Table of Contents

Summary ....................................................................................................................................................... 2
Empirical Observations .................................................................................................................................. 3
Overview of Merton Model ........................................................................................................................... 3
Extension of Merton Model to Index Level ................................................................................................. 5
Data Collection ............................................................................................................................................ 5
SPX-CDX Overlap ........................................................................................................................................... 6
Trading Model Framework ........................................................................................................................... 6
Input Variable Optimization ........................................................................................................................ 9
Merton Model Trading Results .................................................................................................................... 11
Statistical Enhancement to Merton Model .................................................................................................. 13
Enhanced Merton Model Trading Results ................................................................................................. 14
Optimization of Enhanced Model ............................................................................................................. 15
Conclusion and Areas for Further Research ............................................................................................. 15
References .................................................................................................................................................. 16
Summary
Index-based products have enjoyed a recent boom in both variety and trading volume. From virtually all subsectors of the equity market to fixed income asset classes, such as investment grade credit and mortgage-backed credit (in addition to tranches thereof), if the asset class exists, there is probably an index for it. These index products differ tremendously in investor type. For example, institutional fixed income investors utilize the CDX indices to hedge their credit exposures, while retail investors exert a relatively stronger influence on the S&P indices through exchange-traded funds. Another difference is the liquidity of index products; indices such as VIX, for which exchange-traded products exist, enjoy substantially larger trading volumes than those traded over-the-counter such as tranched CDX and LCDX.

Such differences motivate the development of cross-asset class trading strategies that take advantage of historical statistical relationships, fundamental capital structure-based economic relationships, and combinations thereof. In this paper, we develop long-short trading strategies derived from the work of Merton [1974], which provides theoretical relationships between equity, equity volatility, and credit. We then apply the strategies to index products structured primarily based on U.S. investment grade assets.

We find that an optimized Merton-based strategy results in significant trading profits when applied over the span of time for which data is available. Furthermore, we find that trading profits can be enhanced by incorporating information derived from short-term volatility. Given the unlimited number of index combinations spanning different asset classes, geographies and tranche levels, we recommend that further work be allocated to the promising area of capital-structure arbitrage implied index trading.

Figure 1: A Recent History of U.S. Investment Grade Indices
Empirical Observations

Figure 2: Relationship Between Credit, Equity, and Volatility

Figure 2 shows a graphical summary of the relationship among the CDX Investment Grade index, S&P 500, and the VIX. There are a few general qualitative observations from this graph. First of all, high levels of CDX IG are typically accompanied by high levels of volatility. This suggests that volatility may be a reasonable predictor for the credit spread. Another important observation is that the slope of CDX.IG as a function of SPX changes over time and over different market conditions. The time-varying relationship is expected, since credit spread is fundamentally a stationary process, while the equity index is obviously non-stationary. This time-varying relationship makes it difficult to directly use the slope as the hedge ratio in the credit-index index arbitrage.

Overview of Merton Model

In a seminal paper, Merton [1974] proposed a structural model that provides a theoretical relationship between a firm’s equity value and its credit risk. The key concept behind the Merton model is that default occurs when the firm’s asset value falls below its debt value. Hence, investment in a firm’s equity can be viewed as purchasing a call option on the firm’s assets, with the value of the debt as the strike price. The Merton model makes the same assumptions as in the Black-Scholes options pricing framework, such as the log-normal distribution of asset value.
With such assumptions, computing the default probability becomes an exercise in manipulating the Black-Scholes option pricing formula. More specifically, the Merton model uses the Black-Scholes formula to derive two equations with the asset value and volatility as the two unknowns.

\[
(1) \quad E = A\Phi(d_1) - e^{-r\tau}D\Phi(d_2)
\]

\[
(2) \quad \sigma_E = \frac{A}{E}\Phi(d_1)\sigma_A
\]

\[
d_1 = \frac{\ln(A/E) + (r + \sigma_A^2/2)\tau}{\sigma_A\sqrt{\tau}} \quad d_2 = d_1 - \sigma_A\sqrt{\tau}
\]

\(\Phi(.)\): cumulative distribution function of a normal distribution

where the inputs are:

- \(E\): the firm’s equity value
- \(\sigma_E\): the firm’s equity volatility
- \(D\): the firm’s debt value
- \(r\): risk-free rate
- \(\tau\): time to maturity of the firm’s debt

and the unknowns are:
\[ A:\text{the firm's asset value} \]

\[ \sigma_A:\text{the firm's asset volatility} \]

By solving the two nonlinear equations (1) and (2) simultaneously, the firm’s asset value and asset volatility can then be used to compute the distance to default (DD), the default probability (DP), and the credit spread implied by the equity market (\( S_{\text{Merton}} \)).

\[
DD = \ln\left(\frac{A}{E}\right) + \left(r - \frac{\sigma_A^2}{2}\right)\tau - \frac{\sigma_A\sqrt{\tau}}{2}
\]

\[
DP = \Phi(-DD)
\]

\[
S_{\text{Merton}} = \frac{DP}{\tau} \times 10000 \text{ (in basis points)}
\]

**Extension of Merton Model to Index Level**

Since the Merton model in its original formulation was intended to model the credit risk of a single firm, adjustments are necessary when applying the Merton model to the index level. Our methodology treats the S&P 500 index as a single firm. The firm’s equity value and debt are modeled as the aggregated market capitalization and debt of the constituents firms, respectively. Given that volatility skew is an established phenomenon in the options market, we sought to incorporate the volatility measurement associated with extreme downside scenarios. For this reason, the implied volatility of long-dated out-of-the-money put options on the S&P index is used as a proxy for the equity volatility. The time to maturity of debt is computed as the average duration of debt of all constituent firms. Finally, the risk-free interest rate is approximated by the Treasury yield from the corresponding time to maturity.

**Data Collection**

The following describes how we obtained the key inputs to our trading model and assumptions we made:

1. **Debt Time Horizon** – due to the lack of data on precise weighted average debt maturities, we used the following approximation based on available Bloomberg data for each constituent firm:

   \[
   \text{Average debt maturity} = 0.5(\text{Debt Due within 1 year}) + 3(\text{Debt Due between 1 and 5 years}) + 10(\text{Debt Due in more than 5 years})
   \]

2. **Risk Free Rate** – The average debt maturity of the S&P 500 index ranged from a low of 5.6 to a high of 6.4 between April 2003 and May 2011. Consequently, we calculated the appropriate US Treasury yield for each trading day based on an interpolation of 5-year and 7-year US Treasury yields.
3. Equity Market Capitalization – we aggregated the daily market capitalization of each S&P 500 constituent.

4. Debt Value – we summed the total debt value for each trading day based on the most recent quarter-end filings of each S&P 500 constituent.

5. Implied Equity Volatility – we calculated the implied equity volatility by taking the average Black-Scholes implied volatility of all long-dated (360 day+ to expiration) S&P 500 index options with maximum 80% moneyness (ratio of strike price to current index level).

**SPX-CDX Overlap**

Figure 4 illustrates the overlap of total debt and market capitalization between the SPX and CDX-IG indices, calculated based on the daily overlap of the indices at the holding company level over time. Although the overlap in total debt between the two indices has gradually declined over time, the overlap in market capitalization has remained fairly constant, at an average of 15%.

**Figure 4: Calculated Overlap in Total Debt and Market Capitalization between the SPX and CDX-IG**

Trading Model Framework

Figure 5 exhibits the spreads calculated by our Merton model compared to the historical actual spreads of the on-the-run 5-year CDX investment grade index. While absolute differences in spread exist, the two series appear to exhibit significant similarities in relative changes over time.
We apply the following formula to calculate the divergence value (DV) for each trading day in the series.

\[
DV = \ln \frac{S_{\text{Market}}}{\bar{S}_{\text{Market}}^{\lambda_{\text{days}}}} - \ln \frac{S_{\text{Merton}}}{\bar{S}_{\text{Merton}}^{\lambda_{\text{days}}}}
\]

where:

- \(S_{\text{Merton}}\) and \(S_{\text{Market}}\) are the current Merton model spread and actual market 5-year on-the-run CDX-IG spread, respectively; and
- \(\bar{S}_{\text{Merton}}^{\lambda_{\text{days}}}\) and \(\bar{S}_{\text{Market}}^{\lambda_{\text{days}}}\) are the moving average of the Merton model and actual spreads, respectively, over the prior \(\lambda_{\text{days}}\) calendar days.

Figure 5: Calculated Merton model-based spreads versus historical spreads

Figure 6: Divergence values over time for the Merton model-based trading strategy
We subsequently define the indicator function for the event $LC_t^{in}$ corresponding to the entrance of a long credit / short equity trade on day $t$ as follows:

$$LC_t^{in} = 1\left\{ \left( \frac{DV}{\sigma_{DV}} _t \right) > \sigma_{Thresh}, \left( \frac{DV}{\sigma_{DV}} _{t-1} \right) \leq \sigma_{Thresh}, \rho_{\delta_{Days}} \geq \rho_{Thresh} \right\} \times \left( 1 - LC_{t-1} \right)$$

where:

- $\sigma_{DV}$ = the standard deviation of the DV value over the past 180 calendar days;
- $\sigma_{Thresh}$ = the threshold for a trade entrance event;
- $\rho_{\delta_{Days}}$ = the correlation between CDX and SPY prices over the prior $\delta_{Days}$ calendar days; and
- $\rho_{Thresh}$ = the minimum correlation threshold.

We also define the indicator function for event $LC_t^{out}$ corresponding to the exit of a long credit / short equity trade on day $t$ as follows:

$$LC_t^{out} = \left[ 1\left\{ \left( \frac{DV}{\sigma_{DV}} _t \right) < 0, \left( \frac{DV}{\sigma_{DV}} _{t-1} \right) \geq 0 \right\} OR 1\left\{ LC_{days} \geq 180 \right\} \right] \times (LC_{t-1})$$

where:

- $LC_t$ = 1 if a long credit / short equity trade existed at the end of day $t$ and 0 otherwise; and
- $LC_{days}$ = calendar days for which long credit trade has been outstanding.

Similarly, events corresponding to the entry and exit of long equity / short credit trades are defined as follows:

$$LE_t^{in} = 1\left\{ \left( \frac{DV}{\sigma_{DV}} _t \right) < -\sigma_{Thresh}, \left( \frac{DV}{\sigma_{DV}} _{t-1} \right) \geq -\sigma_{Thresh}, \rho_{\delta_{Days}} \geq \rho_{Thresh} \right\} \times \left( 1 - LE_{t-1} \right)$$

$$LE_t^{out} = \left[ 1\left\{ \left( \frac{DV}{\sigma_{DV}} _t \right) > 0, \left( \frac{DV}{\sigma_{DV}} _{t-1} \right) \leq 0 \right\} OR 1\left\{ LE_{days} \geq 180 \right\} \right] \times (LE_{t-1})$$

On each day in which a trade entry event $LC_t^{in}$ or $LE_t^{in}$ occurs, we purchase or sell a number of shares of the SPDR S&P 500 ETF (NYSE: SPY) equal to an assumed fixed trade amount of $1 million divided by the dividend-adjusted closing price.

We then sell or purchase contracts of the current on-the-run 5-year investment grade CDX.IG in an amount equal to (number of SPY shares) X (hedge ratio $HR$), in which
Input Variable Optimization

The profit and loss of our strategy is not a continuous function of the different parameter inputs. Consequently, we analyzed the sensitivity of the strategy to different parameter inputs in order to determine optimal ranges for each input. We discretized the parameters into intervals of approximately 5 days for $\lambda_{Days}$ and $\delta_{Days}$ and 0.05 for $\rho_{Thresh}$ and $\sigma_{Thresh}$. First, we ran a global optimization in which we maximized the final cumulative P&L subject to three constraints: 1) positive gross P&L, 2) positive P&L in at least 5 of 9 trading years, 3) maximum loss of no more than 10% in any single year. From this result, the best parameters were the following:

<table>
<thead>
<tr>
<th>$\lambda_{Days}$</th>
<th>$\delta_{Days}$</th>
<th>$\rho_{Thresh}$</th>
<th>$\sigma_{Thresh}$</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>180</td>
<td>0.5</td>
<td>0.25</td>
<td>82.7%</td>
</tr>
</tbody>
</table>

Then, we ran a walk-forward backtesting. We selected the parameters that maximized profit for a given year and then tested to see what the P&L would be for the following year. Our results are as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>$\lambda_{Days}$</th>
<th>$\delta_{Days}$</th>
<th>$\rho_{Thresh}$</th>
<th>$\sigma_{Thresh}$</th>
<th>Return</th>
<th>Year</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>25</td>
<td>90</td>
<td>0.5</td>
<td>0.35</td>
<td>29.4%</td>
<td>2004</td>
<td>-3.8%</td>
</tr>
<tr>
<td>2004</td>
<td>25</td>
<td>70</td>
<td>0.05</td>
<td>0.20</td>
<td>10.5%</td>
<td>2005</td>
<td>6.4%</td>
</tr>
<tr>
<td>2005</td>
<td>25</td>
<td>90</td>
<td>0.40</td>
<td>0.10</td>
<td>13.3%</td>
<td>2006</td>
<td>9.2%</td>
</tr>
<tr>
<td>2006</td>
<td>60</td>
<td>70</td>
<td>0.50</td>
<td>0.30</td>
<td>14.8%</td>
<td>2007</td>
<td>3.0%</td>
</tr>
<tr>
<td>2007</td>
<td>30</td>
<td>80</td>
<td>0.45</td>
<td>0.20</td>
<td>8.9%</td>
<td>2008</td>
<td>20.3%</td>
</tr>
<tr>
<td>2008</td>
<td>30</td>
<td>90</td>
<td>0.50</td>
<td>0.35</td>
<td>29.1%</td>
<td>2009</td>
<td>0.4%</td>
</tr>
<tr>
<td>2009</td>
<td>60</td>
<td>120</td>
<td>0.05</td>
<td>0.10</td>
<td>42.2%</td>
<td>2010</td>
<td>4.0%</td>
</tr>
<tr>
<td>2010</td>
<td>35</td>
<td>70</td>
<td>0.50</td>
<td>0.20</td>
<td>17.7%</td>
<td>2011</td>
<td>-2.1%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total</td>
<td></td>
<td>37.4%</td>
</tr>
</tbody>
</table>
Last, we ran 30,800 iterations across four parameters in discrete intervals as shown:

<table>
<thead>
<tr>
<th>( \lambda_{\text{Days}} )</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_{\text{Days}} )</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>100</td>
<td>110</td>
<td>120</td>
<td>150</td>
<td>180</td>
<td>270</td>
<td>360</td>
</tr>
<tr>
<td>( \rho_{\text{Thresh}} )</td>
<td>.05</td>
<td>.10</td>
<td>.15</td>
<td>.20</td>
<td>.25</td>
<td>.30</td>
<td>.35</td>
<td>.40</td>
<td>.45</td>
<td>.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_{\text{Thresh}} )</td>
<td>.05</td>
<td>.10</td>
<td>.15</td>
<td>.20</td>
<td>.25</td>
<td>.30</td>
<td>.35</td>
<td>.40</td>
<td>.45</td>
<td>.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We assessed the “profitability ratio” of each parameter value, defined as the number of profitable scenarios divided by total number of scenarios for a given parameter. For \( \lambda_{\text{Days}} \), the 22 discrete levels each had 1,400 combinations of the other three parameters. We observe that a range of 25 to 85 days yields the highest profitability ratios, of between 79.4% and 99.9%, with the maximum profitability ratios at 30 and 35 days.

**Figure 7: Profitability Percentage versus Moving Average Calculation Days**

![Figure 7: Profitability Percentage versus Moving Average Calculation Days](image)

We then examined the impact of the \( \rho_{\text{Thresh}} \) and \( \sigma_{\text{Thresh}} \) constraints. A minimum \( \rho_{\text{Thresh}} \) of 0.05 was found to have a significantly lower profitability ratio (51.8%) than higher thresholds, which is consistent with the model’s premise of correlation between credit and equity. Adjusting \( \sigma_{\text{Thresh}} \) did not seem to affect profitability ratios, although this constraint may impact the level of profit generated by the strategy.

Finally, we assessed the impact of the hedge ratio lookback period, \( \delta_{\text{Days}} \). A window of less than 60 days produced unfavorable profitability ratios of between 29.3% and 43.6%. Parameter values for \( \delta_{\text{Days}} \) of greater than 60 days were generally favorable, with the most favorable windows observed between 70 and 120 days and between 270 and 360 days. These specific time windows became even more favorable when we restricted our analysis to scenarios that involved \( \lambda_{\text{Days}} \) of 25 to 85 days and a minimum \( \rho_{\text{Thresh}} \) of 0.10. Collectively, profitability ratios increased 13 to 19 percentage points on the
shorter window and 5 to 6 percentage points for the longer window. The following chart shows the profitability ratio as a function of $\delta_{\text{Days}}$.

**Figure 8: Profitability Percentage versus Hedge Ratio Calculation Days**

For the final step of our sensitivity analysis, we restrict the range of $\lambda_{\text{Days}}$ to between 30 and 80 days, $\delta_{\text{Days}}$ to between 70 and 360 days, and $\rho_{\text{Thresh}}$ to greater than 0.05. This combination of parameters resulted in 9,900 scenarios out of the original 31,800 iterations that we ran. Of these 9,900 scenarios, 94.9% are profitable, and the median cumulative return is 25.9%. Subject to the aforementioned parameter ranges, our results suggest that the general profitability (i.e. profit > $0$) of our Merton Model strategy is relatively robust to the specific choice of parameter inputs, although specific choice of inputs may impact the amount of profit achieved.

**Merton Model Trading Results**

We utilized the following model input values to illustrate a typical trading strategy implemented over time:

$\sigma_{\text{Thresh}} = 0.4$

$\sigma_{\text{days}} = 80$

$\lambda_{\text{days}} = 30$

$\rho_{\text{Thresh}} = 0.10$

The strategy appears to generate reasonably consistent profits throughout the past 7+ years despite the significant change in market environment over this time.
Figure 9: Cumulative Log Return of the Merton model-based trading strategy

Figure 10: Illustrative statistics for the Merton model-based trading strategy

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of long credit / short equity trades</td>
<td>54</td>
</tr>
<tr>
<td>Number of long equity / short credit trades</td>
<td>54</td>
</tr>
<tr>
<td>Average trade duration</td>
<td>16 days</td>
</tr>
<tr>
<td>Active trading days</td>
<td>70%</td>
</tr>
<tr>
<td>Cumulative log return</td>
<td>59.9%</td>
</tr>
<tr>
<td>Highest log return</td>
<td>6.9%</td>
</tr>
<tr>
<td>Lowest log return</td>
<td>-5.7%</td>
</tr>
<tr>
<td>Median log return</td>
<td>-0.37%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2.16%</td>
</tr>
<tr>
<td>% of profitable trades</td>
<td>60%</td>
</tr>
</tbody>
</table>

Figure 11: Per-trade log returns for the Merton model-based trading strategy
Statistical Enhancement to Merton Model

After detailed analysis of the trading results from the preliminary strategy, we made two observations that provided us with clues on the direction of potential improvements. First, there is a considerable mismatch between the predicted and actual spreads, even though the predicted spread matches the general movement of the actual spread fairly closely. Second, the model is not sufficiently sensitive to movements in the equity market, in particular during certain periods of high, short-term volatility where arbitrage opportunities were abundant.

To improve the shortcomings of the current strategy based on the simple Merton model, we propose a statistical model enhancement that incorporates a short-term volatility measure:

\[ S_{\text{Enhanced}} = \beta_1 \times S_{\text{Merton}} + \beta_2 \times VIX + \epsilon \]

This simple linear regression model offers two purposes. The first regression coefficient serves as a scaling factor to adjust the predicted level from the Merton model to better match the actual level. The second regression coefficient introduces a short-term reaction component to the prediction by regressing against the VIX.

Overall, the enhanced model improves the fit to the actual model. As shown in Figure 12 and Figure 13, the predicted spread matches the actual spread much more closely compared to the simple Merton model. The divergence value based on the enhanced model also appears to be more stationary, i.e. the variance appears to be constant across time.

Figure 12: Merton model, Enhanced Merton model and historical spreads
Enhanced Merton Model Trading Results

We also backtested the trading strategy using the enhanced model. The enhancement successfully improves the shortcomings of the original strategy by responding more rapidly to short-term market movement. As a result, the enhanced strategy generates more trading opportunities, which makes the results more statistically robust. In addition, it also improves the overall profitability, as evidenced by the higher percentage of profitable trades and higher median return, at the expense of slight increase in volatility.

Figure 14: Cumulative Log Return, Enhanced Merton versus Merton
Figure 15: Illustrative statistics for the Enhanced Merton model-based trading strategy

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of long credit / short equity trades</td>
<td>65</td>
</tr>
<tr>
<td>Number of long equity / short credit trades</td>
<td>77</td>
</tr>
<tr>
<td>Average trade duration</td>
<td>12 days</td>
</tr>
<tr>
<td>Active trading days</td>
<td>70%</td>
</tr>
<tr>
<td>Cumulative log return</td>
<td>70.3%</td>
</tr>
<tr>
<td>Highest log return</td>
<td>22.6%</td>
</tr>
<tr>
<td>Lowest log return</td>
<td>-20.1%</td>
</tr>
<tr>
<td>Median log return</td>
<td>-0.56%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>3.60%</td>
</tr>
<tr>
<td>% of profitable trades</td>
<td>64%</td>
</tr>
</tbody>
</table>

Figure 16: Per-trade log returns for the Enhanced Merton model-based trading strategy

![Per-trade log returns for Enhanced Merton model-based trading strategy](image)

Optimization of Enhanced Model

We ran an optimization of the enhanced Merton strategy to see how sensitive the profitability of the strategy is to parameter selection. Overall, the enhanced Merton strategy was profitable in 99.7% of all 30,800 iterations of the parameters ($\lambda_{Days}$, $\delta_{Days}$, $\rho_{Thresh}$, and $\sigma_{Thresh}$). This is a substantial improvement over the 77.3% profitability ratio of our original Merton strategy. Moreover, the median return across all iterations improved from 16.1% to 46.4%, which suggests that the enhanced model is especially insensitive to the precise set of parameters used.

Conclusion and Areas for Further Research

In this paper, we have demonstrated that a long-short trading strategy based on the divergence between equity-implied credit risk and U.S. investment grade credit index can produce significant
trading profits over a variety of market environments. We believe that this is a promising area of research and recommend that resources be devoted to extending the work performed in this paper.

We have not adequately incorporated transaction costs and funding costs into these results. In practice, these costs will erode the returns of the strategy, though the cumulative returns would still be positive. Furthermore, we postulate that similar trading strategies on other geographic area (i.e. utilizing European and Asian equity and credit indices) may result in even more robust profits due to lower liquidity and efficiency in those markets. Finally, while we have focused on the most basic trading instruments (outright long/short of the credit and equity indices), we recognize that utilizing other instruments such as tranched indices or volatility indices may allow one to express more nuanced views.

References

Wharton Research Data Services (WRDS) was used in preparing this piece. Implied volatilities of options on the S&P 500 index and the constituents of the S&P 500 index were obtained from WRDS. CDX constituents, market capitalizations, debt levels, and other fundamental data were obtained from Bloomberg.


