1 Thermal Dose (6 points)

The region 1 shown in figure below has been treated at 58°C for 30 seconds. This has resulted in a change in attenuation coefficient of this region shown in the figure on the right. Calculate the extra source power (as compared to if region 1 did not have the thermal dose delivered) required to deliver 240 CEM at region 2 in 30 seconds. The transducer is unfocused, has an area of 10 cm².

\[ \text{Thermal dose} = \int_{t_0}^{30} R^{(43 - T)} \, dt \]  

Since we are concerned with power, we can ignore integration for now and only compute: 

\[ R^{43-57} = \left( \frac{1}{2} \right)^{43-57} = 16384. \] 

\[ \log(\text{thermal dose}) = \log(16384) = 4.22 \] 

The new attenuation therefore, is: 6 dB/cm, in 30 seconds. The transducer is unfocused and has an area of 10 cm².

\[ I_{\text{des}} = e^{(-2\alpha d)} I_0 \] 
\[ I_{\text{des}} = e^{(-2 \times 2\alpha d)} I_1 \] 
\[ \frac{I_1}{I_0} = e^{(2\alpha d)} \]
2 Ultrasound Imaging (6 points)

An ultrasound image is shown in the figure below.

(a) What kind of image is this? M mode

(b) Estimate the maximum velocity of the structure see about 10 cm into the image. $\frac{1 \text{ cm}}{0.2 \text{ sec}}$

3 Hydrophone Measurements and Temperature (18 points)

A hydrophone is placed 7.68 cm from a transducer in water. The temperature of the water is varied and at each new temperature a measurement is taken. The file temperature.mat contains a matrix, y, of this ultrasound data. Each column represents an acquisition. The sampling rate is 50 MHz and there is a 41 microsecond delay between the beginning of the sonication and the beginning of the acquisition. Hint: Use the peak of the envelope function to define arrival time. Don’t forget to filter!

(a) Find the temperature at which each measurement was taken. You may assume that the speed of sound in water is given by $1402.4 + 5.0 \times T - 0.058 \times T^2$, where $T$ is the temperature of the water. Please attach your scatter plot.

(b) What difficulty do you encounter when trying to find the temperature? What physical property of water leads to this ambiguity?

There is an ambiguity – two temperatures are possible for each measurement. This is because of the hydrogen bonding in water – it causes the compressibility to increase then decrease.

(c) Given that the temperature of the water was continuously increased, can you eliminate the ambiguity? There is still some ambiguity but since it looks pretty linear we are probably safe to assume it is as shown in Figure 3.
Figure 1: Sample traces

Figure 2: Raw temperature estimates (3.b)

Figure 3: Anticipated true temperatures (3.c)
4 Scattering Cross-section (6 points)

Using the following equation, compute the scattering cross section for a steel sphere in water assuming a frequency of 1 MHz and a diameter of 2 \( \mu m \). Repeat for a micro-bubble (made of air) in water.

\[
\sigma_s = \frac{4\pi k^4 a^6}{9} \left( \left| G_w - G \right|^2 + \frac{1}{3} \left| 3\rho_w - 3\rho \right|^2 \right)
\]

\( k = \frac{2\pi}{\lambda} \) and \( a \) is the radius.

<table>
<thead>
<tr>
<th></th>
<th>density (g/cm(^3))</th>
<th>G (cm/ \text{dyne})</th>
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<tbody>
<tr>
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<td>7.04x10(^{-7})</td>
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<tr>
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</tr>
<tr>
<td>water</td>
<td>1</td>
<td>4.54x10(^{-11})</td>
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- \( \rho_{\text{steel}} = 7.60 \ \text{gr/cm}^3 \)
- \( \rho_{\text{water}} = 1 \ \text{gr/cm}^3 \)
- \( \rho_{\text{air}} = 1.14 \times 10^{-3} \ \text{gr/cm}^3 \)
- \( k = \frac{2\pi}{\lambda} = \frac{\pi f}{c} = \frac{2\pi(1 \ \text{MHz})}{148000 \ \text{cm/s}} = 42.45 \ \frac{1}{\text{cm}} \)
- \( \alpha = \frac{2 \mu m}{2} = 1 \times 10^{-4} \ \text{cm} \)

\[
\sigma_{\text{steel}} = 6.67 \times 10^{-18} \ \text{cm}^2
\]
\[
\sigma_{\text{air}} = 1.09 \times 10^{-9} \ \text{cm}^2
\]

Air has the larger cross section and is therefore better.