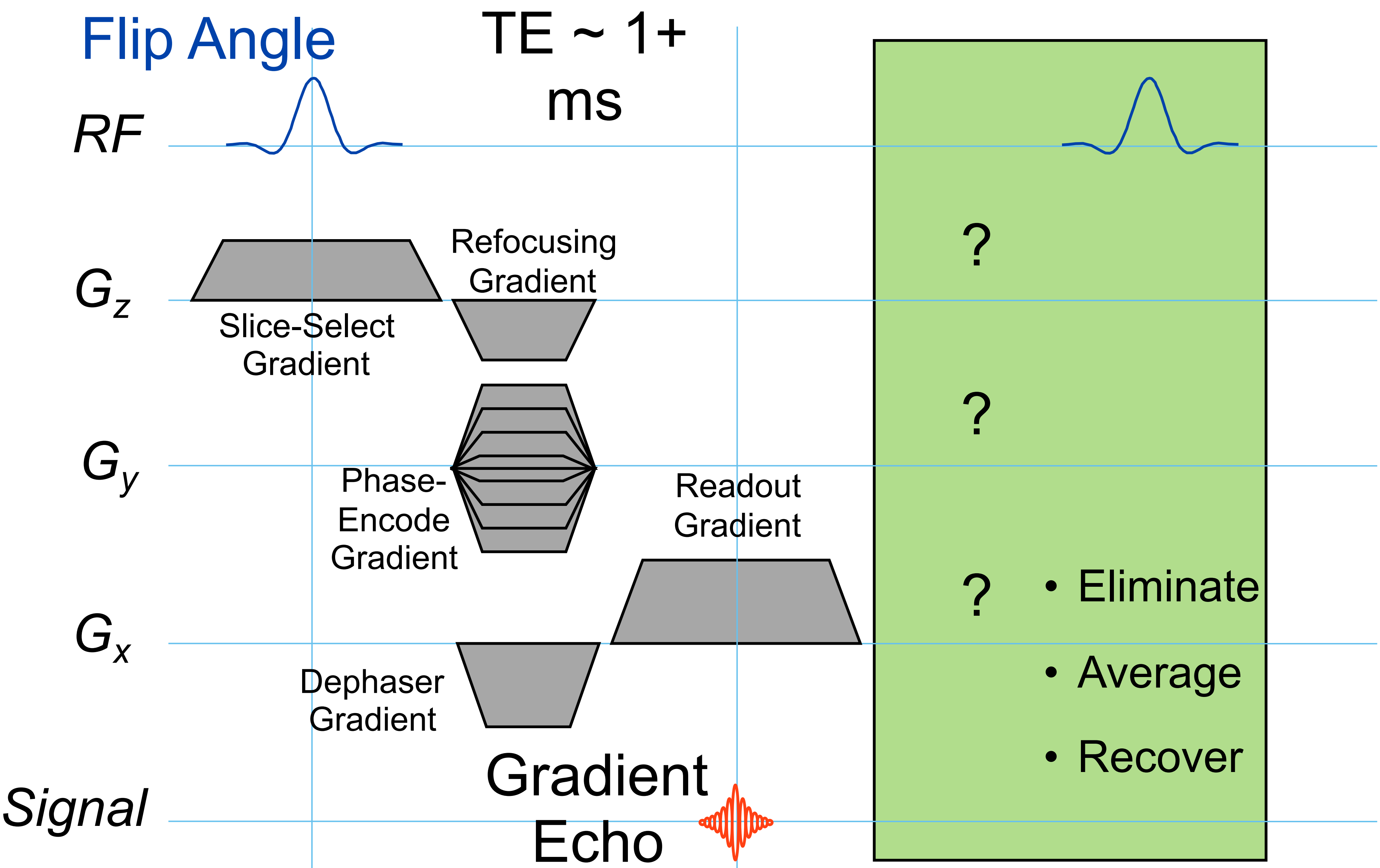
Gradient Echo Sequences

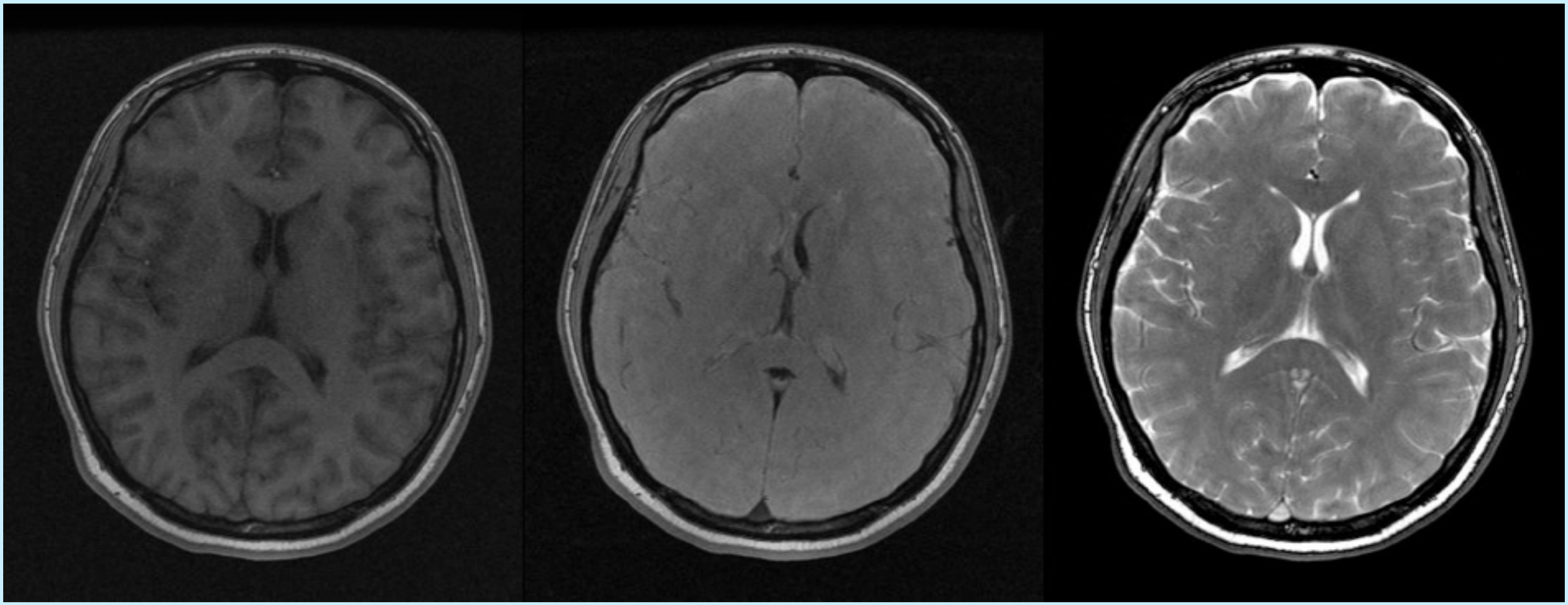
- Balanced SSFP
- Gradient Spoiled Sequences
- RF Spoiled Sequences
- Variations
  - Double-echo, Reversed
  - UTE, BOLD, T2*

Gradient Echo Pulse Sequence



Contrast Example

- Contrast based <u>solely</u> on end-of-TR action



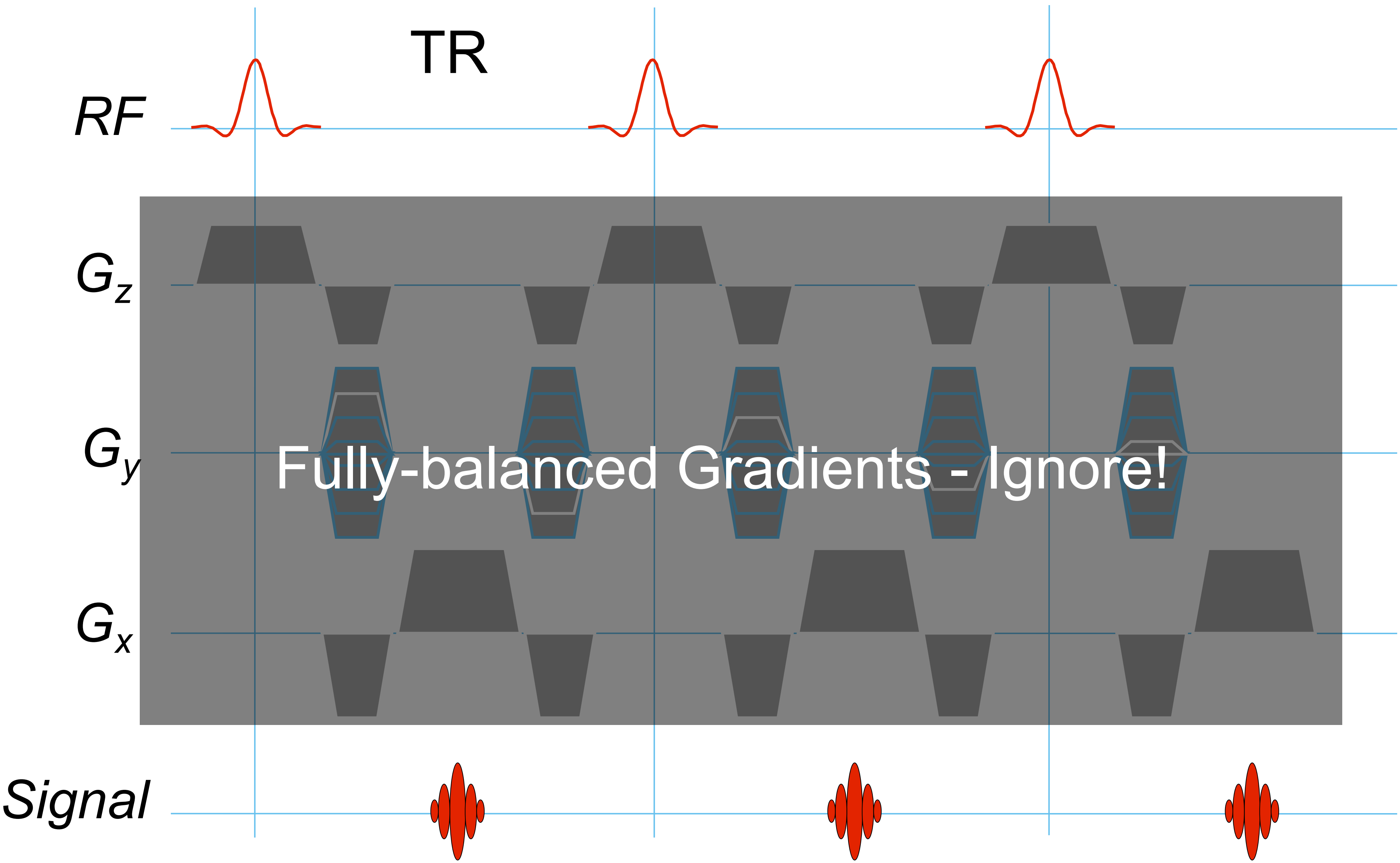
Balanced SSFP

- Goal: Maximize signal
  - Steady-state on-resonance
  - Steady-state off-resonance
  - Flip-angle effects
  - Transients

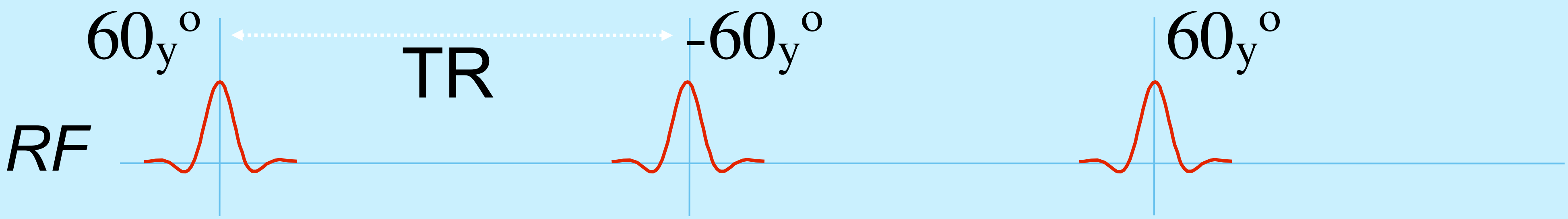
True-FISP, FIESTA, balanced FFE, BASG

Oppelt 1986, Duerk 1997

Balanced SSFP

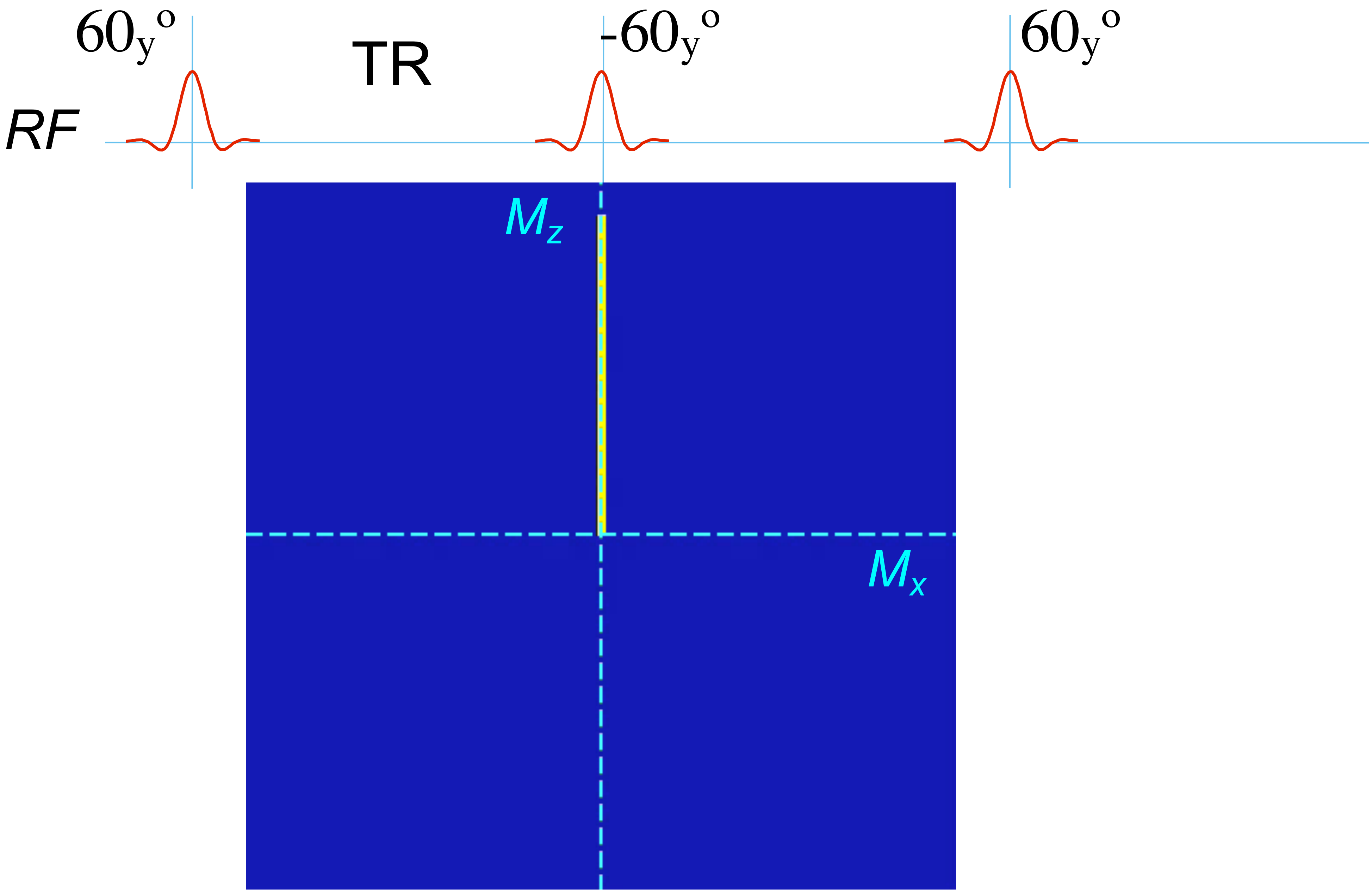


Balanced SSFP

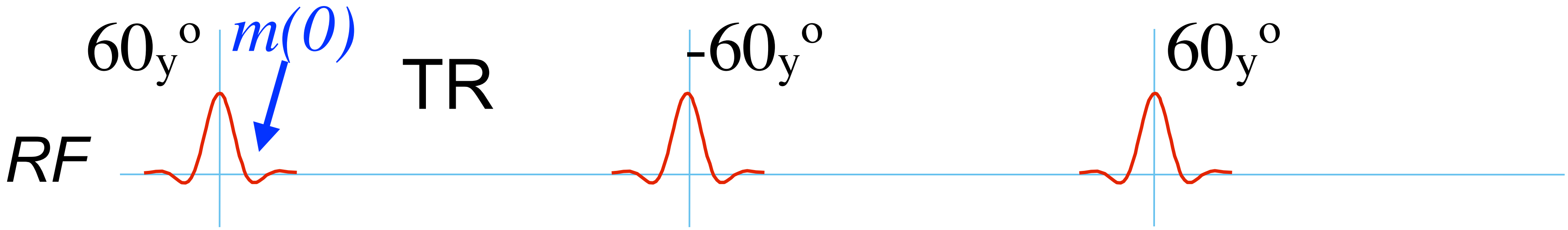


- So what happens here?

Balanced SSFP



Matrix Derivation



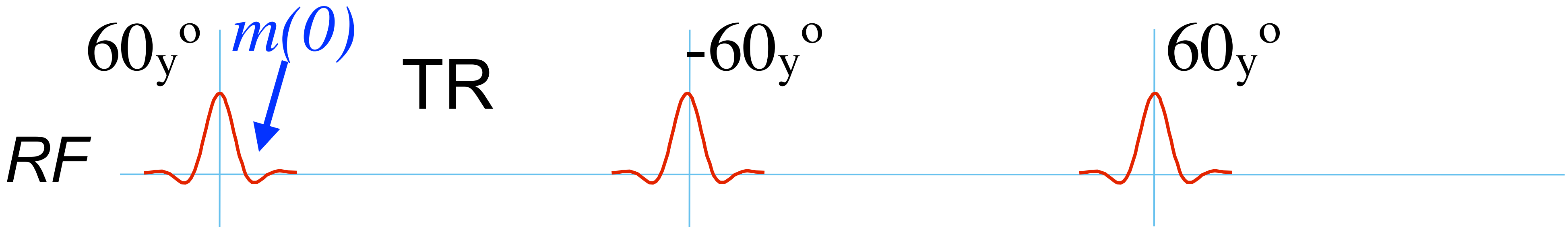
- Sequence: $E_{Relaxation}$ — $R_z(180^o)$ — $R_{-y}(\alpha)$
  - Note $R_z(180^o)$ lets us do this over only 1TR
- $m(0) = R_{-y}(\alpha)\, R_z(180^o)\, E\, m(0) + R_{-y}(\alpha)\, [0;0;1\text{-}E_1]$
  - Note: Could do as 2x2 formulation (no m_y)

$$\begin{bmatrix} 1+E_2\cos\alpha & 0 & -E_1\sin\alpha \\ 0 & 1+E_2 & 0 \\ -E_2\sin\alpha & 0 & 1-E_1\cos\alpha \end{bmatrix} m(0) = \begin{bmatrix} \sin\alpha(1-E_1) \\ 0 \\ \cos\alpha(1-E_1) \end{bmatrix}$$

Some algebra…

$$m(0) = \frac{1-E_1}{1+\cos\alpha(E_2-E_1)-E_1E_2} \begin{bmatrix} \sin\alpha \\ 0 \\ E_2+\cos\alpha \end{bmatrix}$$

Matrix Derivation (cont)

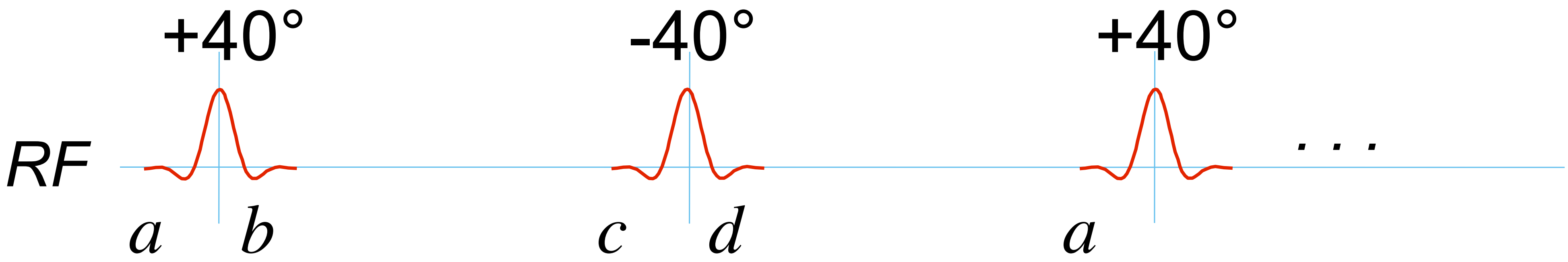


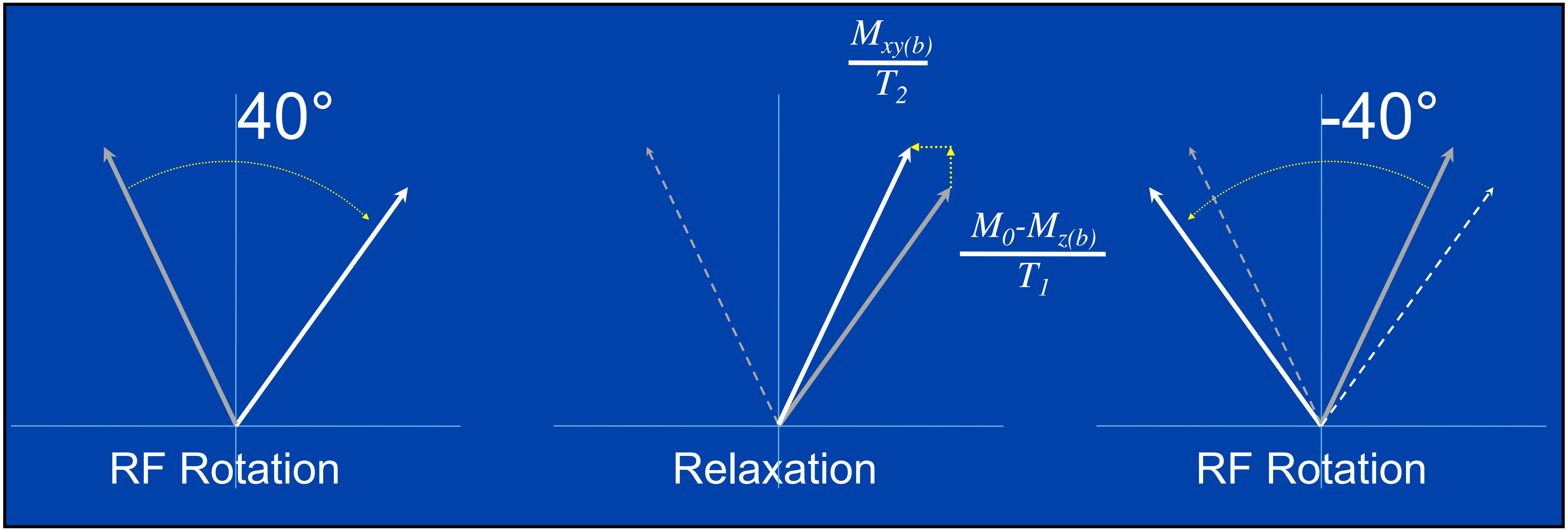
$$m(0) = \frac{1-E_1}{1+\cos\alpha(E_2-E_1)-E_1E_2}\begin{bmatrix}\sin\alpha \\ 0 \\ E_2+\cos\alpha\end{bmatrix}$$

$$(E_2 \approx 1) \qquad \frac{\sin\alpha}{E_2+\cos\alpha} = \tan(\alpha/2) = \frac{m_x}{m_z}$$

- Steady-state $m(0)$ is tilted by α/2 (good!)
- If E_2=0, same as excitation-recovery (good!)

Example: Alternating RF, No precession



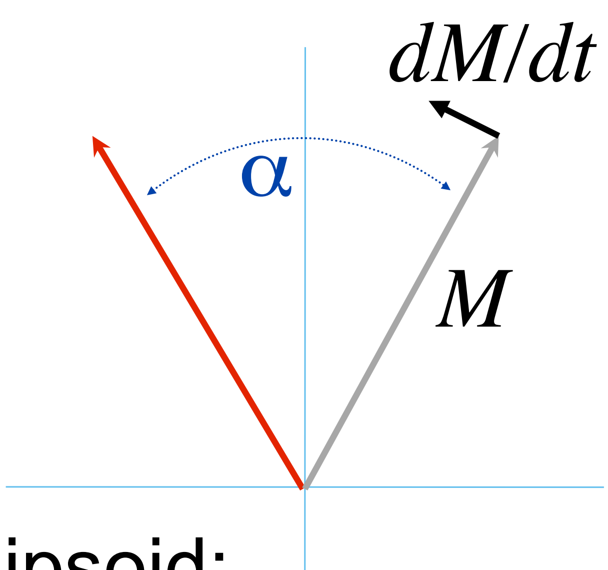


- Intuition: Relaxation does not change length much over TR

Length Solution

- If Relaxation does not change length:

$$dM/dt \cdot M = 0$$



$$\frac{-M_x}{T_2}M_x + \frac{-M_y}{T_2}M_y + \frac{M_0 - M_z}{T_1}M_z = 0$$

x T_1 & Rearrange...

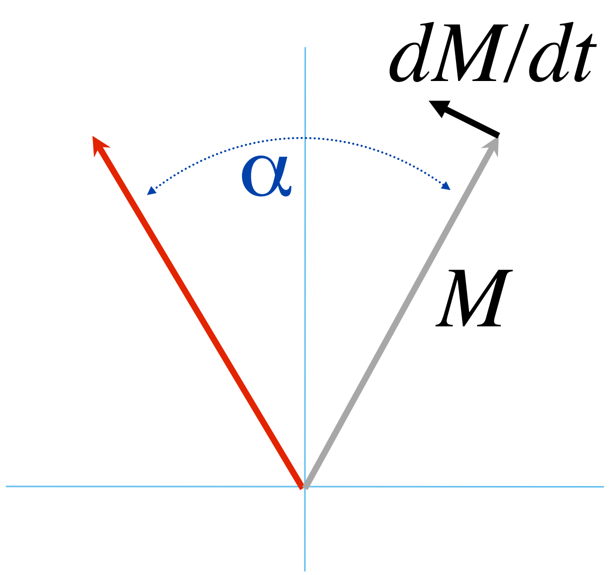
$$\left(M_z - \frac{M_0}{2}\right)^2 + \frac{M_x^2 + M_y^2}{T_2/T_1} = \left(\frac{M_0}{2}\right)^2$$

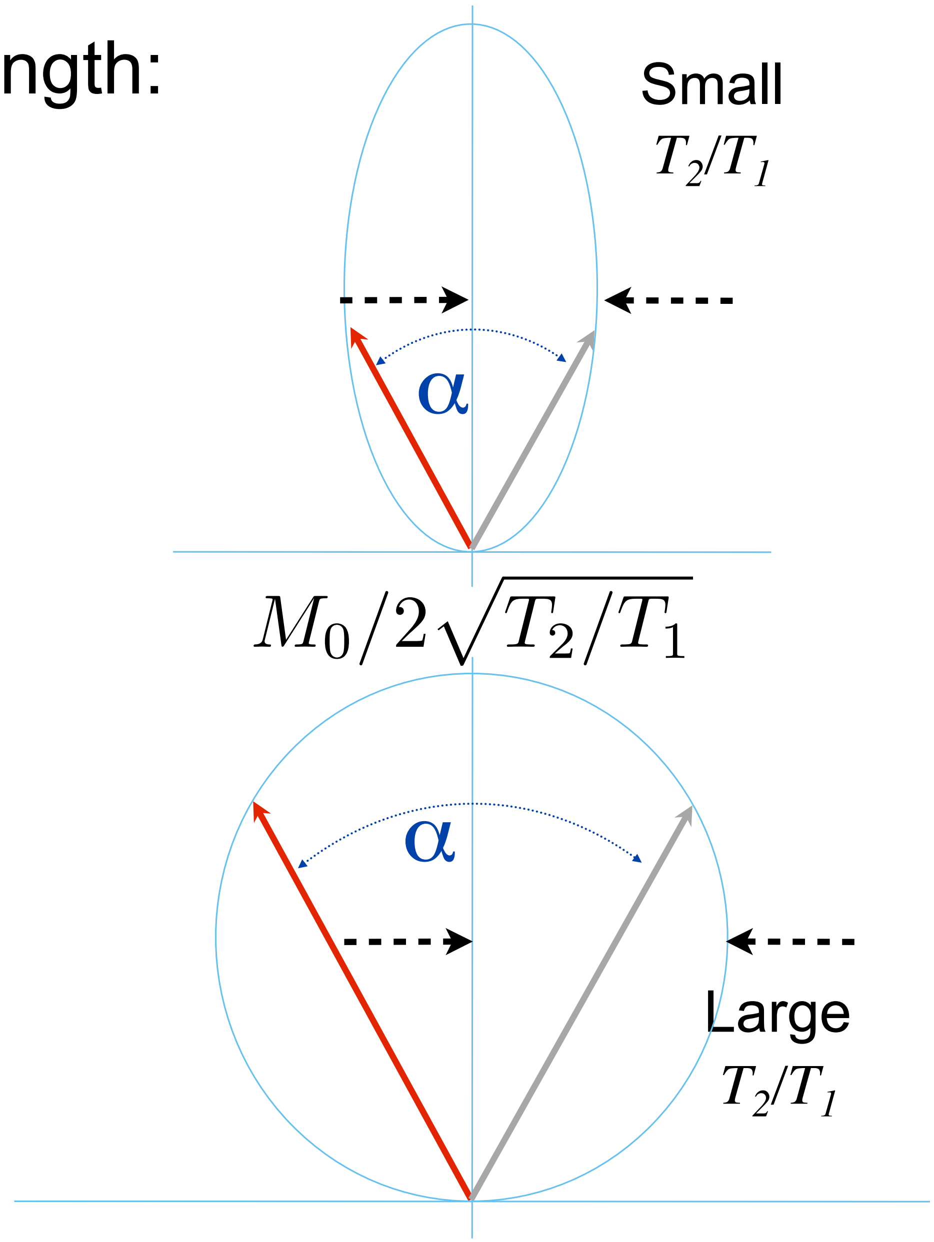
- Ellipsoid:
    - height M_0, half-width (signal) $M_0/2\sqrt{T_2/T_1}$
- T_2/T_1 contrast!

Full Solution

- If Relaxation does not change length:

$$dM/dt \cdot M = 0$$

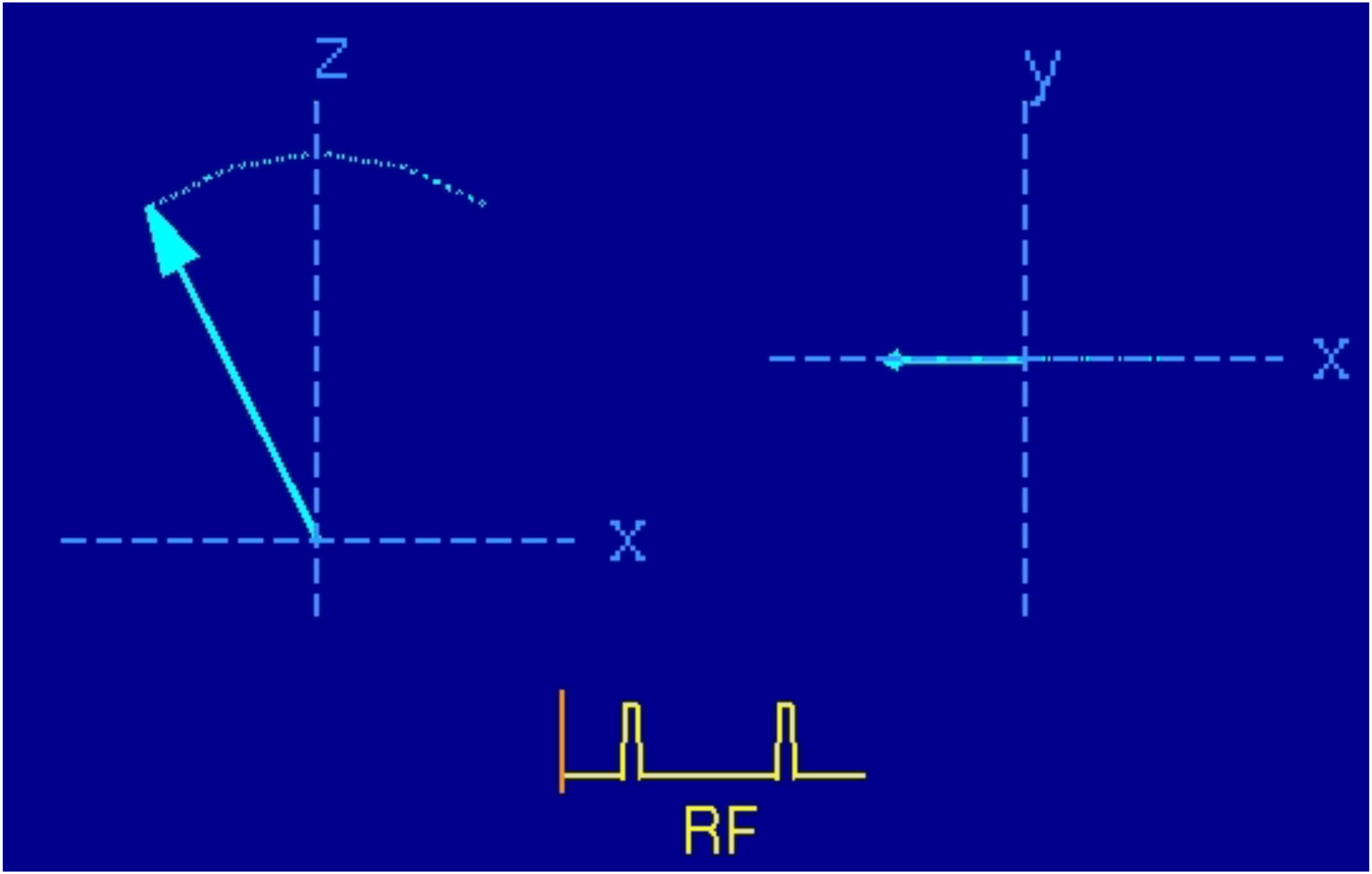




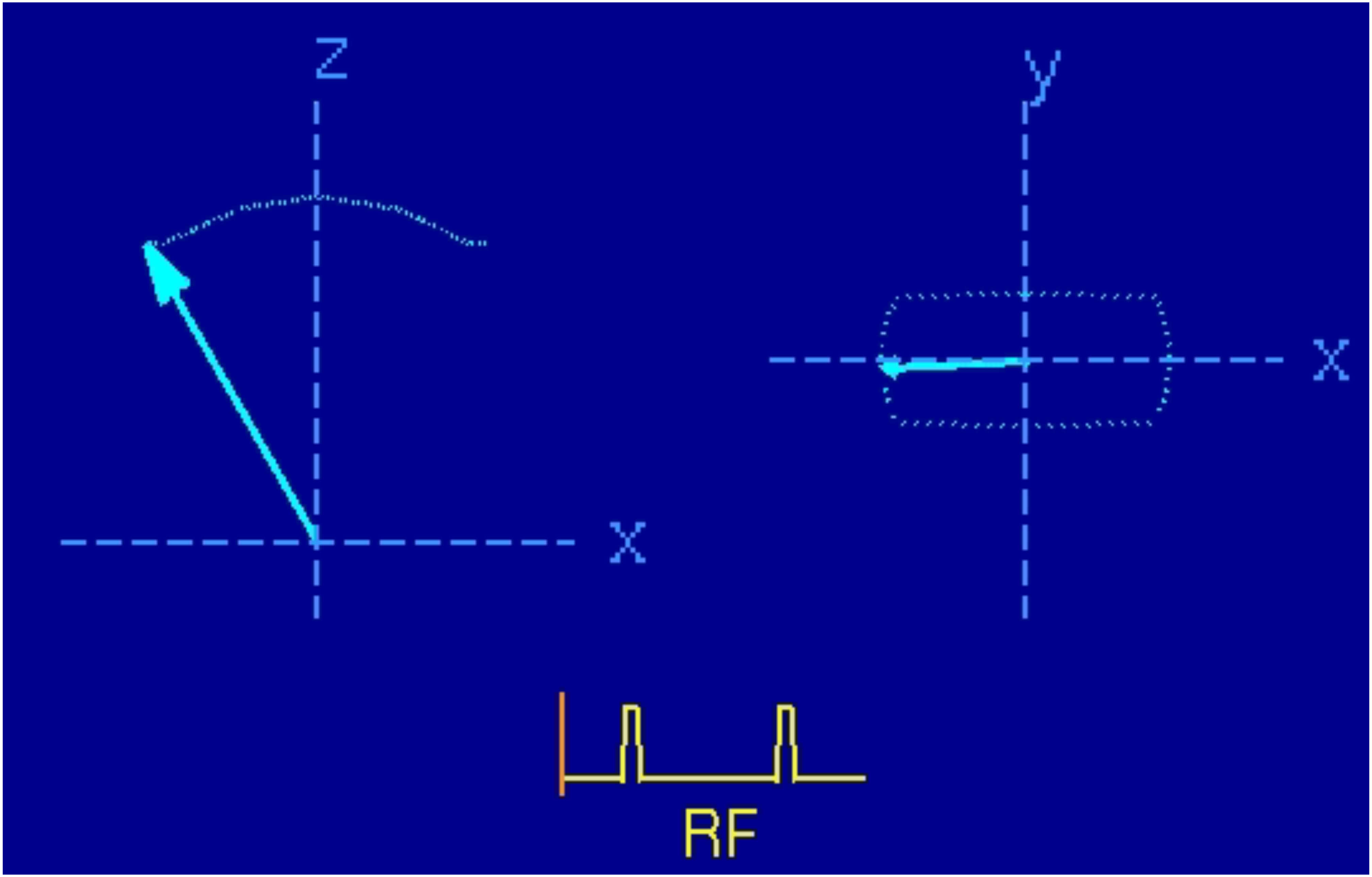
- Length: Ellipsoid intersection
- Direction: $\alpha/2$ angle from M_z

Signal Question

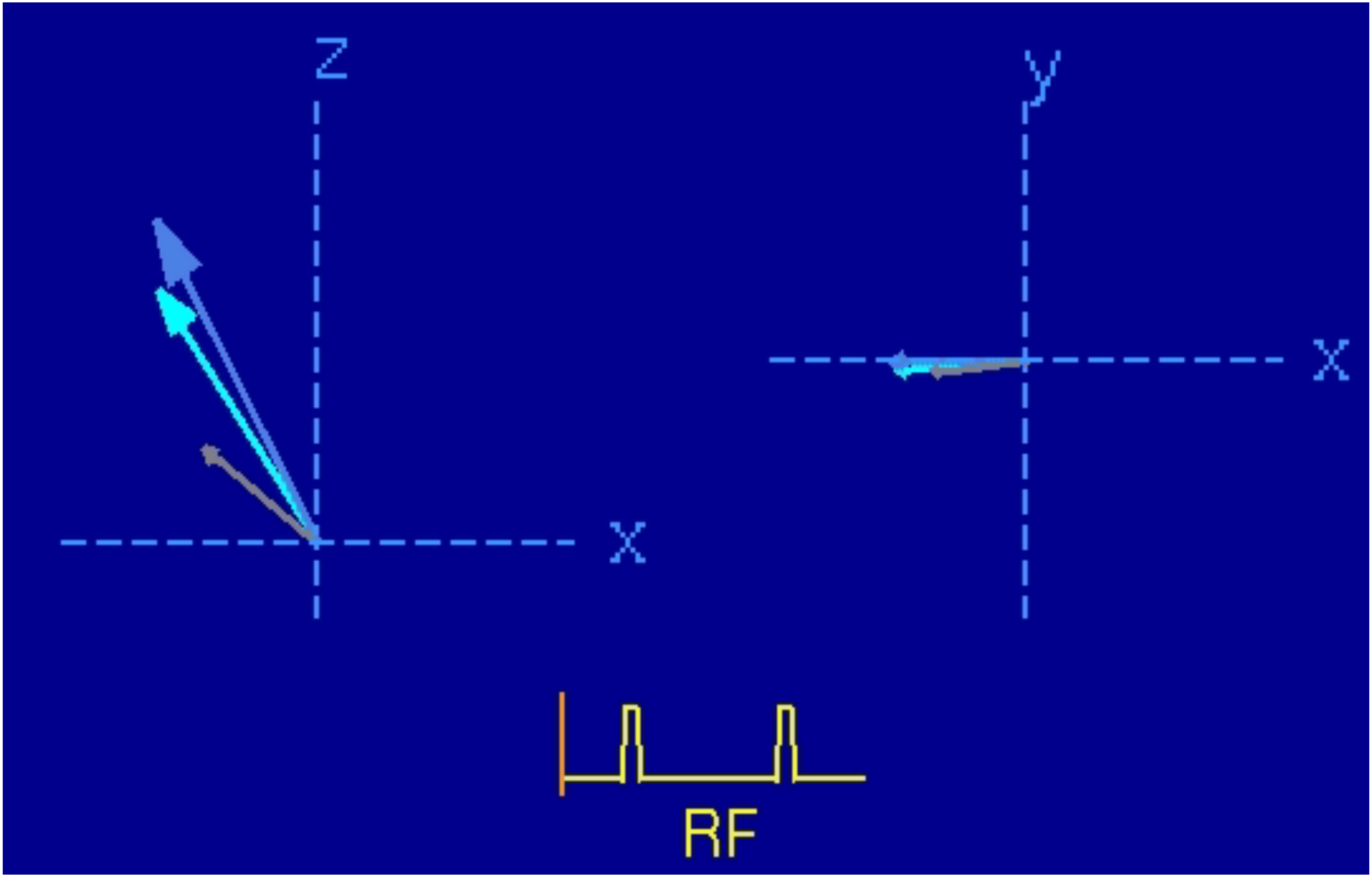
Steady State: No Precession



Off-Resonance: Precession

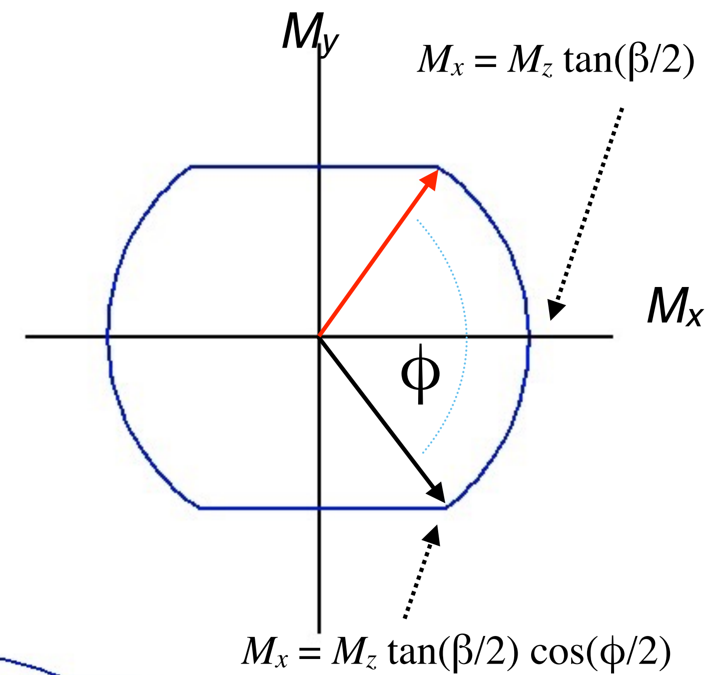


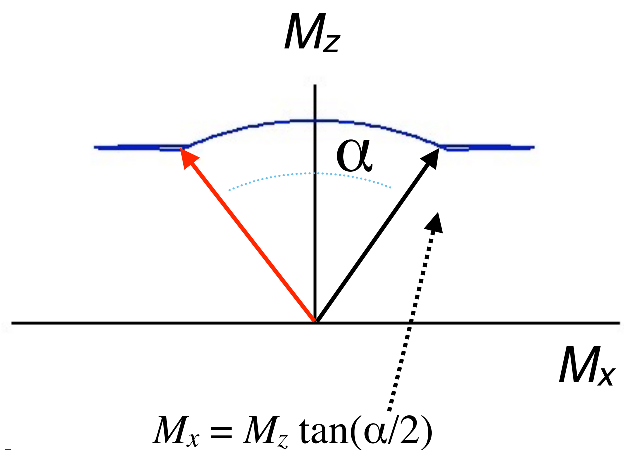
Increasing Precession

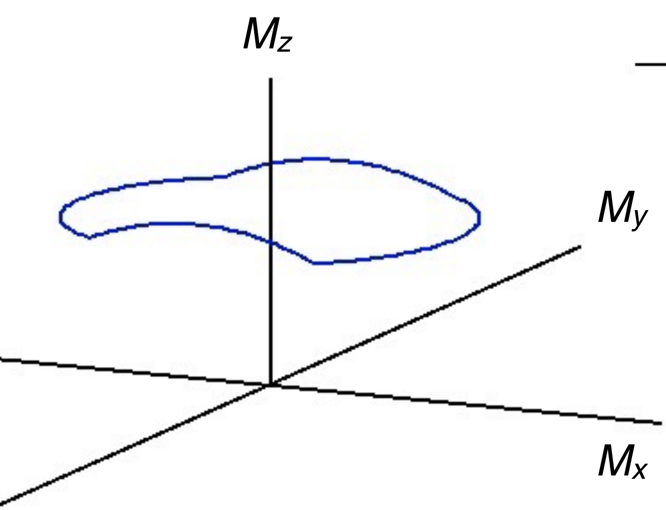


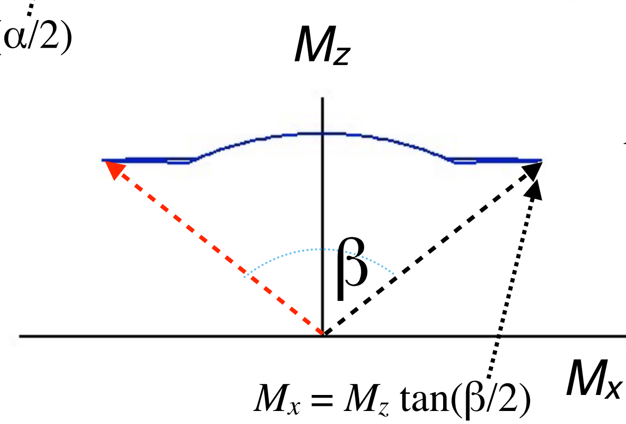
bSSFP: RF, Relaxation and Precession

- RF is balanced by relaxation and precession
- Length is still relatively unchanged over TR
- Ignore relaxation for now…



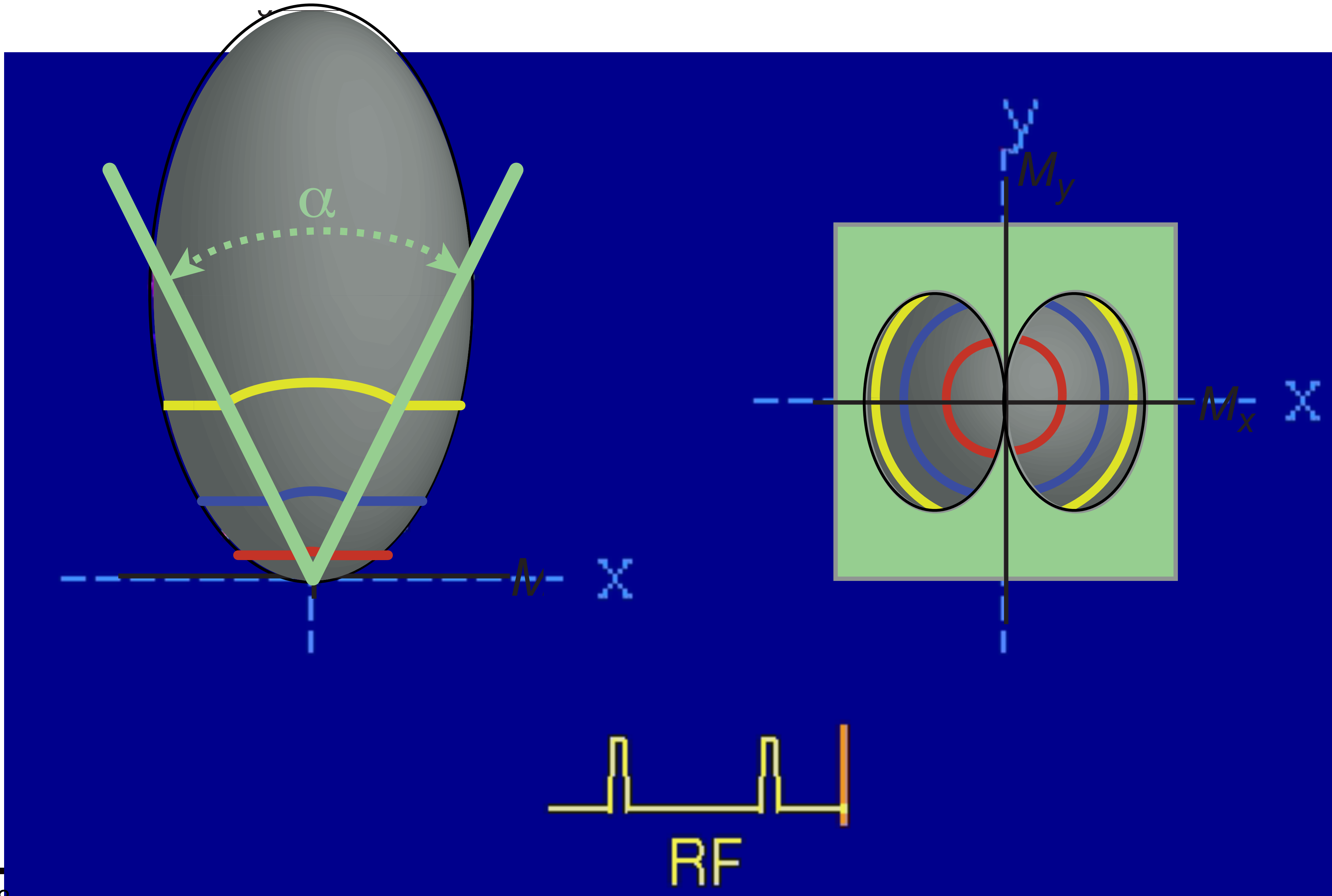




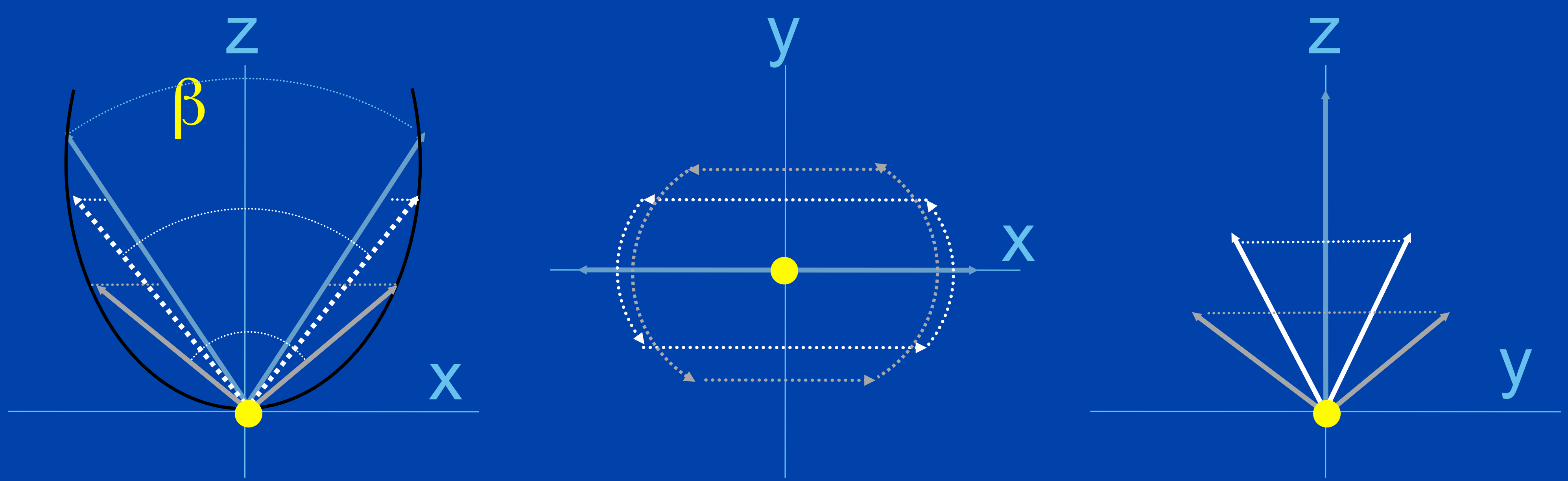


$$\tan(\alpha/2) = \tan(\beta/2)\cos(\phi/2)$$

Balanced SSFP (FIESTA)



Precession and "Effective flip angle"



- Larger precession gives a larger "effective flip," β
- $\tan(\alpha/2) = \tan(\beta/2)\cos(\phi/2)$
- $\beta >= \alpha$
- Can replace flip (α) with effective flip (β) for all calculations
- Limiting case (•) where $\beta = 180°$

(Schmitt MRM 2006, Zun, ISMRM 2006)

"Ellipsoid" Derivation of Signal

- Starting with ellipsoid: $$\left(M_z - \frac{M_0}{2}\right)^2 + \frac{M_x^2 + M_y^2}{T_2/T_1} = \left(\frac{M_0}{2}\right)^2$$

$M_{xy} = M \sin(\beta/2)$

$M_z = M \cos(\beta/2)$

$$\left[M\cos(\beta/2) - \frac{M_0}{2}\right]^2 + \frac{M^2\sin^2(\beta/2)}{T_2/T_1} = \left[\frac{M_0}{2}\right]^2$$

$$M^2\cos^2(\beta/2) + (T_1/T_2)M^2\sin^2(\beta/2) = M M_0\cos(\beta/2)$$

$$M\sin(\beta/2) = \frac{M_0\cos(\beta/2)\sin(\beta/2)}{\cos^2(\beta/2) + (T_1/T_2)\sin^2(\beta/2)}$$

$$\boxed{S = \frac{M_0}{\cot(\beta/2) + (T_1/T_2)\tan(\beta/2)}}$$

- Signal drops with increasing T_1/T_2
- At β=180º signal is 0. At β=90°, T_1=T_2, S=M_0/2

Ellipsoid vs Matrix Derivation

Ellipsoid:

$$S = \frac{M_0}{\cot(\beta/2) + (T_1/T_2)\tan(\beta/2)}$$

Matrix:

$$m(0) = \frac{1-E_1}{1+\cos\alpha(E_2-E_1)-E_1E_2}\begin{bmatrix}\sin\alpha \\ 0 \\ E_2+\cos\alpha\end{bmatrix}$$

Approximate $E_{1,2} = \exp(-TR/T_{1,2}) \sim (1-TR/T_{1,2})$, neglect $TR^2/(T_1T_2)$

=> If $\beta=\alpha$ then the two equations (for $S=M_{xy}$) give the same result.

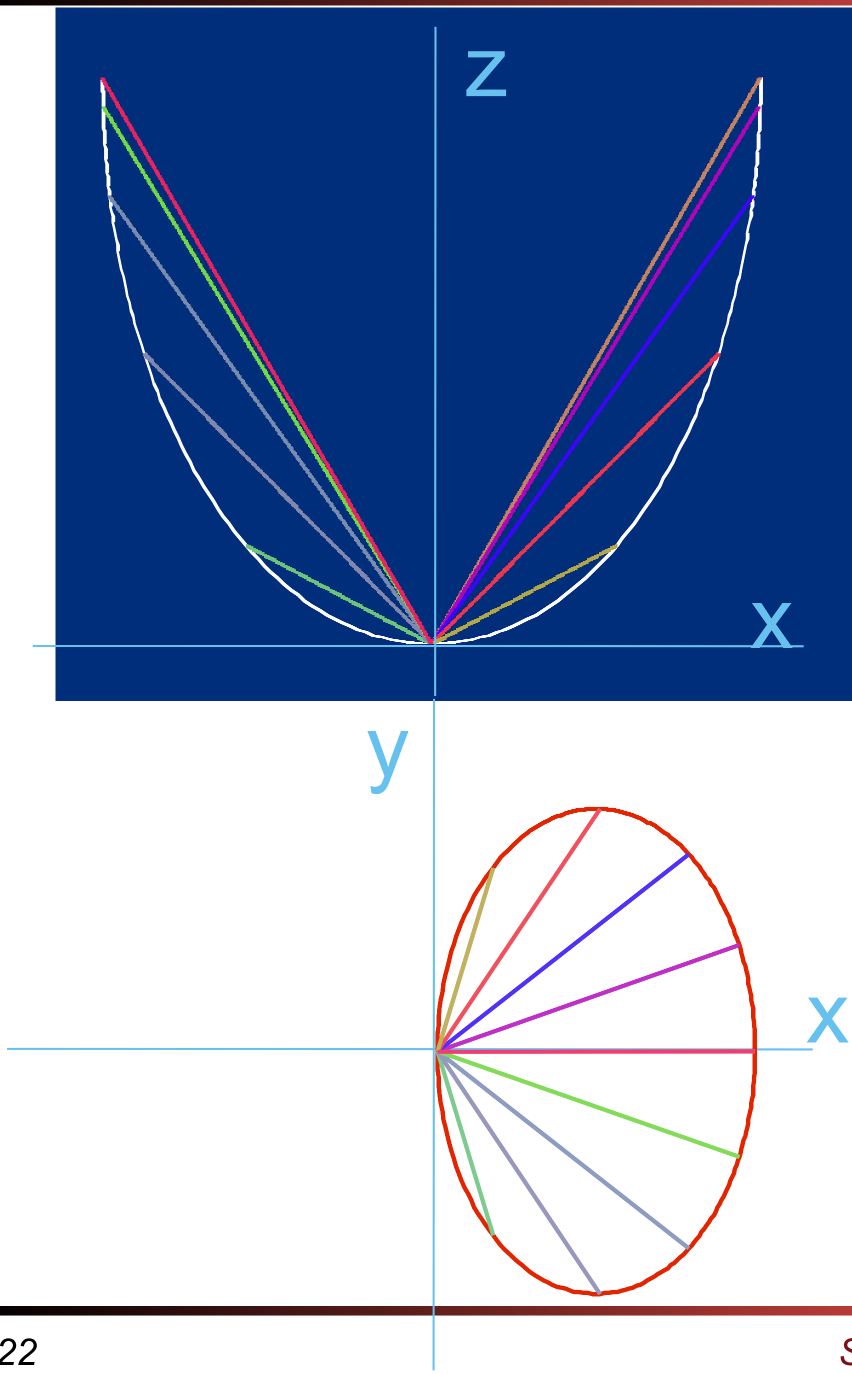
Substitute $E_{1,2}$,
Divide out TR/T_1

$$m(0) = \frac{\sin\alpha}{(1+\cos\alpha)+(T_1/T_2)(1-\cos\alpha)}$$

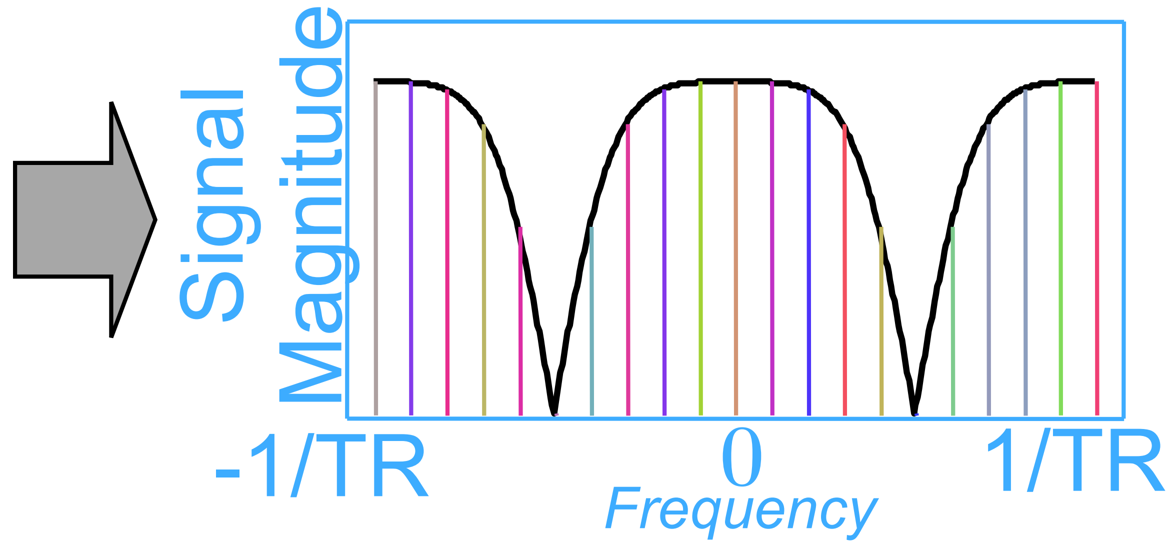
Divide out $\sin(\alpha/2)\cos(\alpha/2)$:

$$m(0) = \frac{2\sin(\alpha/2)\cos(\alpha/2)}{2\cos^2(\alpha/2)+(T_1/T_2)2\sin^2(\alpha/2)}$$

Signal vs Precession/Frequency



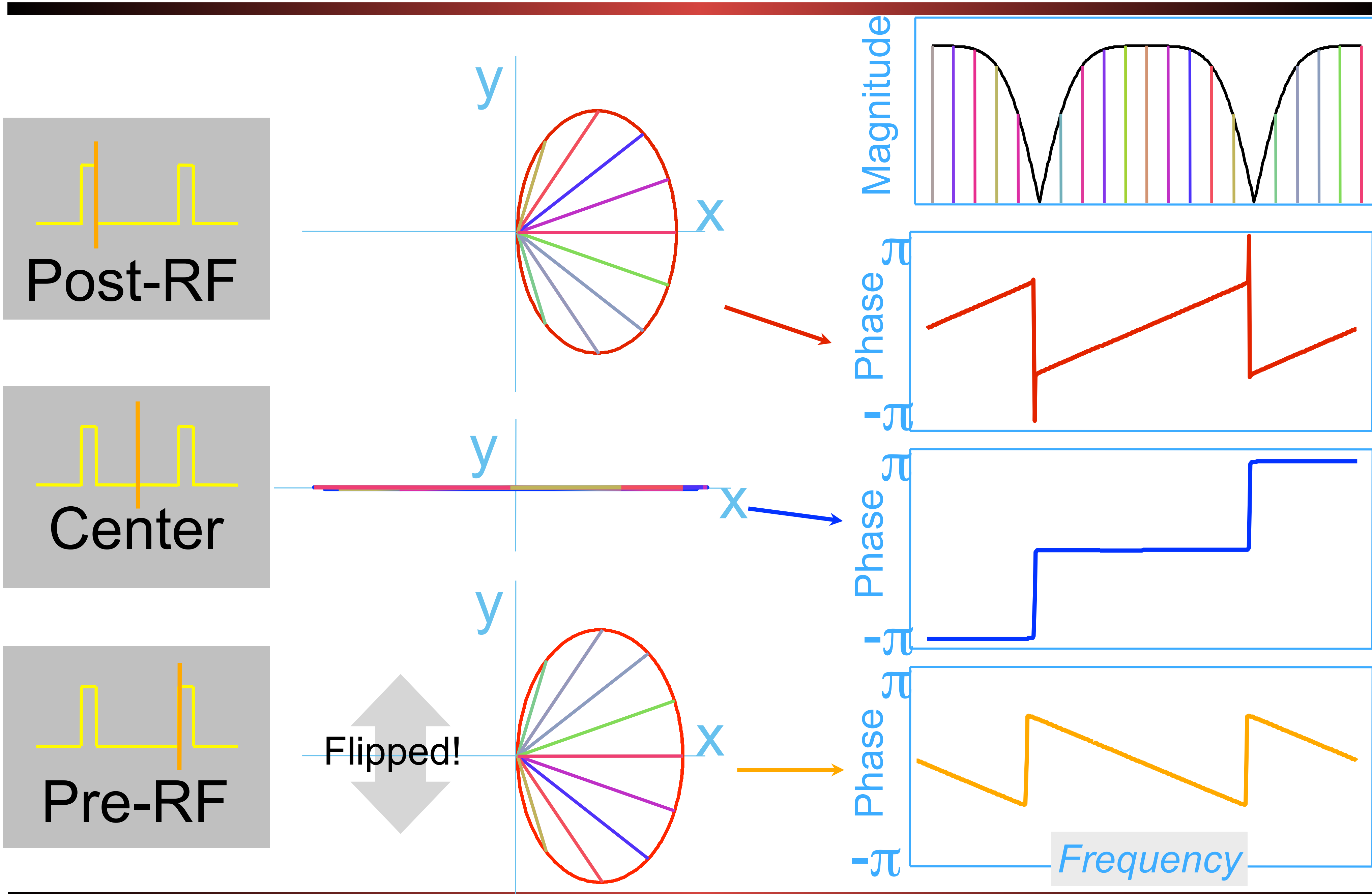
Freeman 1971



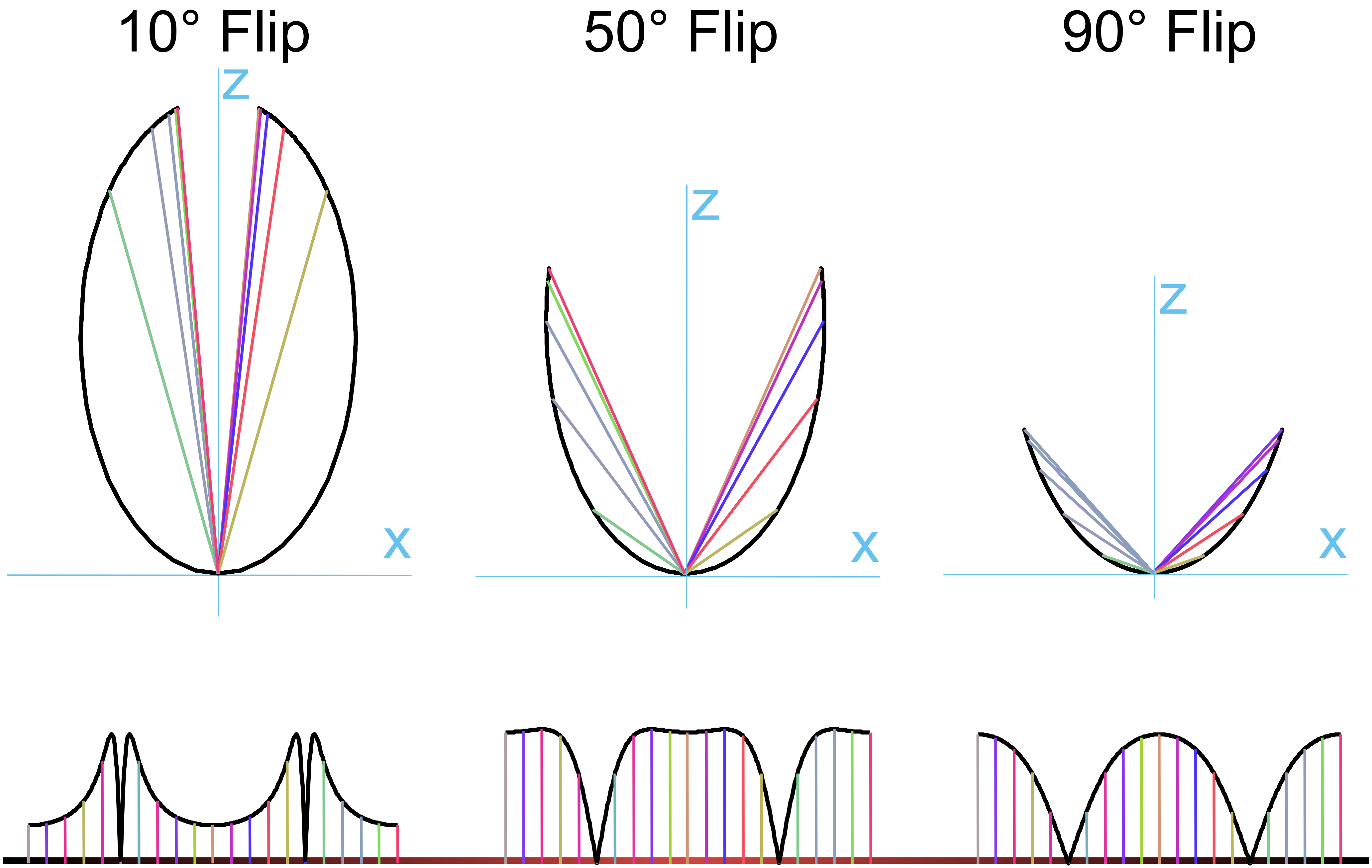
Signal depends on many factors:

- *Resonant frequency*
- T1, T2 (contrast)
- TR, TE
- RF flip / phase

Signal vs Frequency: Phase



Flip angle effects

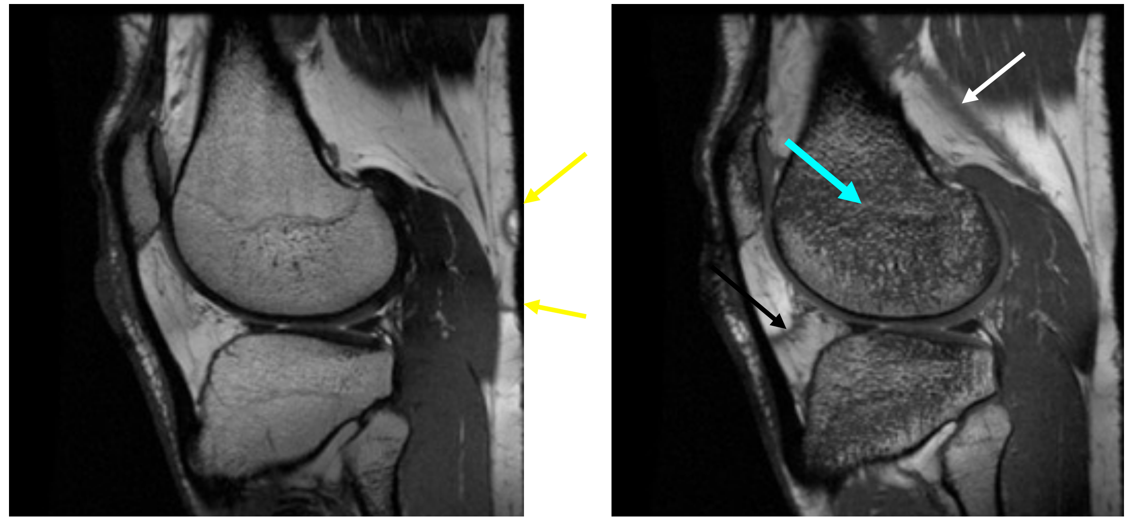


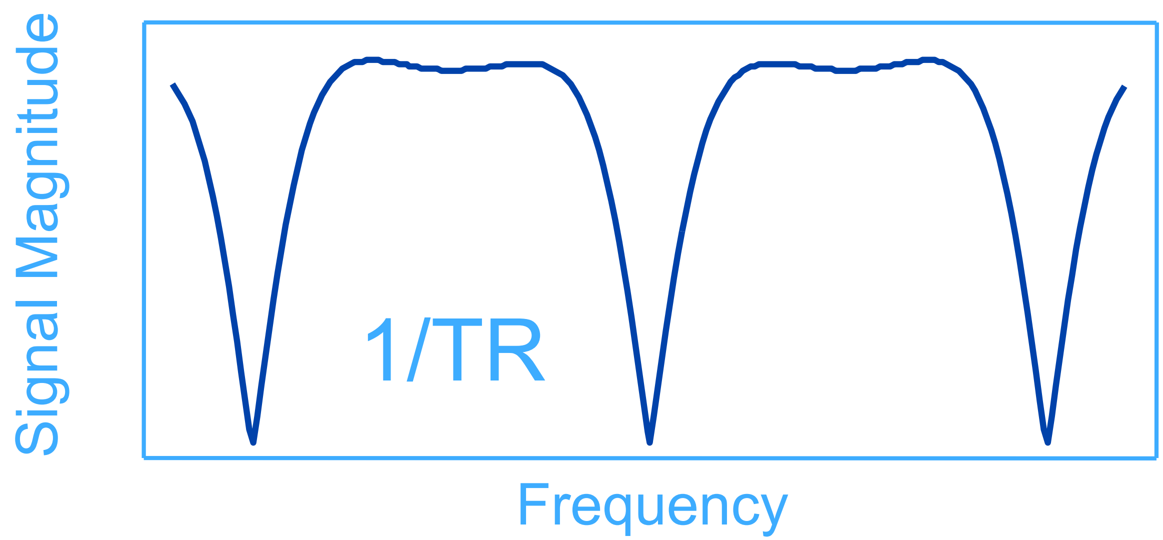
bSSFP Dark Bands

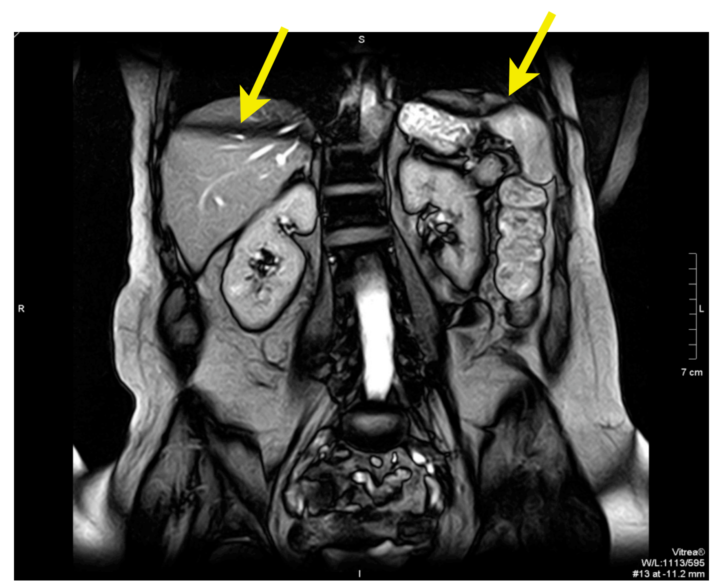
Must limit precession:

Short TR

Limits resolution

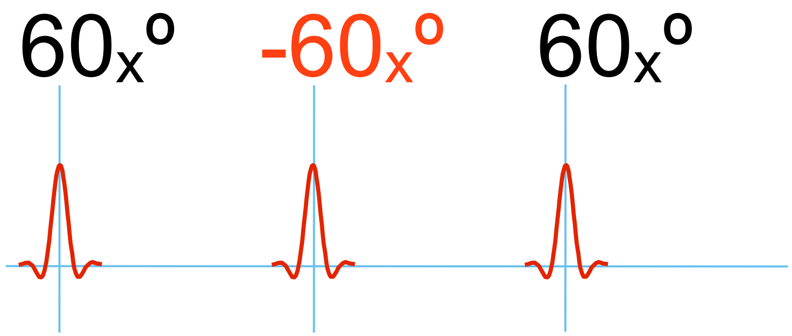


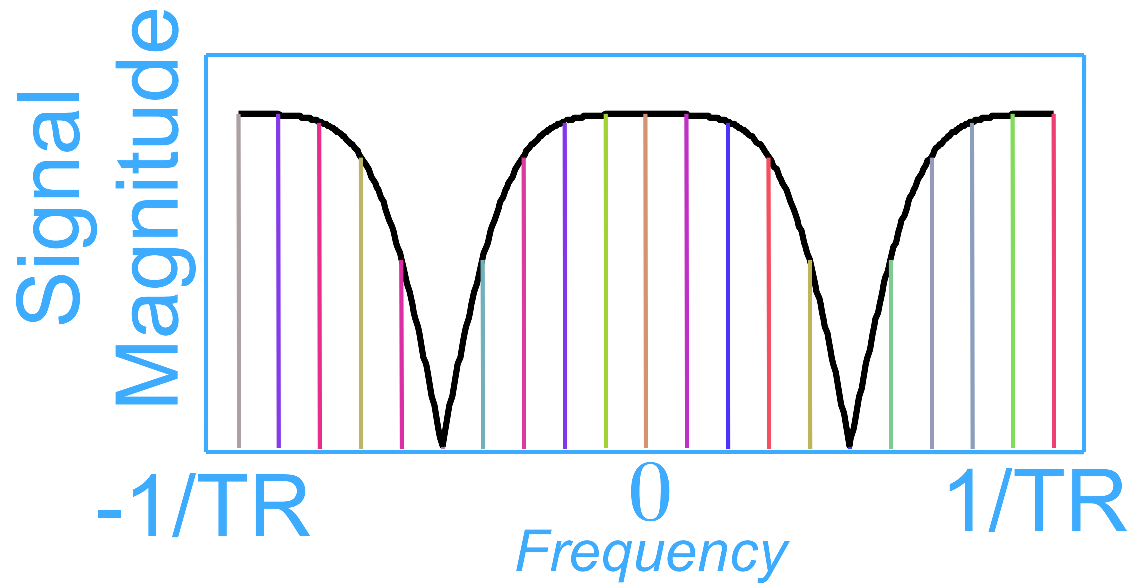


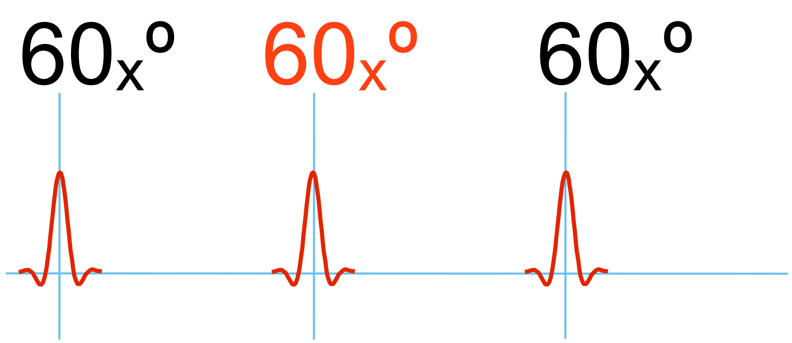


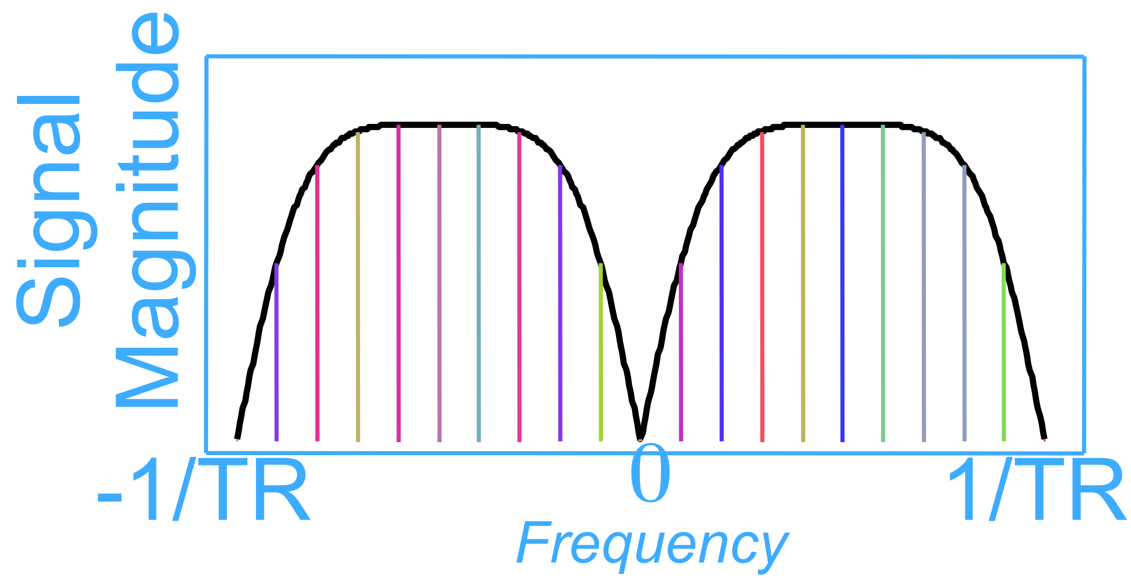
Phase Cycling

Hinshaw 1976





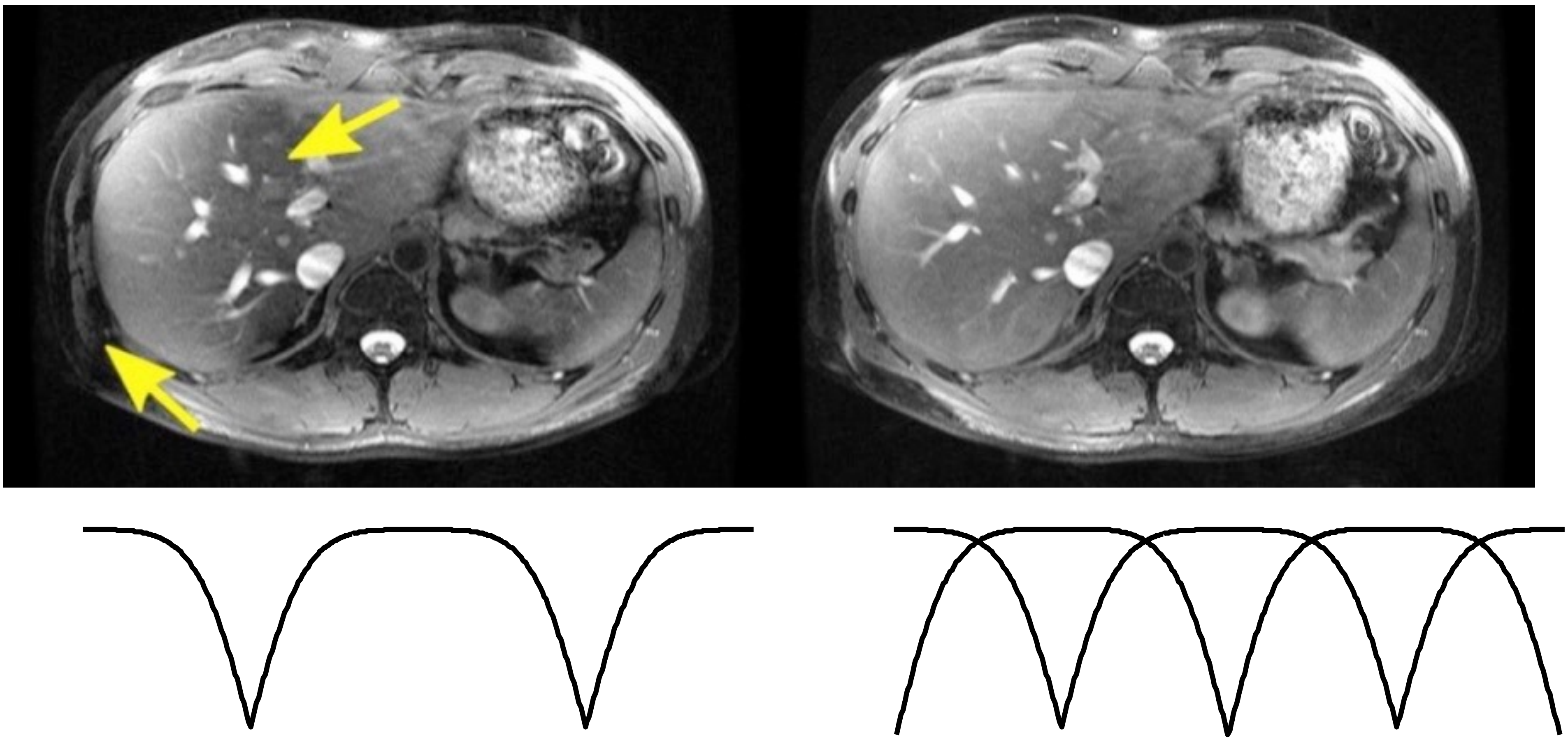




Review Question: Phase Cycling

Balanced SSFP Abdomen Example

Alternating RF | Combined Acquisition

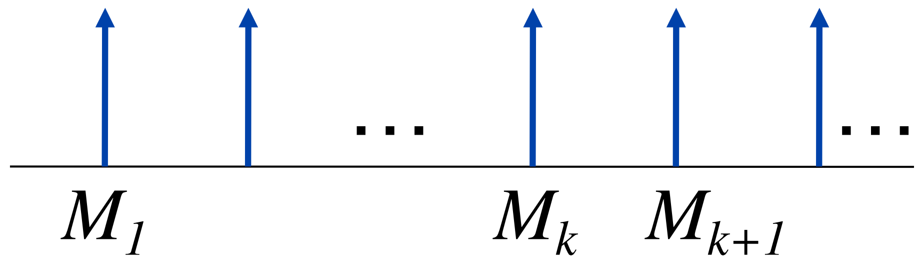


Matrix Solutions

(Jaynes 1955)

Apply 3x3 matrix scheme:

$$M_{k+1} = A\,M_k + B \qquad [1]$$



In steady-state:

$$M_{ss} = AM_{ss} + B \qquad [2]$$

$$M_{k+1} - M_{ss} = A(M_k - M_{ss}) \qquad [1\text{-}2]$$

Consider Eigenvector Decomposition:

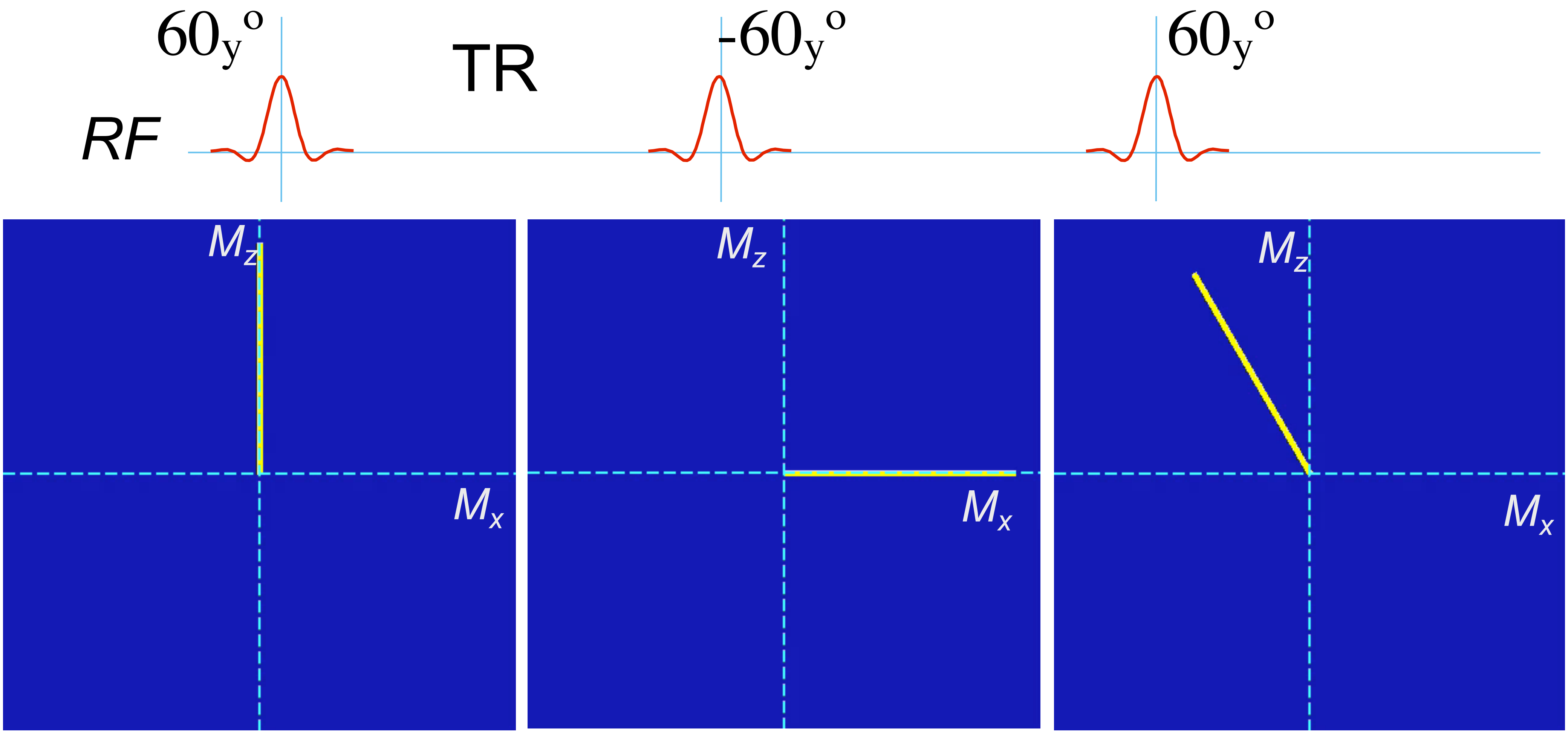
$$A = V \Lambda V^{-1} \qquad M_{k+1} - M_{ss} = (V\Lambda)\ V^{-1}(M_k - M_{ss})$$

At least one eigenvector/value is real.

Others often oscillatory and die out in steady state

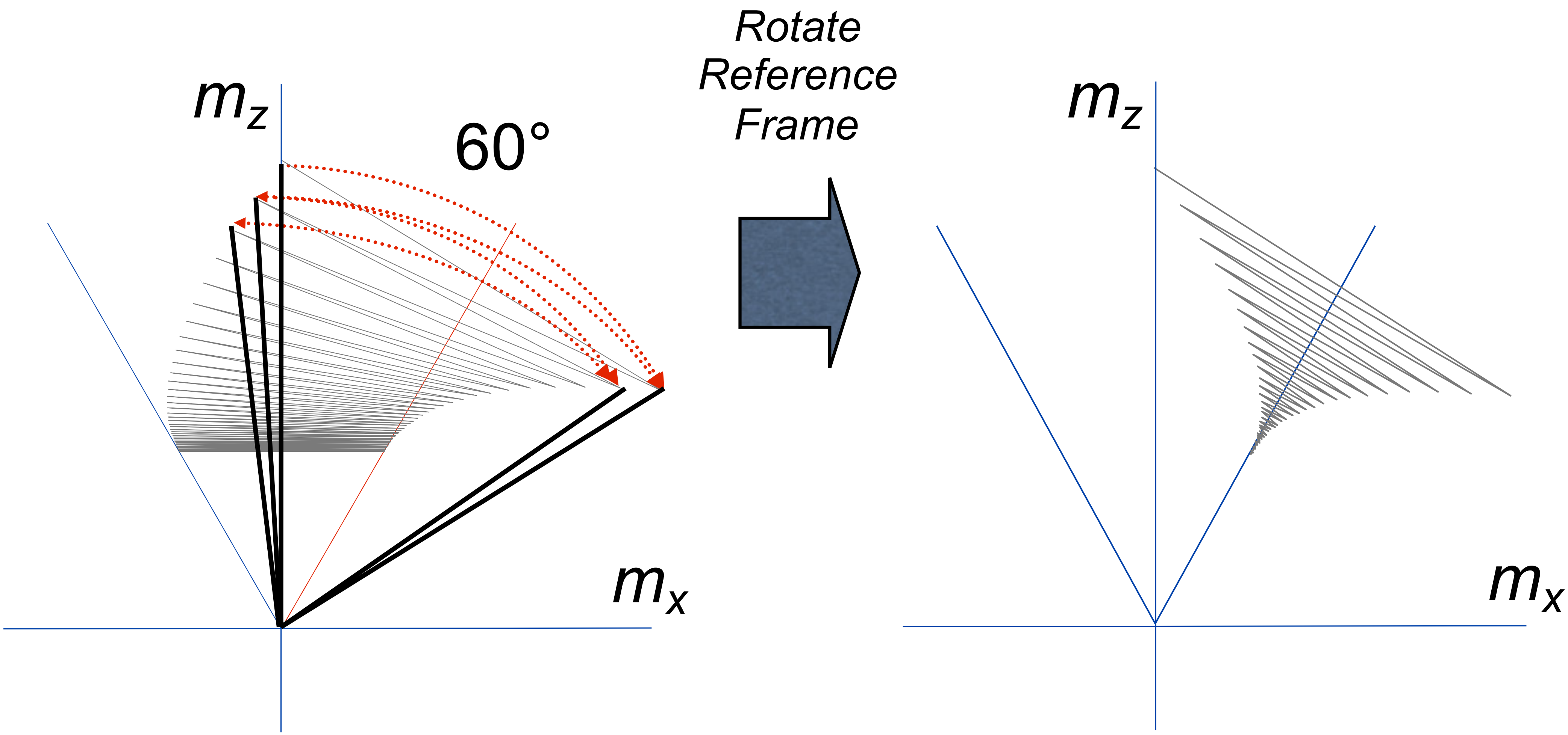
Note in [2] A is mostly rotation, B is small, so M_{ss} lies almost along the real-valued eigenvector

bSSFP: Transients and Steady States



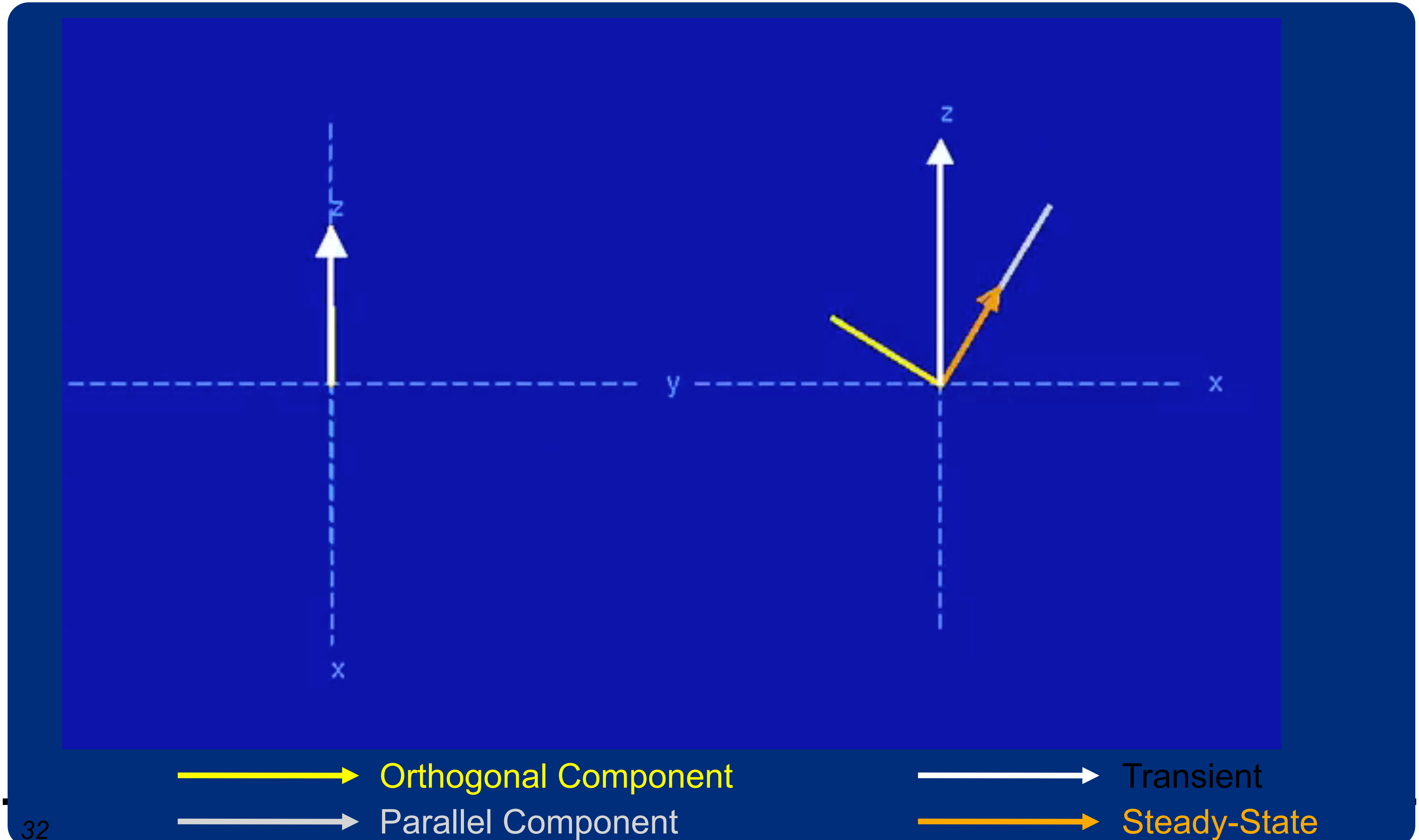
- Same periodic steady state
- Transient paths differ based on initial state

Balanced SSFP: Transient



Transient: Off-resonance

Viewed along Steady-State Vector | Orthogonal View



Transients (General)

- Generally include two components:
  - Smooth exponential (useful!)
  - Oscillatory (problematic)

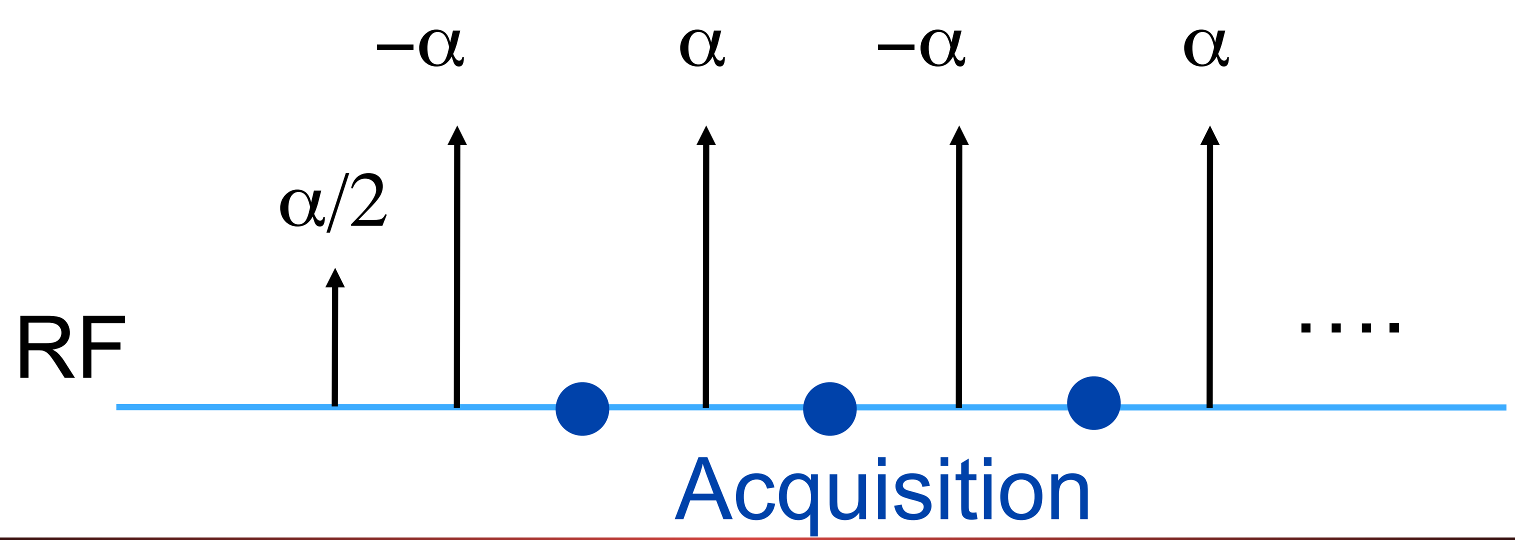
- Smooth transient is along steady-state direction
- Manipulate to steady-state direction to avoid oscillations

Review Question: Transients

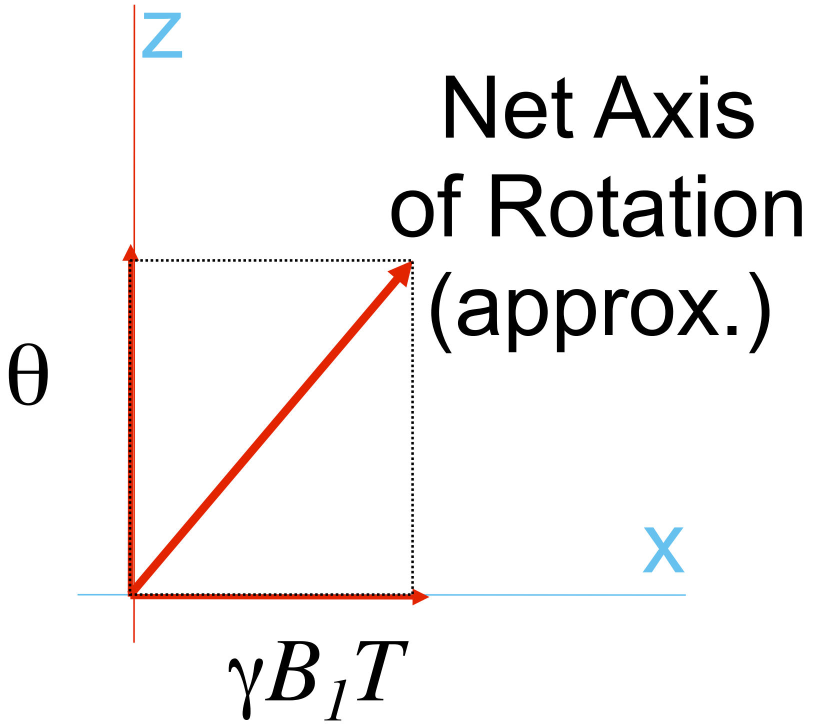
Half-TR, $\alpha/2$ Setup

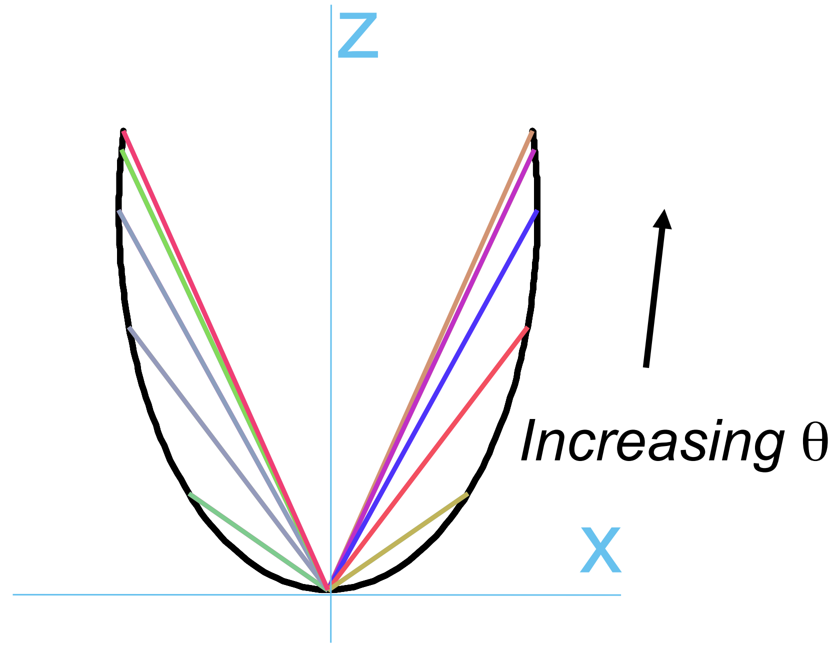
Deimling and Heid, 1994

- First RF pulse has half-amplitude
- Pulse applied TR/2 before next RF pulse
- (More complicated schemes exist)



bSSFP Direction: Some Intuition





- Magnetization aligns to rotation axis
- Rotation θ includes RF phase-increment
- As θ approaches 0°, axis is transverse, signal dies out
- Negative θ means steady state on -x

bSSFP Steady-State: Summary

- Ellipsoidal distribution: shape given by T_2/T_1
- Path depends on flip angle and precession
- Signal very sensitive to resonant frequency