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Problem Set #3
BioE 326B/Rad 226B

1. Solomon equations
2. Time-dependent perturbation theory
3. T_1 and T_2 of bone
4. Temperature mapping

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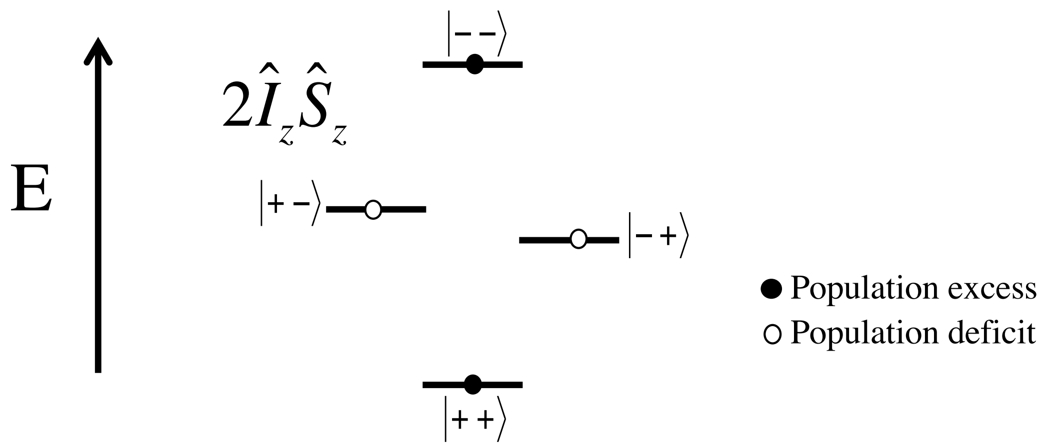
Solomon equations

Using the Solomon equations, derive the relaxation rate of longitudinal two-spin order $2\hat{I}_z\hat{S}_z$.

Hint, the energy diagram for this coherence is:



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Time-dependent Perturbation Theory

Given a Hamiltonian of the form $\hat{H}(t) = \hat{H}_o + \hat{H}_1(t)$ where $\hat{H}_1(t)$ is a perturbation small compared to $\hat{H}_0$ and $|m_n\rangle,\ n = 1\ldots N$ are the eigenkets of the unperturbed Hamiltonian $\hat{H}_0$ with eigenvalues $E_n/\hbar$, the goal is to show that if the system starts at time *t*=0 in the state $|m_j\rangle$, then the probability of finding the sytem in state $|m_k\rangle$ at time *t* is given by:

$$\mathcal{P}_{kj} = \left| \int_o^t \langle m_k(0)|\hat{H}_1(t')|m_j(0)\rangle e^{-i(E_j - E_k)t'/\hbar} dt' \right|^2$$

a) Consider an arbitrary wavefunction $|\psi\rangle = \sum_{n=1}^{N} c_n(t)|m_n\rangle$.

Using Schrodinger's equation: $i\dfrac{\partial|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle$,

show $\dot{c}_k(t) = -i\sum_{n=1}^{N} c_n(t)\langle m_k|\hat{H}_1(t)|m_n\rangle$.

b) Given the state of the system at $t = 0$ is specified by $c_{j=1}$, $c_{n\neq j}= 0$, the perturbation assumption is that the $\hat{H}_1(t)$ will only have a small effect on the dynamics, i.e.

$$c_n(t) << c_j(t) \quad \text{for} \quad n \neq j \quad \text{and} \quad c_j(t) \approx 1.$$

Using these assumptions and the results from (a), show

$$\mathcal{P}_{kj} = \left| \int_o^t \langle m_k(0)|\hat{H}_1(t')|m_j(0)\rangle e^{-i(E_j - E_k)t'/\hbar} dt' \right|^2$$

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T_1 and T_2 of bone (from de Graaf, problem 3.3)

a. Given a longitudinal relaxation time constant T_1 of 4.0 s for free water ($\tau_c = 10^{-11}$ s) at 7.05 T, calculate the T_1 for bone ($\tau_c = 10^{-6}$ s) under the condition of pure dipolar relaxation. Assume equal dipolar distances r for all compounds.

b. Calculate the minimum T_1 relaxation time constant at 7.05 T as a result of pure dipolar relaxation.

c. Calculate the transverse relaxation time constants T_2 for water and bone at 7.05 T.

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Temperature mapping (from de Graaf, problem 2.1)

In a proton spectrum acquired from rat brain at 7.05 T, the water resonance appears on-resonance while the NAA methyl resonance appears –801 Hz off-resonance. On a phantom the relation between the temperature T (in Kelvin) and the chemical shift difference (in ppm) between water and NAA, □water-NAA, was established as:

$$T = -95.24\delta_{water-NAA} + 564.15$$

a. Calculate the brain temperature (in Kelvin) using the phantom calibration data.

b. Following a period of ischemia a proton spectrum is acquired in which the water appears +11 Hz off-resonance. The NAA methyl resonance now appears at −796 Hz off-resonance. Calculate the brain temperature (in Kelvin).