1 Frequency as a Function of Temperature (12 points)

(a) Using the table below, estimate the fraction of fat in the slightly fatty liver (use 30°C) and the very fatty liver (use 34°C).

<table>
<thead>
<tr>
<th>Speed of Sound</th>
<th>30°C</th>
<th>34°C</th>
<th>37°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Liver</td>
<td>1584</td>
<td>1589</td>
<td>1592</td>
</tr>
<tr>
<td>Slightly Fatty Liver</td>
<td>1574</td>
<td>1575</td>
<td>1581</td>
</tr>
<tr>
<td>Very Fatty Liver</td>
<td>1527</td>
<td>1524</td>
<td>1522</td>
</tr>
<tr>
<td>Fat</td>
<td>1490</td>
<td>1390</td>
<td>1345</td>
</tr>
</tbody>
</table>

(b) Ebbini’s formula (below) relates the change in ultrasound frequency to the change in tissue temperature:

$$\Delta f = \frac{1}{2d} \left[ \frac{\Delta C(T)}{\Delta T} - \alpha c_0(T) \right] \Delta T$$

$$\alpha$$ is the coefficient of thermal expansion, $$c_0(T)$$ is the speed of sound in each tissue type at a given temperature. If we assume the coefficient of thermal expansion is the same for all at 0.0001 /°C, how does Ebbini’s change in fundamental frequency change with temperature for normal, slightly fatty, and fatty liver? Assume $$d = 1 \text{ mm}$$.

2 Precessing Protons (9 points)

(a) How fast are water hydrogen protons rotating in a 1.5 T magnetic field?

(b) If you want water hydrogen protons to precess at 100 MHz, how strong of a field (in units of Gauss) would you need?

(c) How fast are the hydrogen protons of fat rotating in a 3 T magnetic field?
3 Magnetization Excitation Calculation (15 points)

The RF pulse (also called the $B_1$ field) tips the magnetization from the longitudinal axis into the transverse plane. The transverse magnetization is then measured and used to create images.

(a) Why does the RF pulse need to be applied at the Larmor frequency? What happens to the magnetization if our RF pulse isn’t applied at the Larmor frequency? Briefly justify your answer. Try to use intuition instead of a mathematical derivation!

Assuming we are in the rotating reference frame, the magnetization tips/rotates from the longitudinal axis to the transverse plane about the $B_1$ axis according to the left-hand rule. The degree of tipping that occurs is called the flip angle. A 90° flip angle (see diagram) would fully rotate the magnetization $M$ (with equilibrium magnitude $M_0$) from the longitudinal direction into the transverse plane. However, flip angles other than 90° are frequently used! When the magnetization is not fully tipped into the transverse plane, signal is only detected from the transverse component of the tipped magnetization.

For the following questions, assume all RF pulses are ideal and instantaneous. Ignore any magnetization relaxation effects.

(b) Suppose our magnetization starts at equilibrium. After applying an RF pulse in the $x$ direction to achieve a 30° flip angle, what is the measured signal?

(c) How about if we use a 135° flip angle?

(d) Assume our magnetization starts at equilibrium. Suppose we apply an RF pulse in the $x$ direction with a 90° flip angle. Next, we apply a second RF pulse with the same tip angle, but this time in the $y$ direction. What is the detected signal after both RF pulses are applied?
(e) Suppose that our magnetization now recovers longitudinally. Sketch the approximate longitudinal magnetization recovery curves for parts b to d. Assume that all RF pulses are applied instantaneously at time $t = 0$.

4 Optimal TE (6 points)

In an MR temperature acquisition, there are two effects: 1) the magnetization accumulates phase linearly with temperature change and 2) the magnetization is decreasing exponentially in amplitude with a time constant called $T_2^*$. Just considering these two effects, derive the optimal time (TE) to measure the signal.