download them).

6) **Long Dated option-implied volatility data:** Data for long-dated option–implied index (SPX and RAY) volatility data was used to construct the forward volatility assumption out to the durations of CDSs (see section V-B2 for details). Time series run from Jan 27, 2010 to May 4, 2011 and were provided by Overland Advisors.

As mentioned at the beginning of this paper, all the data above except for long dated option–implied volatilities is from Jan 03, 2005 to Apr 14, 2011. A selection of 57 stocks was analyzed. These stocks were chosen to reflect different sectors and different credit ratings. A few stocks were from Europe, although the majority was from the USA and European stocks were eventually excluded from the analysis. In choosing the different sectors, care was taken to include all the specific sectors mentioned in the CreditGrades Technical Document. However, actual proportions were different because of constraints in getting reasonable amount of data for all such companies.

Towards reconciling data pooled from different sources, few adjustments were made before analysis. Some stocks had missing entries for all the fields of entire trading days or for specific field(s) of a particular day. For such fields, the data was imputed from the latest available trading day.

Finally, the programs were written in MATLAB and ran on the Stanford computing systems with MATLAB version 7.9.0 (R2009b).

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REFERENCES


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