Lecture #19
Fast Spin Echo, CPMG, and J coupling

• Spin echo vs Fast Spin Echo imaging
• Spin locking
• Decoupling
• References
  – van de Ven, Chp 3.9, 4.9.
Analysis of J Coupling-Induced Fat Suppression in DIET Imaging

L. A. Stables,* R. P. Kennan,† A. W. Anderson,* † R. T. Constable, † and J. C. Gore* †

*Department of Applied Physics and †Department of Diagnostic Radiology, Yale University School of Medicine, New Haven, Connecticut 06520-8042
Spin Echo Imaging

• One k-space line collected each TR

Cardiac MRI example:

After n lines of data (e.g. 256) are acquired, a FT creates the image.
Fast Spin Echo

• A train of 180s is used to acquire multiple k-space lines each TR

Cardiac MRI example:

• Provides the ability to acquire images much faster, while retaining $T_2$ weighting if desired.
FSE Neuro Example

$T_2$-weighted images in much less time ($3T$, $TE/TR = 80/2000$ ms)

Spin Echo

FSE

Acq. time $\approx 16$ min

Acq. time $\approx 1$ min

echo train length (ETL) = 16
Why is fat bright in FSE images?

Lipid $T_2 = ?$

SE: lipid $T_2 \sim 35$ ms
FSE: lipid $T_2 \sim 135$ ms
Multi-Spin Systems

• Ignoring relaxation, the Hamiltonian has the following general form:

\[ \hat{H} = \hat{H}^0 + \hat{H}_{Rf} \]

\[ \hat{H}^0 = \sum_{j=1}^{j} \delta_j \hat{I}_z + \sum_{j<k} 2\pi J_{jk} \hat{\tilde{I}}_j \cdot \hat{\tilde{I}}_k \]

\[ \hat{F}_x = \sum_j \hat{I}_{xj} \quad \hat{F}_y = \sum_j \hat{I}_{yj} \quad \hat{F}_z = \sum_j \hat{I}_{zj} \]

• For a system with multiple spins, the total x, y, and z coherences are

• We wish to consider two cases.

  1. Magnetically equivalent spins, i.e. all \( \delta_j \)s are equal.
  2. Non-equivalent spins, i.e. unequal \( \delta_j \)s.
Equivalent Spins

- Multi-spin system for equivalent spins

\[ \hat{H}^0 = \sum_j \hat{I}_{zj} + \sum_{j<k} 2\pi J_{jk} \hat{I}_j \cdot \hat{I}_k = \hat{H}_1 + \hat{H}_2 \]

One can show \[ \sum_j \hat{I}_{pj} \cdot \hat{I}_k = 0, \quad p = x, y, z \]

2-spin example

\[ \begin{align*}
[\hat{I}_z + \hat{S}_z, \hat{I}_x \hat{S}_x + \hat{I}_y \hat{S}_y + \hat{I}_z \hat{S}_z] &= \hat{I}_y \hat{S}_x - \hat{I}_x \hat{S}_y + \hat{I}_x \hat{S}_y - \hat{I}_y \hat{S}_x = 0 \\
[\hat{I}_x + \hat{S}_x, \hat{I}_x \hat{S}_x + \hat{I}_y \hat{S}_y + \hat{I}_z \hat{S}_z] &= \hat{I}_z \hat{S}_y - \hat{I}_y \hat{S}_z + \hat{I}_y \hat{S}_z - \hat{I}_z \hat{S}_y = 0 \\
[\hat{I}_y + \hat{S}_y, \hat{I}_x \hat{S}_x + \hat{I}_y \hat{S}_y + \hat{I}_z \hat{S}_z] &= \hat{I}_z \hat{S}_x - \hat{I}_x \hat{S}_z + \hat{I}_x \hat{S}_z - \hat{I}_z \hat{S}_x = 0
\end{align*} \]

- Theorem: Let \( \hat{H} = \hat{H}_1 + \hat{H}_2 \) and \( [\hat{H}_1, \hat{H}_2] = [\hat{F}_x, \hat{H}_2] = [\hat{F}_y, \hat{H}_2] = [\hat{F}_z, \hat{H}_2] = 0 \) then the observed signal is independent of \( \hat{H}_2 \)

Proof:

\[
\text{Tr} \left[ \hat{F}_p e^{-i\hat{H}_t} \sigma(0) e^{i\hat{H}_t} \right] = \text{Tr} \left[ \hat{F}_p e^{-i(\hat{H}_t + \hat{H}_2 t)} \sigma(0) e^{i(\hat{H}_t + \hat{H}_2 t)} \right] = \text{Tr} \left[ \hat{F}_p e^{-i\hat{H}_2 t} e^{-i\hat{H}_2 t} \sigma(0) e^{i\hat{H}_2 t} e^{i\hat{H}_2 t} \right] = \text{Tr} \left[ e^{i\hat{H}_2 t} \hat{F}_p e^{-i\hat{H}_2 t} e^{-i\hat{H}_2 t} \sigma(0) e^{i\hat{H}_2 t} \right] = \text{Tr} \left[ \hat{F}_p e^{i\hat{H}_2 t} e^{-i\hat{H}_2 t} e^{-i\hat{H}_2 t} \sigma(0) e^{i\hat{H}_2 t} \right] = \text{Tr} \left[ \hat{F}_p e^{-i\hat{H}_2 t} \sigma(0) e^{i\hat{H}_2 t} \right] \quad \text{Independent of } \hat{H}_2!
\]

Why is this step legitimate?
The two water $^1$H spins are equivalent, hence show no effects due to J-coupling.
Lipids

- Lipids consist of multiple J-coupled resonances, and lipid $^1$Hs are not equivalent!

\[
\hat{H}^0 = \sum_j \delta_j \hat{I}_z^j + \sum_{j<k} 2\pi J_{jk} \hat{I}_j \cdot \hat{I}_k
\]

$\delta_j$s not all equal, thus

\[
\left[ \sum_j \delta_j \hat{I}_p^j, \hat{I}_j \cdot \hat{I}_k \right] \neq 0
\]

Hence J-coupling can NOT be ignored.

Carr-Purcell-Meiboom-Gill (CPMG)

- Consider a multi-spin system with $j$ spins, coupling constants $J_{jk}$, rotating frame resonance frequencies $\delta_j$, and the following pulse sequence:

![Pulse Sequence Diagram]

- Assuming hard RF pulses with $\omega_1 >> \delta_j, J_{jk}$ for all $j$, find the Hamiltonian for each time interval.

- $90_y$ Rf pulse: $\hat{H}_1 = \omega_1 \sum_j \hat{I}_{yj} = \omega_1 \hat{F}_y$ where $\omega_1 = -\gamma B_1 >> \delta_j, J_{jk}$ for all $j,k$.

- $180_x$ Rf pulses: $\hat{H}_2 = \omega_1 \hat{F}_x$

- between Rf pulses: $\hat{H}_3 = \sum_j \delta_j \hat{I}_{zj} + \sum_{j<k} 2\pi J_{jk} \hat{I}_j \cdot \hat{I}_k$
CPMG Product Operator Analysis

- Ignoring relaxation...

At thermal equilibrium: \( \hat{\sigma}_0 \propto \hat{F}_z \)

After the 90\( _y \): \( \hat{\sigma} \propto \hat{F}_x \)

Before the first 180\( _x \): \( \hat{\sigma} \propto e^{-i\hat{H}_3t} \hat{F}_x e^{i\hat{H}_3t} \)

After the first 180\( _x \): \( \hat{\sigma} \propto e^{-i\pi} \hat{F}_x e^{-i\hat{H}_3t} \hat{F}_x e^{i\hat{H}_3t} e^{-i\pi} \hat{F}_x \)

\( \hat{\sigma} \propto \hat{B}\hat{A}\hat{F}_x \hat{A}^{-1} \hat{B}^{-1} \) where \( \hat{A} = e^{-i\hat{H}_3t} \) and \( \hat{B} = e^{-i\pi} \hat{F}_x \)

At Echo 1: \( \hat{\sigma}_1 \propto (\hat{A}\hat{B}\hat{A})\hat{F}_x (\hat{A}\hat{B}\hat{A})^{-1} \)

Continuing, the spin density at the \( n \)th echo will be:

\( \hat{\sigma}_n \propto (\hat{A}\hat{B}\hat{A})^n \hat{F}_x (\hat{A}\hat{B}\hat{A})^{-n} \)
CPMG Product Operator Analysis

• Let examine $\hat{\sigma}_n \propto \left( \hat{A} \hat{B} \hat{A} \right)^n \hat{F}_x \left( \hat{A} \hat{B} \hat{A} \right)^{-n}$ more closely for the case where $\tau$ is short, i.e. $|J_{jk} \tau|, |\delta_j \tau| << 1$ for all spin groups $j,k$.

• Expanding $\hat{A}$ to first order in a Taylor series yields

$$\hat{A} = e^{-\frac{i}{2} \hat{H}_3 \tau} \approx 1 - \frac{i}{2} \hat{H}_3 \tau = 1 - \frac{i}{2} \sum_j \delta_j \hat{I}_j \tau - \frac{i}{2} \sum_{j<k} 2\pi J_{jk} \hat{I}_j \cdot \hat{I}_k \tau$$

Substituting… $\hat{A} \hat{B} \hat{A} = e^{-\frac{i}{2} \hat{H}_3 \tau} e^{-i\pi \hat{F}_x} e^{-\frac{i}{2} \hat{H}_3 \tau} \rightarrow \left(1 + i \sum_{j<k} 2\pi J_{jk} \hat{I}_j \cdot \hat{I}_k \tau \right) e^{-i\pi \hat{F}_x}$

However, $\hat{F}_x$ and $e^{-i\pi \hat{F}_x}$ commute…

Therefore, in a CPMG sequence with $|J_{jk} \tau|, |\delta_j \tau| << 1$, $\hat{\sigma}_n \rightarrow \hat{F}_x$ short $\tau$

Independent of $n$, $\delta_j$, and $J_{jk}$

• Therefore, for this rapidly refocused CPMG sequence, lipids will decay with their true $T_2$s free from the additional dephasing due to multiple J-couplings.
FSE Simulations

**FIG. 1.** The effect of $J$ coupling on a strongly coupled $A_1B_2$ spin system. The plot shows signal vs echo number for CPMG sequences where $\tau$, the spacing between echoes, is 4, 10, or 30 ms. $J_{AB} = 6$ Hz, $\delta_{AB} = 40$ Hz. Intrinsic $T_2$ relaxation is neglected. Note that as $\tau$ increases, $J$ coupling becomes more effective at suppressing the NMR signal.

Dual Interval Echo Train (DIET) FSE

• Goal: lipid suppression

• The relatively long first echo allows for J-coupling-induced dephasing to occur.
• This signal loss is not recovered in the remaining echo train.

DIET Results

DIET Results

Results

Spin Locking

• The suppression of J-coupling during a FSE sequence is considered a nuisance. However, the effect can also have advantages.

• Spin locking: the application of a long, strong continuous Rf pulse along a specified axis, e.g. x, in the rotating frame.
  – Chemical shift is suppressed, and spins are rendered effectively equivalent
  – Coherences along x are retained, those along y are dephased due to Rf inhomogeneity

\[
\begin{align*}
\hat{I}_x + \hat{S}_x & \xrightarrow{\text{spin lock}} \hat{I}_x + \hat{S}_x \\
\hat{I}_x - \hat{S}_x & \xrightarrow{\text{spin lock}} \left( \hat{I}_x - \hat{S}_x \right) \cos 2\pi J \tau + \left( 2\hat{I}_y \hat{S}_z - 2\hat{I}_z \hat{S}_y \right) \sin 2\pi J \tau
\end{align*}
\]

• The observable magnetization for truly equivalent spins does not evolve under J coupling, however spins rendered temporarily equivalent can show much more complex behavior, as they an enter the spin-lock period in a variety of initial states.
Decoupling

- Line splitting reduces the ability to detect and quantify in vivo peaks.
- Decoupling involves the use of a long strong Rf pulse on the coupled partner of the spin being observed.

Example

\[ ^{13}\text{C} \quad 90^\circ \quad 180^\circ \]

\[ ^{1}\text{H} \quad \text{acquire} \quad \text{decouple} \]

\[ ^{13}\text{C} \text{ Ethanol (phantom)} \]

No decoupling

with decoupling

Which carbon has the \(^{13}\text{C}\) label?
Decoupling

- Hamiltonian: \( \hat{H} = \hat{H}_{Zeeman} + \hat{H}_{dipole} + 2\pi J \hat{I} \cdot \hat{S} \)

  time average=0 with “decouple” using RF pulses
  rapid molecular tumbling to rapidly flip the S spin

Example

\( ^1H \)

\( ^1H \) (NMR)

\( ^13C \)

\( ^13C \) (RF)

\( ^13C \) (RF)

We don’t need to synchronize if refocusing on time scale << 1/J
Broadband Decoupling

• Problem: long Rf pulses have narrow bandwidths.

• Phase cycling the 180° pulses improves off-resonance behavior.

• Composite 180 pulses are even better:

\[ R = 90° \times 180°_x \times 270°_x = 1\bar{2}3 \]

…and are typically used in supercycles.

\[ 1\bar{2}3 \quad 1\bar{2}3 \quad 1\bar{2}3 \quad 1\bar{2}3 \]

Wideband Alternating Phase Low-power Technique for Zero-residue Splitting

WALTZ
Decoupling in practice

- **Theory**

- **In practice**

- Rf power deposition, typically measured as Specific Absorption Rate (SAR), is usually the limiting factor.

de Graaf, Chpt 8
WALTZ Decoupling

\[ R = 90^\circ, 180^\circ, 270^\circ \]

\[ R = 90^\circ, 180^\circ, 270^\circ \]

A

B

\text{‘WALTZ-16’}

\[ M_z/M_0 \]

\[ \text{frequency offset } \Delta \Omega \text{ / RF amplitude } B_2 \]
In Vitro Example: $^{13}$C MRS

$^{13}$C-labeled glucose

with 1H decoupling

without 1H decoupling

Note, $^1$H decoupling does not eliminate $^{13}$C-$^{13}$C J-coupling!
**$^{13}$C-Glucose Infusion: $^1$H MRS**

Rat brain: 180 $\mu$l, TR/TE=4000/8.5 ms, 9.4 T
2 hrs post [1,6-$^{13}$C$_2$]glucose infusion

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**$^1$H decoying**

**$^1$H-$^{13}$C edited + decoyling**

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*de Graaf, Chpt 8*
In Vivo Human: $^{13}$C MRS

Calf muscle, 4 T, polarization transfer acquisition, natural abundance $^{13}$C

Decoupling without Rf?

What happened to splitting from J-coupling with -OH?

100% Ethanol $\rightarrow$ $^1$H Spectrum = ?

Ethanol + 5% H$_2$O $\rightarrow$ $^1$H Spectrum = ?

Ethanol + 5% D$_2$O $\rightarrow$ $^1$H Spectrum = ?

Hint: Consider the effects of chemical exchange
The End.

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