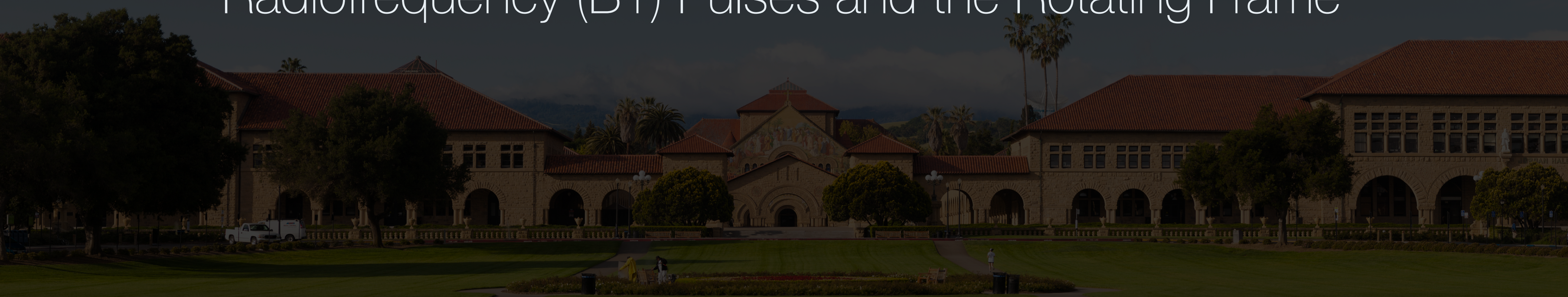
Rad229 – MRI Signals and Sequences

Daniel Ennis & Brian Hargreaves
dbe@stanford.edu –or– bah@stanford.edu

Lecture-01B — The Bloch Equations and Image Contrast

Radiofrequency (B1) Pulses and the Rotating Frame

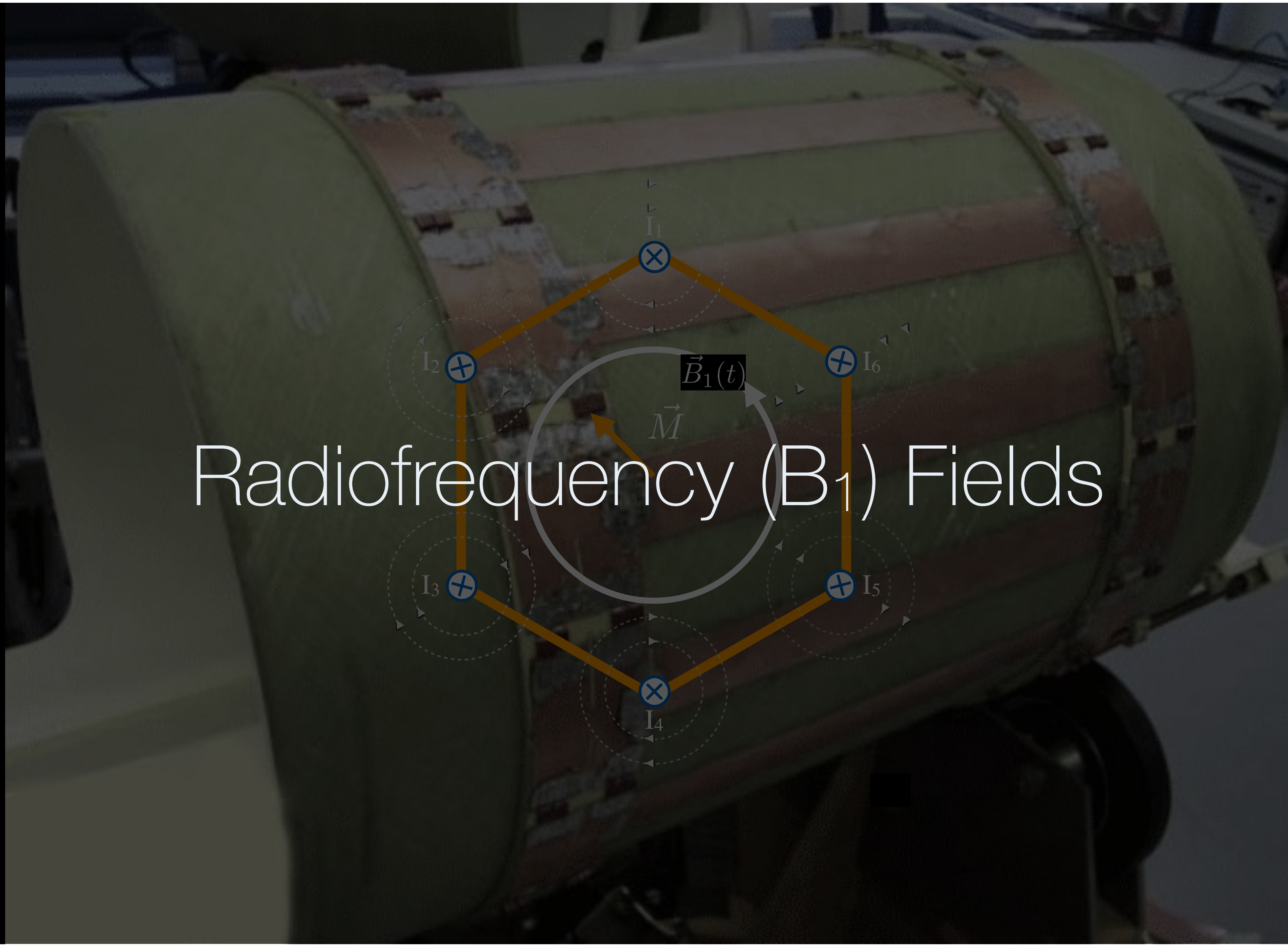


Daniel Ennis
dbe@stanford.edu

Learning Objectives

- Explain the principal function of the RF (B_1) field.
  - Describe the magnitude, spatial, and temporal characteristics of the B_1 field.
  - Define B_1 with a mathematical expression.
- Distinguish between spin, precession, and nutation.
- Differentiate between the lab and rotating frames.
- Remember an expression for a circular polarized RF field.
- Contrast the equation of motion in the lab and rotating frames.

Radiofrequency (B_1) Fields



MRI Hardware

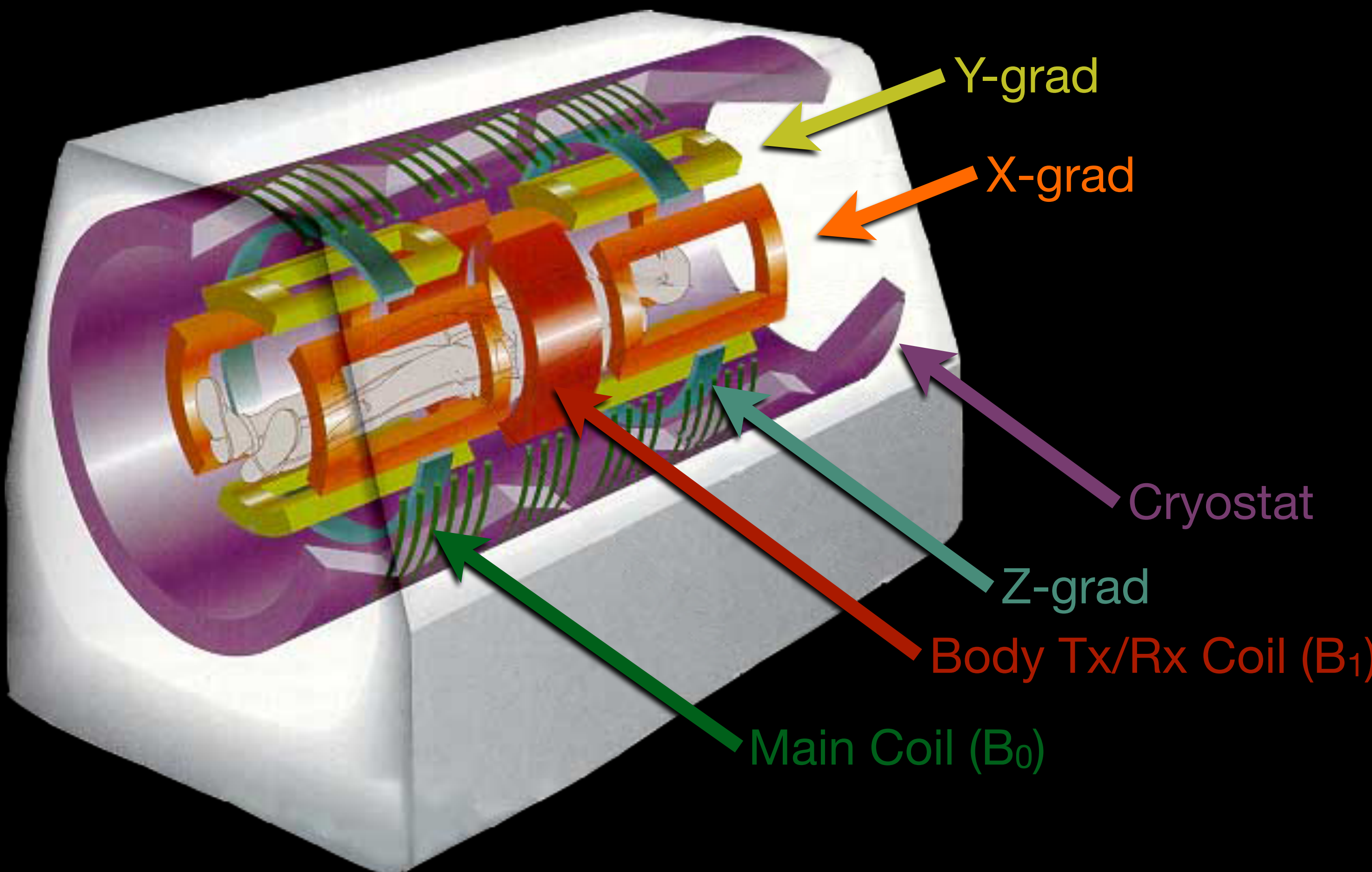
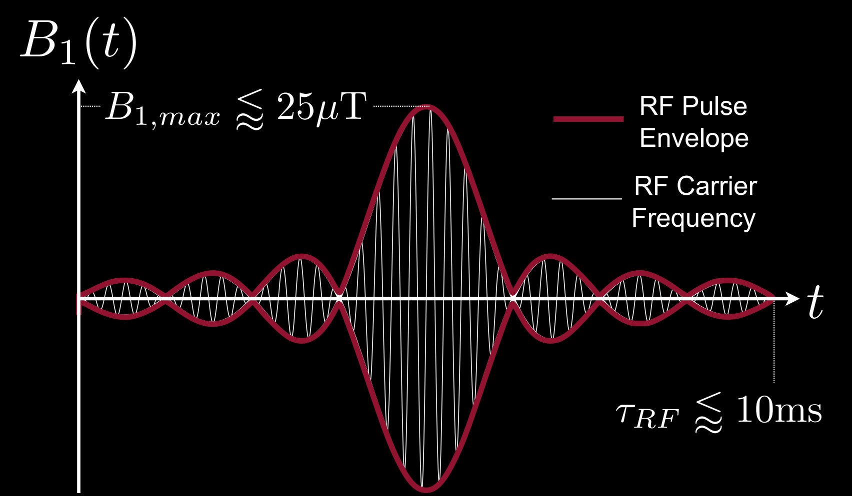


Image Adapted From: http://www.ee.duke.edu/~jshorey

RF (B_1) Field - Characteristics

- B_1 is a
  - radiofrequency (RF)
    - 42.58MHz/T for ^{1}H (63MHz at 1.5T)
  - short duration pulse (~0.1 to 10ms)
  - small amplitude
    - <25 μT
  - shaped by an envelope function
  - circularly polarized
    - rotates at Larmor frequency
  - magnetic field
  - perpendicular to B_0
- Principal use: excitation and refocusing.
  - Image contrast (inversion, saturation)
  - Signal spoiling



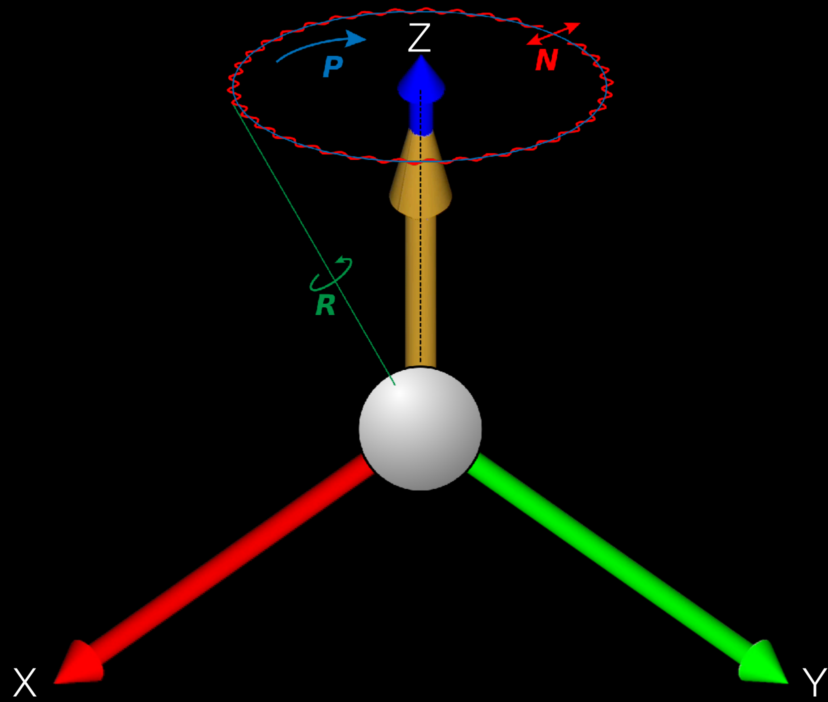
RF Excitation - Lab Frame

^{1}H has intrinsic Spin

$\omega_0 = \gamma B_0$ Precession

Combined with…

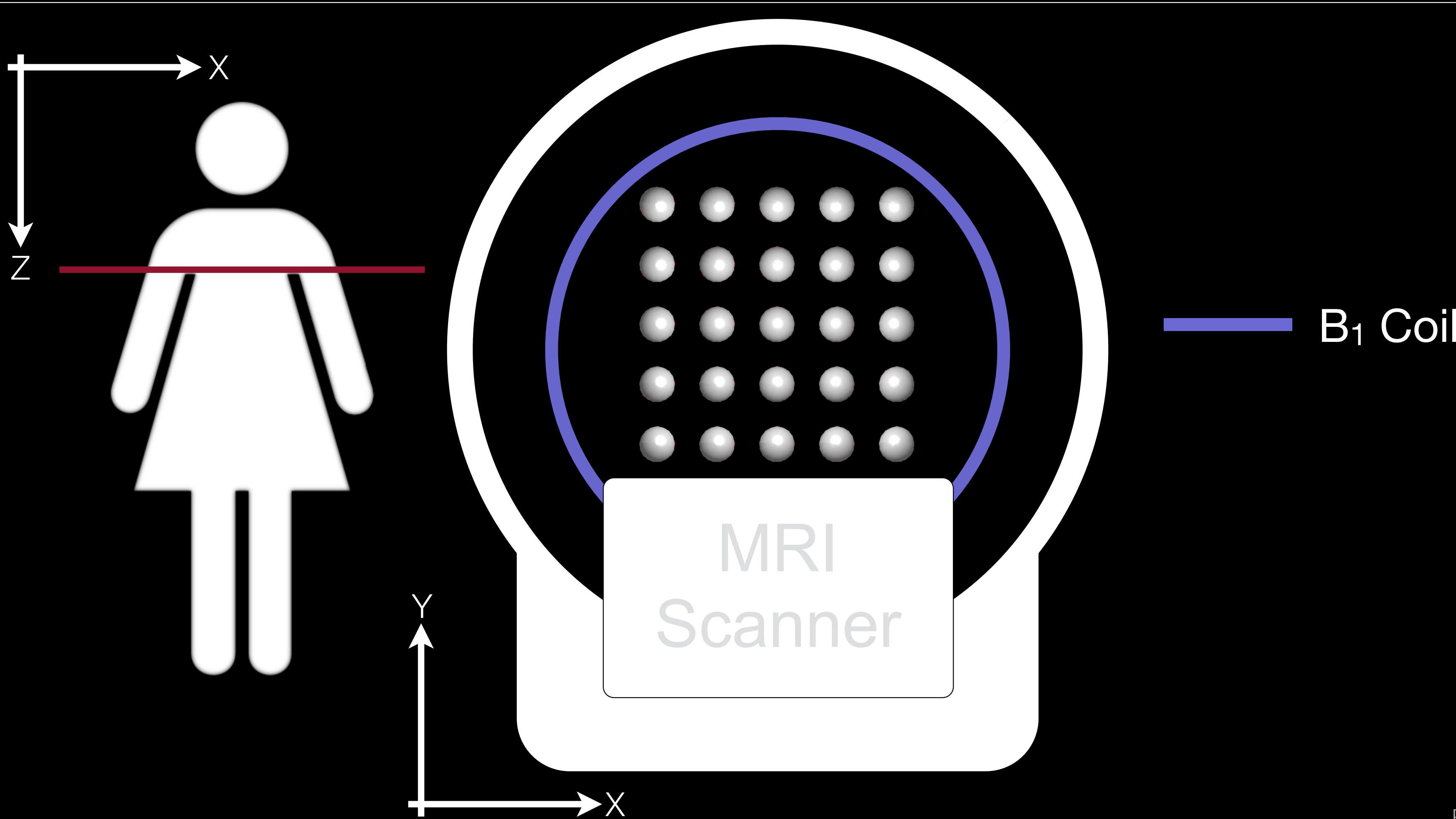
$\omega_1 = \gamma B_1$ Nutation



RF (B_1) pulse impart forced precession.

B_0 causes precession about z-axis. B_1 causes forced precession (i.e. nutation). https://en.wikipedia.org/wiki/Nutation

RF Excitation - Lab Frame



RF pulses can generate transverse magnetization (Mxy).

Basic RF Pulse - Linear Polarized

$$\vec{B}_1(t) = 2B_1^e(t)\cos(\omega_{RF}t + \theta)\hat{i}'$$

- $\vec{B}_1(t)$: applied B_1 field
- $B_1^e(t)$: pulse envelope function
- ω_{RF}: excitation carrier frequency
- θ: initial phase angle
- $\hat{i}'$: linearly polarized

$$\omega_{RF} = \omega_0$$

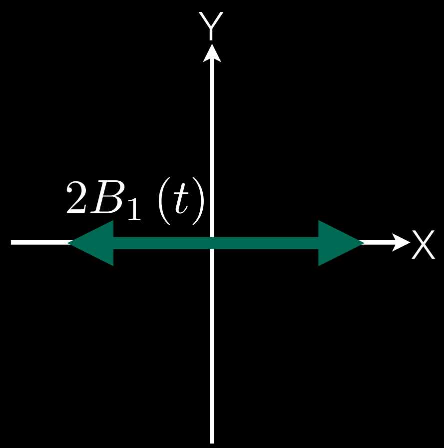
Resonance Condition

B_1 is perpendicular to B_0.

Linear & Circular Polarized B_1 Fields

Linear Polarization

$$\vec{B}_1(t) = 2B_1^e(t)\cos(\omega_{RF}t + \theta)\hat{i}'$$



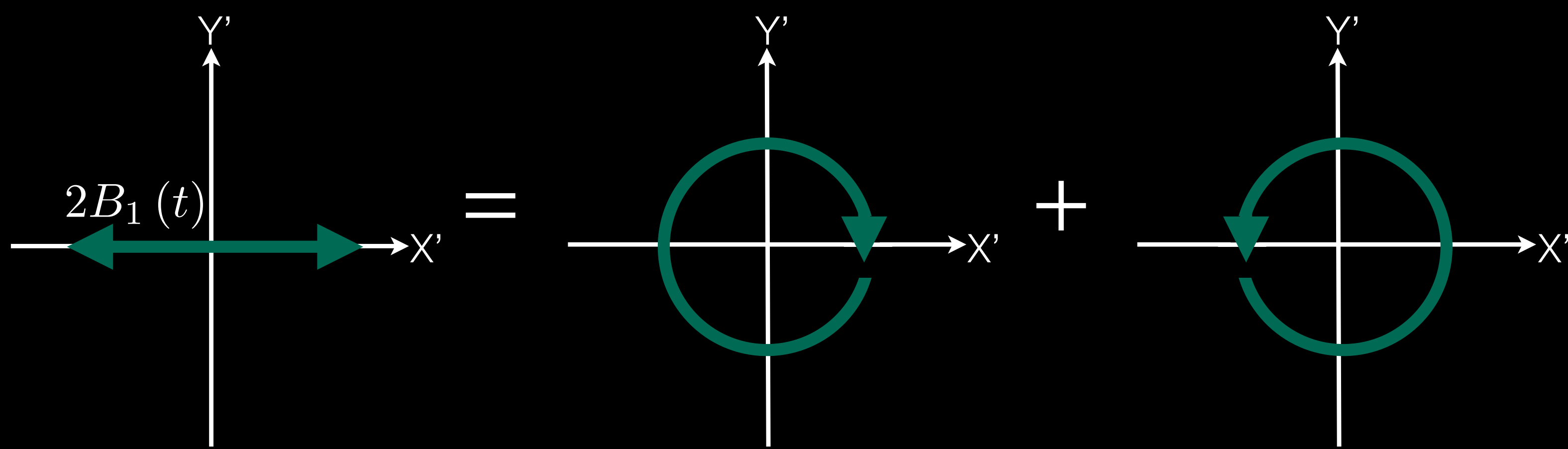
First Generation MRI Systems Used Linear Polarization; Simple and Cheaper, but higher RF power.

Arrow indicates direction of B-field.

Linear & Circular Polarized B_1 Fields

Linear Polarization | CW Circular Polarization | CCW Circular Polarization

$$\vec{B}_1(t) = 2B_1^e(t)\cos(\omega_{RF}t+\theta)\hat{i}' = B_1^e(t)\left[\cos(\omega_{RF}t)\hat{i}' - \sin(\omega_{RF}t)\hat{j}'\right] + B_1^e(t)\left[\cos(\omega_{RF}t)\hat{i}' + \sin(\omega_{RF}t+\theta)\hat{j}'\right]$$



First Generation MRI Systems Used Linear Polarization; Simple and Cheaper, but higher RF power.

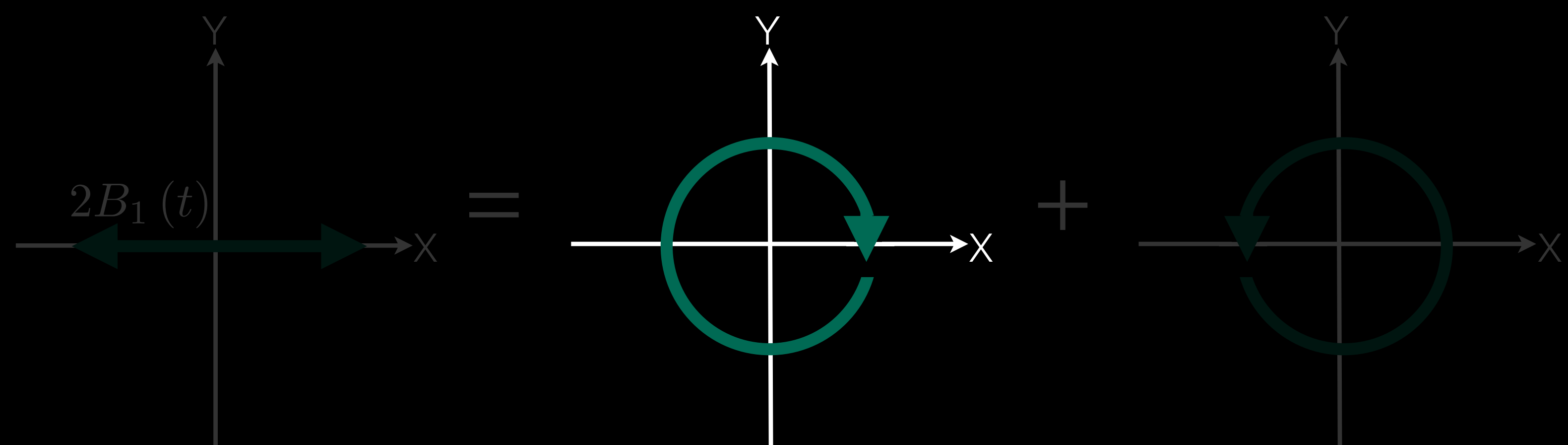
On-Resonance Excitation+Heating

Very Off-Resonance Heating

Arrow indicates direction of B-field.

Linear & Circular Polarized B_1 Fields

Linear Polarization		CW Circular Polarization		CCW Circular Polarization
$\vec{B}_1(t) = 2B_1^e(t)\cos(\omega_{RF}t+\theta)\hat{i}'$	=	$B_1^e(t)\left[\cos(\omega_{RF}t)\hat{i}' - \sin(\omega_{RF}t)\hat{j}'\right]$	+	$B_1^e(t)\left[\cos(\omega_{RF}t)\hat{i}' + \sin(\omega_{RF}t+\theta)\hat{j}'\right]$



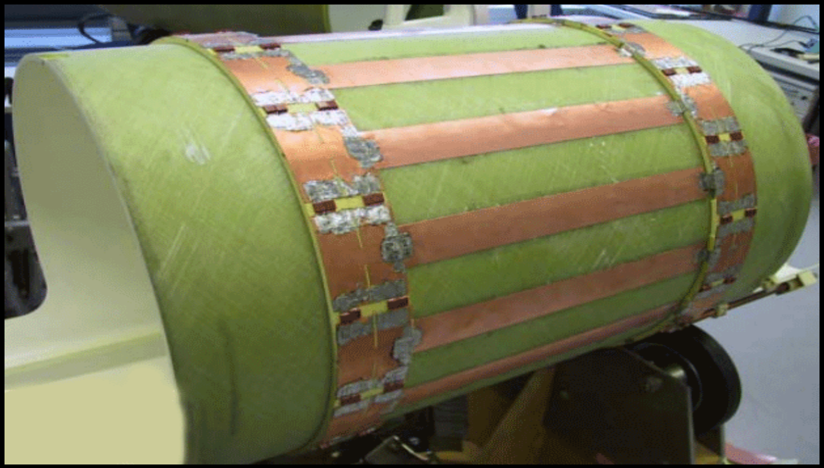
First Generation MRI Systems Used Linear Polarization; Simple and Cheaper, but higher RF power.	On-Resonance Excitation+Heating Modern MRI systems only use CW circular polarization	Very Off-Resonance Heating Modern MRI systems don't apply the CCW field

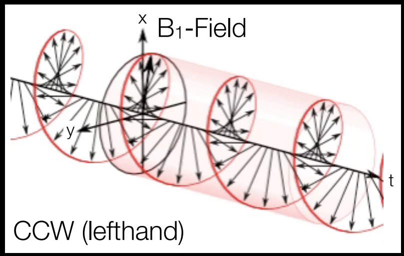
Even with circular polarization many pulse sequences are SAR limited at ≥3T.

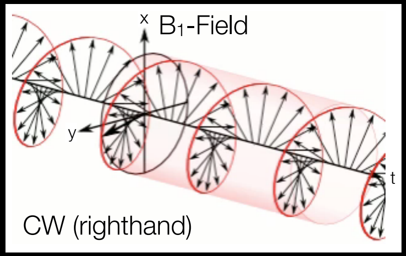
RF (B_1) Field - Generation

- Birdcage coil
  - Most common design
  - Highly efficient
  - Nearly all of the fields produced contribute to imaging
- Very uniform field
  - Especially radially
  - Decays axially
  - Uniform sphere if L≈D
- Generates a “quadrature” field
  - Circular polarization

Birdcage Body Tx/Rx Coil (B_1)







http://mri-q.com/birdcage-coil.html —and— https://en.wikipedia.org/wiki/Circular_polarization

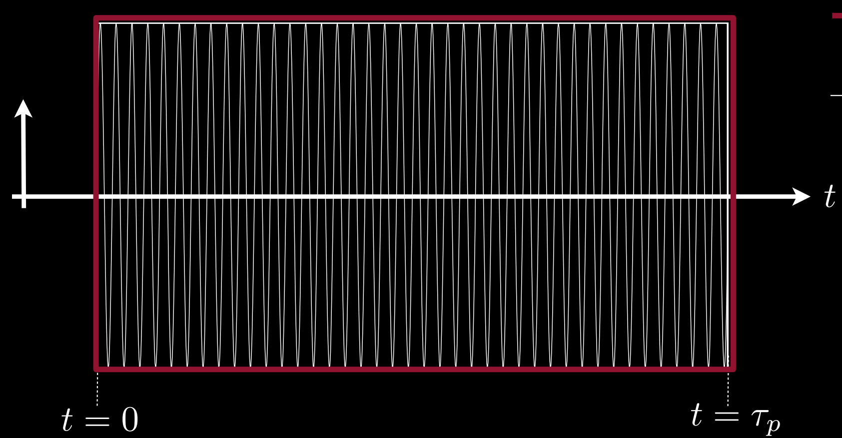
Basic RF Pulse - Circular Polarized

$$\vec{B}_1(t) = B_1^e(t)\left[\cos(\omega_{RF}t)\hat{i}' - \sin(\omega_{RF}t)\hat{j}'\right]$$

Circular Polarized RF Field

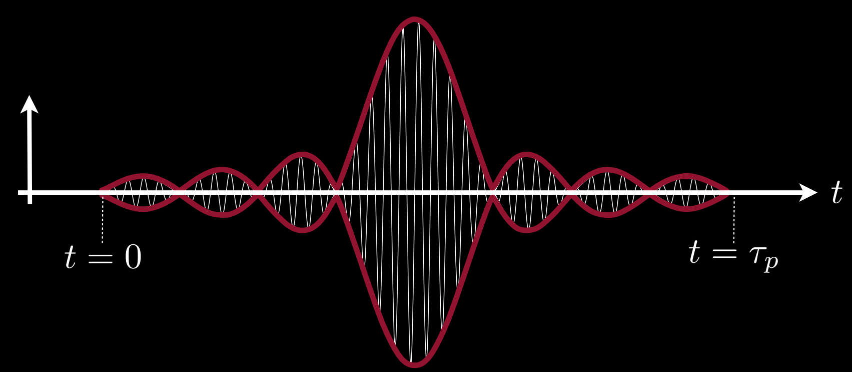
$$B_1^e(t) = B_1 \sqcap\left(\frac{t-\tau_p/2}{\tau_p}\right) = \begin{cases} B_1, & 0 \le t \le \tau_p \\ 0, & otherwise \end{cases}$$

$$B_1^e(t) = \begin{cases} B_1 \text{sinc}\left[\pi f_\omega \left(t - \tau_p/2\right)\right], & 0 \le t \le \tau_p \\ 0, & otherwise \end{cases}$$



RF Pulse Envelope

RF Carrier Frequency



RECT functions are used to excite a "single" frequency. SINC functions are used to excite a narrow band of frequencies.

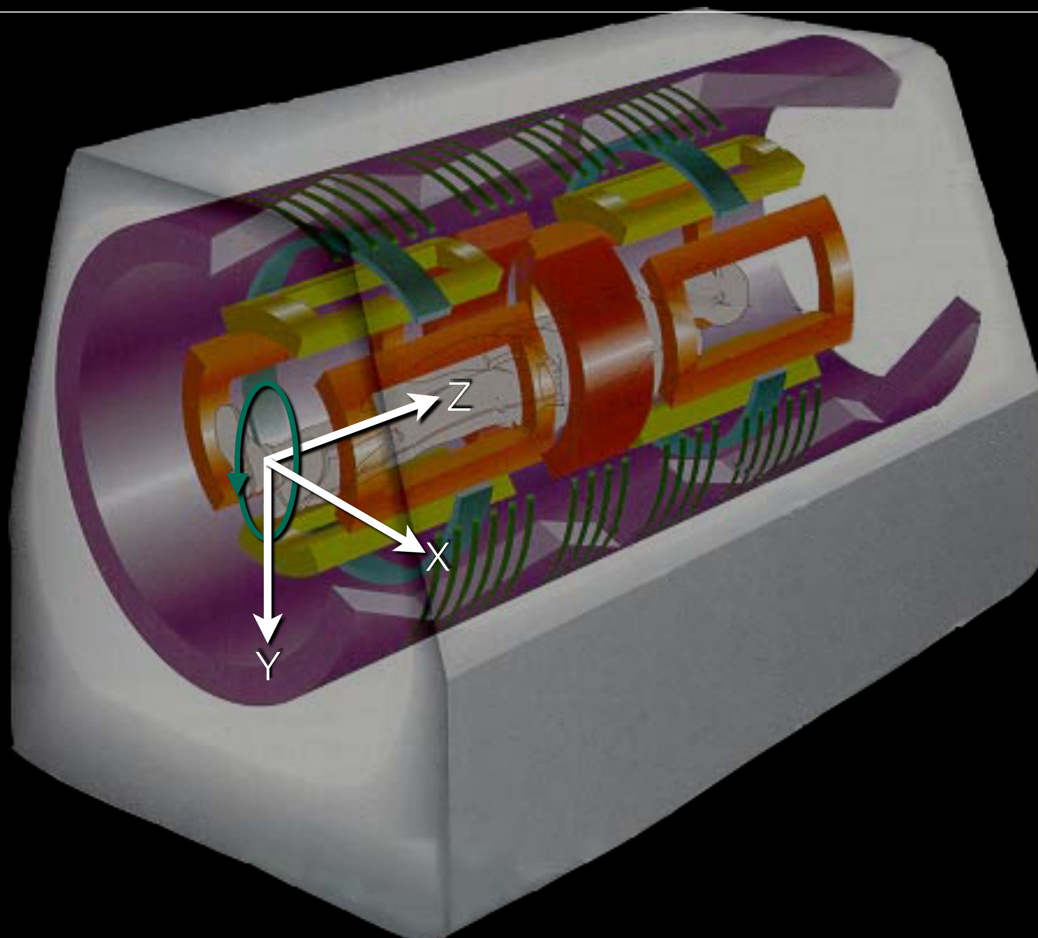
Lab vs. Rotating Frame

Laboratory Frame Coordinates

$$\vec{\omega} = \gamma B_0 \hat{k}'$$

↑

Rotational Angular Velocity



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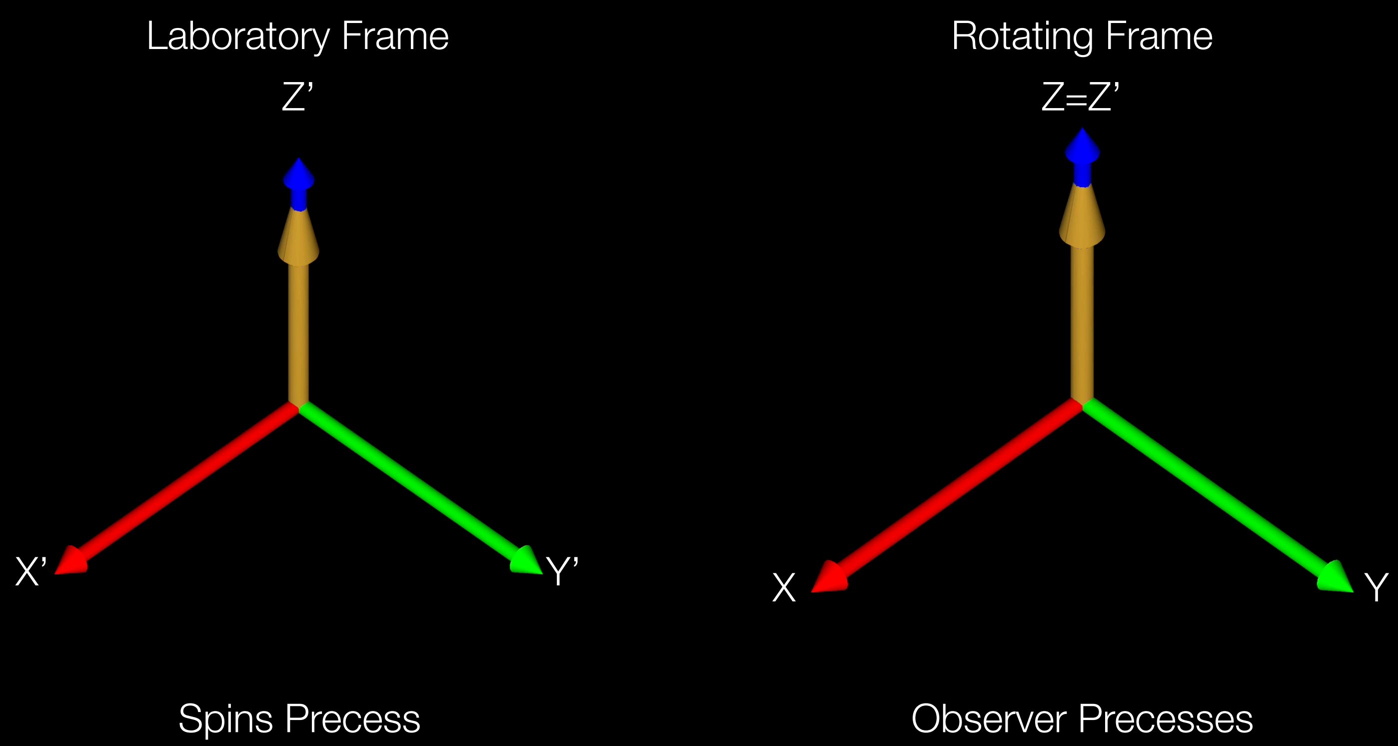
Lab vs. Rotating Frame Coordinates

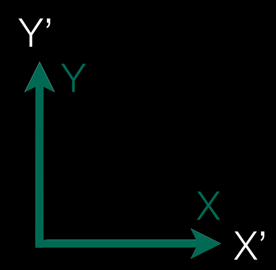
- The rotating frame simplifies the mathematics; more intuitive understanding.



Note: Both coordinate frames share the same z-axis.

Rotating Frame Coordinates

- If the rotational frequency of the rotating frame is matched to the bulk magnetization's precessional frequency, then rotational motion of the bulk magnetization is "removed" or demodulated.
- The rotating frame's transverse xy-plane rotates clockwise (left-handed) at frequency ω relative to the scanners x'y'-plane.



The rotating frame simplifies the mathematical description of spin dynamics and affords a more intuitive understanding.

Equation of Motion

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of motion for an ensemble of spins (isochromats). [Laboratory Frame]

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \left(\frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \right)$$

Equation of motion for an ensemble of spins (isochromats). [Rotating Frame]

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of B_0.

Applied B-field in the rotating frame.

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Equation of motion for an ensemble of spins (isochromats). [Rotating Frame]

Free Precession in the Rotating Frame w/o Relaxation

$$\begin{aligned}\vec{B}_{eff} &= \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \\ &= \frac{-\gamma B_0 \hat{k}'}{\gamma} + B_0 \hat{k}' \\ &= 0\end{aligned}$$

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\frac{d\vec{M}_{rot}}{dt} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ 0 & 0 & 0 \end{vmatrix}$$

$$\frac{dM_x}{dt} = 0$$

$$\frac{dM_y}{dt} = 0$$

$$\frac{dM_z}{dt} = 0$$

$$M_x(t) = M_x^0$$

$$M_y(t) = M_y^0$$

$$M_z(t) = M_z^0$$

Constant components in the rotating frame without relaxation.

The bulk magnetization components in the rotating frame maintain the initial condition for all time in the absence of relaxation.

How do RF pulses tip the magnetization?

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