From Last Time

Why do we use a coupling gel?
Reflection and Refraction

- Reflection
  - normal incidence
  - examples
    - non-normal incidence - Refraction
- Transmission through skull for ET treatment
- Standing Waves
Reflection and Refraction

✦ Reflection
  ✦ normal incidence
  ✦ examples
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✦ Transmission through skull for ET treatment
✦ Standing Waves
# Acoustic Impedance

\[ Z = \rho c \]

<table>
<thead>
<tr>
<th>Material</th>
<th>Density ( \rho ) (kg/m(^3))</th>
<th>Speed of Sound ( c ) (m/s)</th>
<th>Impedance (MRayl) ( Z ) ( \times 10^6 ) kg/m(^2)/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air (25°C)</td>
<td>1.16</td>
<td>344</td>
<td>0.0004</td>
</tr>
<tr>
<td>Water (22°C)</td>
<td>998</td>
<td>1482</td>
<td>1.48</td>
</tr>
<tr>
<td>Blood</td>
<td>1060</td>
<td>1584</td>
<td>1.68</td>
</tr>
<tr>
<td>Skeletal muscle</td>
<td>1041</td>
<td>1580</td>
<td>1.65</td>
</tr>
<tr>
<td>Liver</td>
<td>1050</td>
<td>1578</td>
<td>1.64</td>
</tr>
<tr>
<td>Kidney</td>
<td>1050</td>
<td>1560</td>
<td>1.64</td>
</tr>
<tr>
<td>Fat</td>
<td>928</td>
<td>1430</td>
<td>1.33</td>
</tr>
<tr>
<td>Bone</td>
<td>1600</td>
<td>3360</td>
<td>5.69</td>
</tr>
</tbody>
</table>

Szabo
Boundaries

- Pressure is *not* a conserved quantity
- Pressure on each side of boundary must be equal (the boundary doesn’t supply any resistance)
- Perpendicular velocities must equal (or the particles would pull away from each other)

\[ 1 + R_P = T_P \]

- Power is a conserved quantity

\[ 1 = R_I + T_I \]
Reflected Pressure Ratio, $R_p$

Reflected pressure ratio $R_p$ may be positive or negative $R_p < 0$, inverted phase

$$R_p = \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$
Transmitted Pressure Ratio, $T_p$

- Incident pressure $P_i$
- Reflection pressure $P_r$
- Pressure on each side of the boundary must be equal

Transmission $P_t$

\[ 1 + R_p = T_p \]

$R_p$ may be positive or negative

$T_p$ may be greater than 1

Transmitted pressure ratio

\[ T_p = \frac{Z_2 + Z_1}{Z_2 + Z_1} + \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{2Z_2}{Z_2 + Z_1} \]
Reflected Intensity Ratio, $R_I$

$$R_I = \frac{I_r}{I_i} = \frac{P_r^2 / Z_1}{P_i^2 / Z_1} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

incident $P_i$

reflection $P_r$

transmission $P_t$
Transmission Intensity Ratio, $T_I$

\[
1 = R_I + T_I
\]

\[
T_I = \frac{(Z_2 + Z_1)^2}{(Z_2 + Z_1)^2} - \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} = \frac{4Z_2Z_1}{(Z_2 + Z_1)^2}
\]
Summary

incident \( P_i \)

reflection \( P_r \)

transmission \( P_t \)

reflected

transmitted

\[
1 + R_p = T_p
\]

\[
R_p = \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1}
\]

\[
T_p = \frac{P_t}{P_i} = \frac{2Z_2}{Z_2 + Z_1}
\]

\[
I = \frac{P^2}{Z}
\]

\[
R_I = \frac{I_r}{I_i} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}
\]

\[
T_I = \frac{I_t}{I_i} = \frac{4Z_2Z_1}{(Z_2 + Z_1)^2}
\]

\[
1 = R_I + T_I
\]
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Example: Liver-Fat

incident

$P_i$

reflection

$P_r$

transmission

$P_t$

$Z_1 = 1.38$ (fat)

$Z_2 = 1.65$ (liver)

reflected

transmitted

incident pressure

reflection intensity

transmission intensity

$R_p = \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$

0.09

$T_p = \frac{P_t}{P_i} = \frac{2Z_2}{Z_2 + Z_1}$

1.09

$R_i = \frac{I_r}{I_i} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$

0.01

$T_i = \frac{I_t}{I_i} = \frac{4Z_2Z_1}{(Z_2 + Z_1)^2}$

0.99

>Sums to 1

>1
Example: Fat-Air

\[ Z_1 = 1.38 \text{ (fat)} \]

\[ Z_2 = 0.0004 \text{ (air)} \]

incident
\[ P_i \]

reflection
\[ P_r \]

transmission
\[ P_t \]

\[ R_p = \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]

-0.9999

\[ T_p = \frac{P_t}{P_i} = \frac{2Z_2}{Z_2 + Z_1} \]

0.0001

\[ R_I = \frac{I_r}{I_i} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} \]

0.9998

\[ T_I = \frac{I_t}{I_i} = \frac{4Z_2Z_1}{(Z_2 + Z_1)^2} \]

0.0001

\(<1\)

reflected
transmitted

pressure

intensity
Example: Fat-Bone

incident

reflection

transmission

\[ Z_1 = 1.38 \text{ (fat)} \]

\[ Z_2 = 5.69 \text{ (bone)} \]

\[ \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]

\[ R_P = 0.61 \]

\[ \frac{P_t}{P_i} = \frac{2Z_2}{Z_2 + Z_1} \]

\[ T_P = 1.61 \]

\[ \frac{I_r}{I_i} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} \]

\[ R_I = 0.37 \]

\[ \frac{I_t}{I_i} = \frac{4Z_2Z_1}{(Z_2 + Z_1)^2} \]

\[ T_I = 0.63 \]

Sums to 1

> 1

reflected

transmitted
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Non-normal Incidence

\[ \theta_i = \theta_r \]

\[ \theta_t = ? \]
Snell’s Law

\[ \frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{c_1}{c_2} \]

- Refraction is necessary to maintain alignment of the wavefronts
Snell’s Law

\[
\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{c_1}{c_2}
\]

where

\[
c_1 > c_2
\]

\[
\theta_i > \theta_t
\]

Imagine car, one wheel off road in gravel (slow), will turn into the gravel
Snell’s Law

\[
\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{c_1}{c_2}
\]

\(c_1 < c_2\)

\(\theta_i < \theta_t\)
Snell’s Law

\[ \frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{c_1}{c_2} \]

\[ \theta_i = 10^\circ \]
\[ \theta_{t1} = 20^\circ \]
\[ \theta_t = 10^\circ \]
Non Normal Incidence

Reflection and transmission equations:

\[
R_p = \frac{Z_2 \cos(\theta_i) - Z_1 \cos(\theta_t)}{Z_2 \cos(\theta_i) + Z_1 \cos(\theta_t)}
\]

\[
T_p = \frac{2Z_2 \cos(\theta_i)}{Z_2 \cos(\theta_i) + Z_1 \cos(\theta_t)}
\]

\[
R_I = \left(\frac{Z_2 \cos(\theta_i) - Z_1 \cos(\theta_t)}{Z_2 \cos(\theta_i) + Z_1 \cos(\theta_t)}\right)^2
\]

\[
T_I = \frac{4Z_1 Z_2 \cos(\theta_i) \cos(\theta_t)}{(Z_2 \cos(\theta_i) + Z_1 \cos(\theta_t))^2}
\]

Pressure equations:

\[
1 + R_p = T_p
\]

Intensity equations:

\[
I = \frac{P^2}{Z \cos(\theta)}
\]

\[
T_I = \frac{T_p^2 Z_1 \cos(\theta_t)}{Z_2 \cos(\theta_i)}
\]

\[
1 = R_I + T_I
\]
Critical Angle

![Graph showing critical angles for Fat/Air, Fat/Bone, and Fat/Liver](image_url)
Fat-Skull Bone Interface

\[ 1 + R_p = T_p \]

\[ 1 = R_I + T_I \]

\[ I = \frac{P^2}{Z \cos(\theta)} \]

\[ T_I = \frac{T_p^2 Z_1 \cos(\theta_t)}{Z_2 \cos(\theta_i)} \]
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MRgFUS

- coupled to MRI
- 1000 elements
- cooled circulating water

- focusing/amplification
- ~ 4 mm focal spot
Essential Tremor

10 million Americans or 4% of the population have ET.

~50°C results in reversible symptom changes

FDA Approved Reimbursement 55 systems

Energy Density

Uniform Energy on the Transducer
Energy Density

Uniform Energy on the Transducer
Energy Density

Uniform Energy on the Transducer

Higher Energy Density on this part of the skull (lines are closer together)
Energy Density

Uniform Energy on the Skull

Angled Spot

Uniform on Skull
Angled Hotspot in ET Treatment

Sagittal

Coronal
Incidence Angles

Left Half
Average = 5.2°

Right Half
Average = 3.4°

Average = 4°

Left Half
Average = 4.4°

Right Half
Average = 9.4°

Average = 7°
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Standing Waves

You can get twice the intensity if conditions are right

https://www.acs.psu.edu/drussell/demos/swr/swr.html