Bioe225/Rad225

Intro to Ultrasound Imaging and Therapeutic Applications
Course Goals

✦ to learn the fundamentals of each ultrasound imaging, from wave equations to applications.
✦ to learn the fundamentals of therapeutic applications of ultrasound, including thermal ablation, drug delivery, lithotripsy.
Radiology 225

vs

Bioengineering 225

✦ no difference
✦ check your grading basis
  - you can sign up for regular grades, med center grades, CR/NC
  - only the med students should sign up for med center grades
Lecturers

Kim Butts Pauly, PhD

Kasra
TA

- Kasra Naftchi-Ardebili
- knaftchi@stanford.edu
- office hours TBD

- Kim Butts Pauly
- kbpauly@stanford.edu
- please email to schedule a meeting
Lectures posted on the web in Adobe Acrobat (.pdf) format at:

http://rad225.stanford.edu or http://bioe225.stanford.edu

Password protected page for handouts

All homeworks, announcements etc. will be via website
Textbooks

Zagzebski: very intuitive, diagnostic ultrasound
You should know every line of this book.

Szabo - overview of everything, more in depth,
available to SU for free

Shung: additional in depth source,
available to SU for free

Duck: variety of topics,
available to SU for free

Therapeutic US

Diagnostic US

Biophysics
Grading

- Homework Problem Sets -- including Python/MATLAB (open book)
  - 8 Problem Sets, Due on Fridays at midnight
  - TA review/office hours: TBD

- Exam: 1 Midterm (closed book)
  - Material in lecture, in addition to the homeworks

Grading

- Problem sets: (30% of grade)
- Midterm Exam (35% of grade)
- Final Exam (35% of grade)
Welcome
Feel free to turn in homeworks and take the exam
Request --
please keep your questions to the material presented in lecture,
ask other questions after class
Prerequisites

- basic physics
- some signal processing and/or Fourier transforms
- some scientific programming background with Python/MATLAB
Course Logistics Questions?
Diagnostic Ultrasound

- Cheap
- Portable
- Safe
- Easy
- A lot of information
Fluid doesn’t have speckle
Real-Time Imaging

[Image of an ultrasound scan]
More Fetal Ultrasound Examples
Transducer Converts Electrical Signal to Pressure Wave

1. An oscillating electric field is applied
2. This realigns the dipoles in a piezoelectric crystal
3. Causing the crystal to change shape
4. It bumps up against the tissue
5. Creating a pressure wave
Transducer Creates Pressure Wave

- Placed next to tissue, the transducer creates a pressure wave - the ultrasound pulse.

- Limited duration sine wave

- Common frequencies:
  Diagnostic: 1-10 MHz
  Therapeutic: 200kHz - 10 MHz
Beam Formation

Transducer

Multiple Elements

Spherical pattern

Interference Patterns
Beam Sensitivity

Single Element

5 Elements

5 Elements

Inner Elements Delayed

beam gets wider: resolution worse at depth

Grating Lobes

Thinnest Beam

Class 8 ➔ Beam Formation
Imaging Transducers

Translate which elements are used

Image one line at a time

Linear

Rectangular image
Imaging Transducers

Beam Steering

Phased Array

Fan Shaped Image

Class 8,9,10 ➞ Linear, Phased, and Focused Transducers
How is Imaging Done?
How is Imaging Done?
How is Imaging Done?

- Pulse, listen
- Sound bounces back from reflectors all along the way

Amplitude is given a greyscale value: one line of the image
Repeat with a new line to build up the image

$$\text{distance} = v \times t$$
assume $$v = 1540 \text{ m/s}$$
Doppler

Class 13,14 ➔ Doppler Imaging
Nonlinearity, Tissue Harmonic Imaging, Contrast Agent Imaging

\[ c_0 + \beta u \]
\[ c_0 - \beta u \]

\( u = \) particle velocity

direction of propagation

Class 18 ➔ Nonlinearity
Thermal Effects

- large area ultrasound transducer array outside the body
- focused geometrically or electronically
- amplification
- significant intensities deep within the body, lower intensities in intervening tissues
- ultrasound absorbed as heat, cooks tissue in place
Elastography and Mechanical Interactions

Shear Waves

Acoustic Radiation Force
Lithotripsy

Nonlinear “Shock” Wave

Break up kidney stones

Class 27 ➫ Lithotripsy and Histotripsy
Bubbles

Oscillation = Stable Cavitation

Stable cavitation
Stable Cavitation
Bubbles

Inertial Cavitation

Figure 1: The process of Acoustic Cavitation.

http://www.ems007.com/pages/zone.cgi?a=57265
Jetting

Movie courtesy of Larry Crum

Class 19 ➞ Bubbles, Cavitation, and nonlinearity
Pistol Shrimp

http://www.youtube.com/watch?v=XC6I8iPiHT8

Class 19 »» Bubbles and Cavitation
Safety, Measurement Devices, Phantoms

MI = Mechanical Index
TI = Thermal Index

TI = 1 => exposure leading to 1°C temperature rise
Tls = TI of soft tissue
Tlb = TI of bone

Class 28 ⊳ Safety, Measurement Devices, Phantoms
Fourier Series (review)

Question: can we reconstruct a periodic function such as the one below with a series of complex exponentials?

Fourier series representation of $f(t)$:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{2\pi int/T}$$

Complex Fourier coefficients:

$$c_n = \frac{1}{T} \int_{0}^{T} e^{-2\pi int/T} f(t) dt$$
Fourier Series (review)

A few remarks:

n=0 is essentially the average of f(t) and is called the fundamental:

\[ c_0 = \frac{1}{T} \int_0^T f(t) dt \]

n = +/- 1 is the first harmonic:

\[ c_1 = \frac{1}{T} \int_0^T e^{-2\pi i t/T} f(t) dt \]

n = +/- 2 is the second harmonic:

\[ c_2 = \frac{1}{T} \int_0^T e^{-2\pi i 2t/T} f(t) dt \]

and so on …
Fourier Series (review)

A few remarks:

It is common to write the Fourier coefficients as follows:

$$c_1 = \frac{1}{\sqrt{T}} \int_0^T e^{-2\pi int/T} f(t) dt$$
Fourier Series (review)

We are almost there. We just need to further simplify this infinite Fourier series so that we can start reconstructing the original square wave signal.
Fourier Series (review)

\[ 1 - e^{-\pi in} = \begin{cases} 
0 & n \text{ even} \\
2 & n \text{ odd} 
\end{cases} \]

\[ \sum_{n \neq 0} \frac{1}{\pi in} \left( 1 - e^{-\pi in} \right) e^{2\pi int} = \sum_{n \text{ odd}} \frac{2}{\pi in} e^{2\pi int} \]

Note the summation is over all odd numbers, negative and positive. Using the following identity we can further simplify this expression:

\[ e^{2\pi int} - e^{-2\pi int} = 2i \sin 2\pi nt \]

\[ \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{1}{2k + 1} \sin 2\pi (2k + 1)t \]

And that is it! That is the infinite Fourier series of the square wave function of period 1. Do not worry too much about the math, but focus on the results and specifically for this course, the application.
Fourier Series (review)

\[ f(t) = \begin{cases} 
  +1 & 0 \leq t < \frac{1}{2} \\
  -1 & \frac{1}{2} \leq t < 1 
\end{cases} \]

\[
\sum_{k=0}^{\infty} \frac{1}{2k + 1} \frac{1}{\pi} \sin(2\pi(2k + 1)t)
\]
Important note: now that we have transitioned from Fourier series to Fourier transforms, the objective is no longer to reconstruct a periodic function. We are mainly interested in the Fourier coefficients, which help us understand the frequencies that build up the signal (its **frequency spectrum**)

Question: what if the signal is not periodic? Can we generalize the infinite Fourier series expansion?

Let’s start with an example.
Fourier Transforms (review)

This rect function is not periodic. Therefore, it doesn’t have a Fourier series. However, we can artificially periodize it:
For Fourier Transforms (review)

\[ c_n = \frac{1}{T} \int_0^T e^{-2\pi i n t/T} f(t) dt = \frac{1}{T} \int_{-T/2}^{T/2} e^{-2\pi i n t/T} f(t) dt \]

After doing a few manipulations:

\[ c_n = \frac{\sin(\pi n/T)}{\pi n/T} \]

Note the pattern: as we increase the period, the partitioning between the frequency components becomes finer.

You can imagine that as we increase the period to infinity (effectively reinstating the non-periodic nature of the original signal), the frequency spectrum becomes continuous.
Fourier Transforms (review)

As we let T approach infinity, n/T becomes a continuous variable. Let’s call it s:

\[
\lim_{T \to \infty} c_n = \frac{\sin(\pi s)}{(\pi s)}
\]

This function is called a sinc. It is the Fourier transform of a rect function! There you have it. We did the first Fourier transform through an example :)
Fourier Transforms (review)

Formal definition and some theorems:

Fourier transform of a signal \( f(t) \):
\[
\mathcal{F}f(s) = \int_{-\infty}^{\infty} f(t)e^{-2\pi ist} \, dt
\]

Inverse Fourier transform of \( f(s) \):
\[
\mathcal{F}^{-1}f(t) = \int_{-\infty}^{\infty} f(s)e^{2\pi ist} \, ds
\]

Linearity:
\[
\mathcal{F}(f + g)(s) = \mathcal{F}f(s) + \mathcal{F}g(s)
\]
\[
\mathcal{F}(\alpha f)(s) = \alpha \mathcal{F}f(s)
\]

Shift theorem:
\[
f(t \pm b) \Leftrightarrow e^{\pm 2\pi ist}f(s)
\]

Stretch (similarity) theorem:
\[
f(\alpha t) \Leftrightarrow \frac{1}{|\alpha|} f\left(\frac{s}{\alpha}\right)
\]
Fourier Transforms (review)

Examples that are relevant to the course:

A pulsed ultrasound wave is composed of many frequencies, as revealed by the frequency spectrum on the right:

![Time Domain](image1)

### Frequency Domain

$$a_0 = A \cdot d$$
$$a_n = \frac{2A}{n \pi} \sin(n \pi d)$$
$$b_n = 0$$

(d = 0.27 in this example)

A simple continuous ultrasound beam on the other hand, is composed of one frequency only:

![Time Domain](image2)

### Frequency Domain

$$a_1 = A$$

(all other coefficients are zero)
Fourier Transforms (review)

Some Fourier transform pairs:

- **(a)** Sine/cosine wave
- **(b)** Top hat
- **(c)** Single frequency
- **(d)** Sinc function

As we will see later, lateral pressure profile and aperture function are Fourier transform pairs. The sinc-like lateral pressure profile of a rectangular aperture therefore, is due to the fact that Fourier transform of a rect (top hat) function is a sinc!
Questions?