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Problem Set #5
BioE 326B/Rad 226B

1. The interaction frame
2. Relaxation by random fields

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The interaction frame

Starting with the Liouville-Von Neumann equation:

$$\frac{d\hat{\sigma}}{dt} = -i\hat{\hat{H}}\hat{\sigma}$$

where $\hat{\hat{H}}$ is the superoperator associated with the Hamiltonian

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_1(t).$$

Switching to a frame of reference rotating about $\hat{H}_0$

$$\hat{\sigma}'(t) = e^{i\hat{\hat{H}}_0 t}\hat{\sigma}(t) \text{ and } \hat{H}'(t) = e^{i\hat{\hat{H}}_0 t}\hat{H}(t)$$

a. Show $\dfrac{d\hat{\sigma}'}{dt} = -i\hat{\hat{H}}_1'\hat{\sigma}'$.

b. Given $\dfrac{d\hat{\sigma}'}{dt} = -\hat{\hat{\Gamma}}\left(\hat{\sigma}' - \hat{\sigma}_B\right)$, where $\hat{\hat{\Gamma}}$ is the relaxation superoperator.

Show $\dfrac{d\hat{\sigma}}{dt} = -i\hat{\hat{H}}_0\hat{\sigma} - \hat{\hat{\Gamma}}\left(\hat{\sigma} - \hat{\sigma}_B\right)$

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Relaxation by random fields

Consider a system of spin-1/2 particles subjects to a large main magnetic field $B_0\vec{z}$ plus a small isotropic randomly fluctuating magnetic field, $\Delta\vec{B}(t) << B_0$, given by:

$$\Delta\vec{B}(t) = B_x(t)\vec{x} + B_y(t)\vec{y} + B_z(t)\vec{z}$$

where $\langle B_x^2 \rangle = \langle B_y^2 \rangle = \langle B_z^2 \rangle = \langle B^2 \rangle$.

Assuming an exponential correlation time for the perturbation fields of τ_c, use Redfield theory to show (a):

$$\frac{1}{T_1} = 2\gamma^2 \langle B^2 \rangle \frac{\tau_c}{1+\omega_0^2\tau_c^2}.$$

and (b):

$$\frac{1}{T_2} = \gamma^2 \langle B^2 \rangle \left(\tau_c + \frac{\tau_c}{1+\omega_0^2\tau_c^2} \right).$$