Homework 1 – Due Thursday Oct 7

1. The wave function describing a wave travelling on a string is given by:
   \[ y = 0.1 \sin \left( \frac{20}{17} \pi x - 200 \pi t \right) \]
where \( y \) is the displacement in [mm], \( t \) is time in [sec] and \( x \) is the distance from the origin in [m].
   (a) Find the frequency of the wave in Hz
   (b) Find the wavelength
   (c) What is the phase difference between the wave at \( x=0.25 \) and \( x=1.1 \)?
   (d) Write down the wave equation of a wave travelling in the opposite direction with double amplitude and frequency

2. Calculate the pressure reflection coefficient \( R \) and the ratio of reflected power to incident power for each of the following interfaces (Assuming normal incidence)
   a) Fat to Bone
   b) Muscle to Liver
   c) Fat to Lung

3. Consider the ultrasound imaging configuration shown on the right:

   If at least 20 \( \mu W/cm^2 \) must be returned to the transducer from the reflection at the muscle/ fat interface for a good signal-to-noise ratio, calculate the power density that must be transmitted by the transducer at a frequency of \( f = 2 \) MHz. Neglect any losses due to transverse spreading of the beam. Do this calculation in dB and in nepers and show they are the same.

4. A wave incident from the left of the boundary shown below has a pressure reflection coefficient given by \( R_L \) and a power reflection coefficient given by \( R_L^2 \),
Find the pressure reflection coefficient $R_R$ and the power reflection coefficient for a wave incident on the same boundary from the right. Put both in terms of $R_L$.

5. Here we have a simple model of two ultrasound beams (originating at Tx1 and Tx2) crossing the skull to a point at the center of the skull. The skull has a speed of sound different from the speed of sound in and outside the skull. The skull will aberrate the beam (the two beams will have different phases).

Assume the transducers have a center frequency of 1 MHz.

a. Relative to Tx1, what phase should be applied to Tx2 in order to focus at the center of the circle? Use the following values:
   i. $a = 2$ mm
   ii. $r = 2$ cm
   iii. $d = 2$ cm
   iv. $c_0 = 1540$ m/s
   v. $c_1 = 1900$ m/s

b. What complicating factors would occur if the path between the transducer and the target didn’t intersect the circle at a right angle?

c. Assuming the circle was much more complicated with a variety of thicknesses and sound speeds, can you think of an experimental way to recover the phase that should be applied?
6. Assume you want to measure the speed of sound of a rectangular object of thickness, h. You immerse the object in 20°C water and measure the time it takes for a pulse to arrive at the receiver. You then repeat the experiment without the object.
   a. What is the speed of sound in the object? Use the following values:
      i. h = 1 mm
      ii. arrival time with object present is 26 μs
      iii. arrival time without object present is 24 μs
   b. How does your answer change if you assume the temperature of the water is 37°C?

7. Relate the bulk modulus, density, and speed of sound in both water and fat. Why is the speed of sound lower in fat (is it because of the bulk modulus or the density)? What is it about that parameter that varies significantly between fat and water?