Rad229 – MRI Signals and Sequences

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Lecture-02A — *k*-space and Imaging

Magnetic Field Gradients

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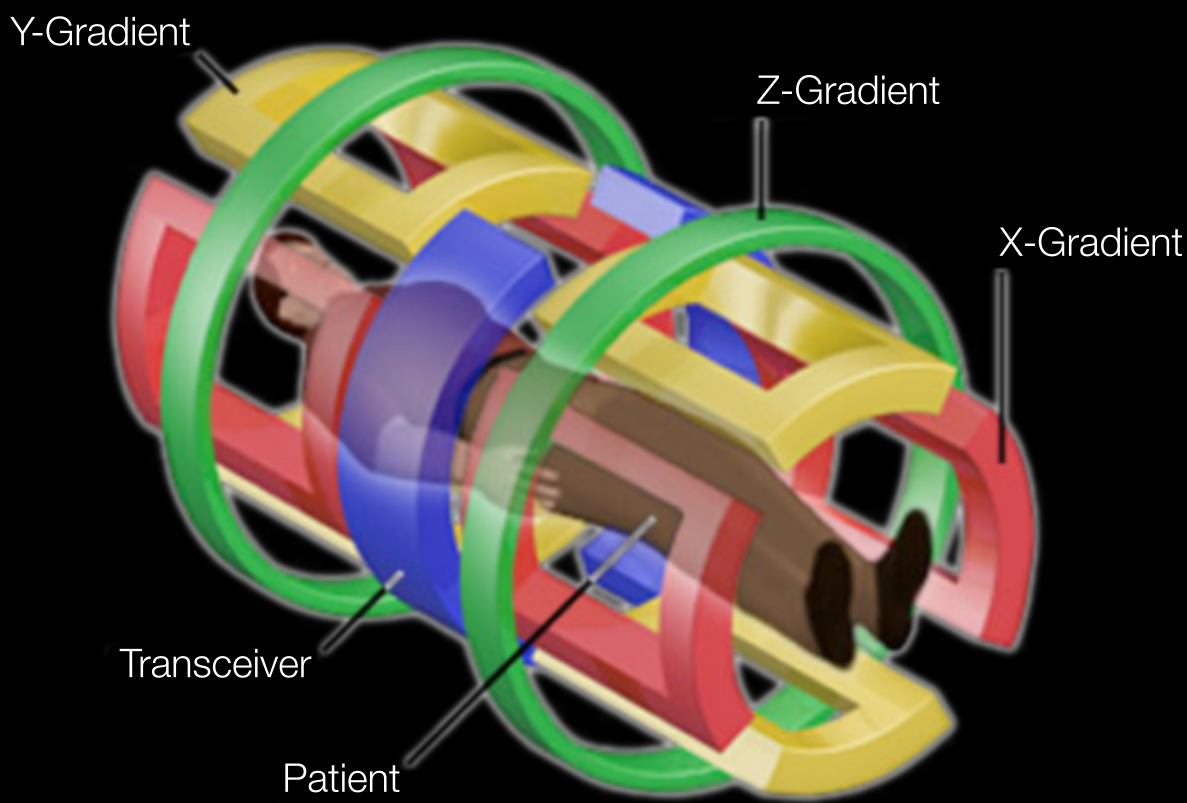
Learning Objectives

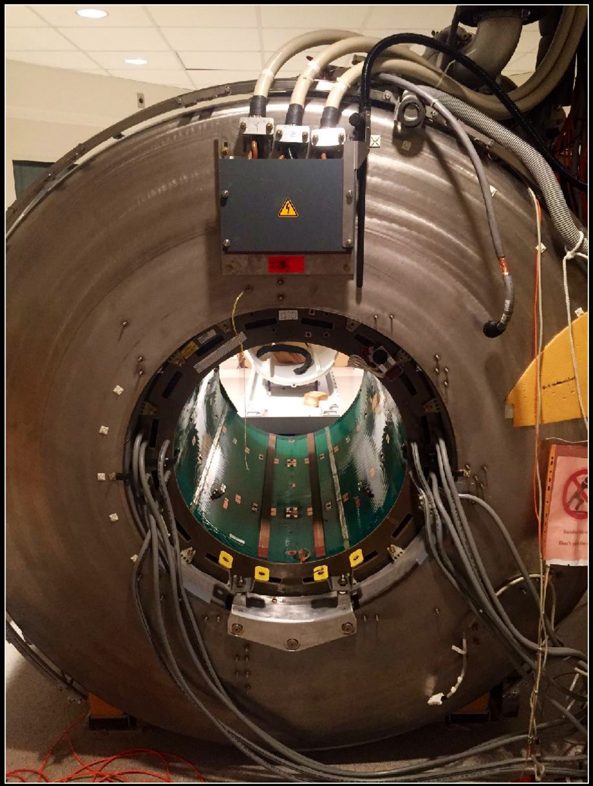
- Describe three different roles for gradients.
- Compare the spatial and temporal characteristics of the gradient fields to both B_0 and B_1.
- Be able to define the magnetic field at any point in space when a gradient is applied.
- Explain how gradients are used to manipulate either spin frequency or phase.
- Compare the assumptions between the B_0, B_1, and gradient fields.

Magnetic Field Gradients (G_x, G_y, G_z)

Gradient Hardware



http://www.magnet.fsu.edu



High voltage, high current, and water cooled.

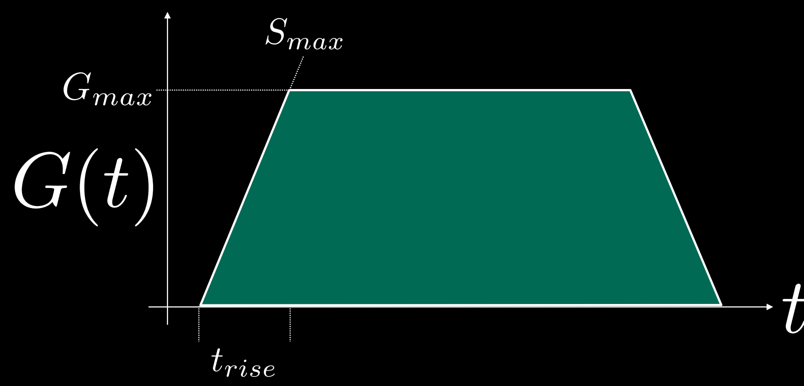
Modern gradient systems operate at the engineering limits of power delivery and heat dissipation.

Gradient - Principles

- Primary function
  - Encode spatial information
    - Slice selection
    - Phase encoding
    - Frequency encoding
- Secondary functions
  - Spin de-phasing to minimize artifacts (crushers & spoilers)
  - Spin re-phasing after slice selection
  - Spin pre-phasing before readout
  - Sensitize/de-sensitize images to motion

Gradient – Characteristics

- Intermediate strength
  - <80-100mT/m (<8-10 G/cm)
    - <0.0150T (150G) @ edge of 30cm FOV
  - Adds/Subtracts to the B_0 field
  - Parallel to B_0; only adds to B_0 in Z-direction
- Spatially varying
  - Linear B-field gradients
- Temporally dynamic (kHz)
  - Slewrate Max. <200T/m/s
    - Risetime ~500µs
- Gradients are NOT:
  - Fields perpendicular to B_0



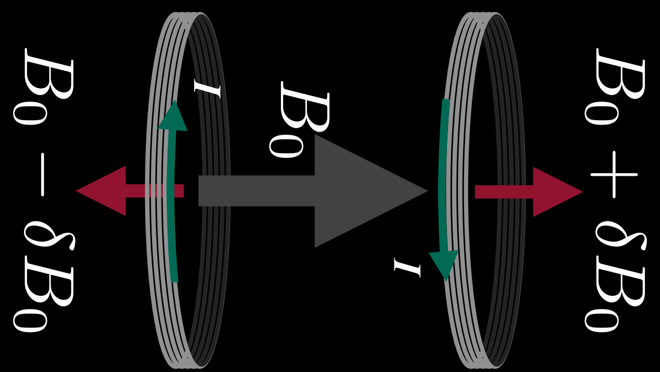
$$t_{rise,min} = G_{max}/S_{max}$$

Gradients are “linear” over ~40-50cm on each axis.

Gradient – Hardware

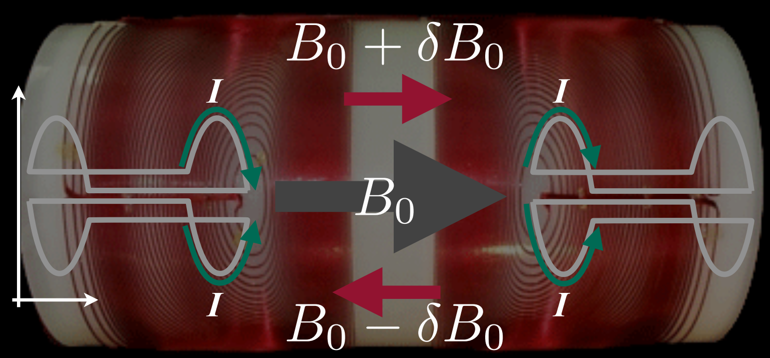
Z-Gradient

- Typically Maxwell pair coil
- Linear B-field variation along *z*-
- Highly uniform in the *xy*-plane
- Currents are equal-and-opposite

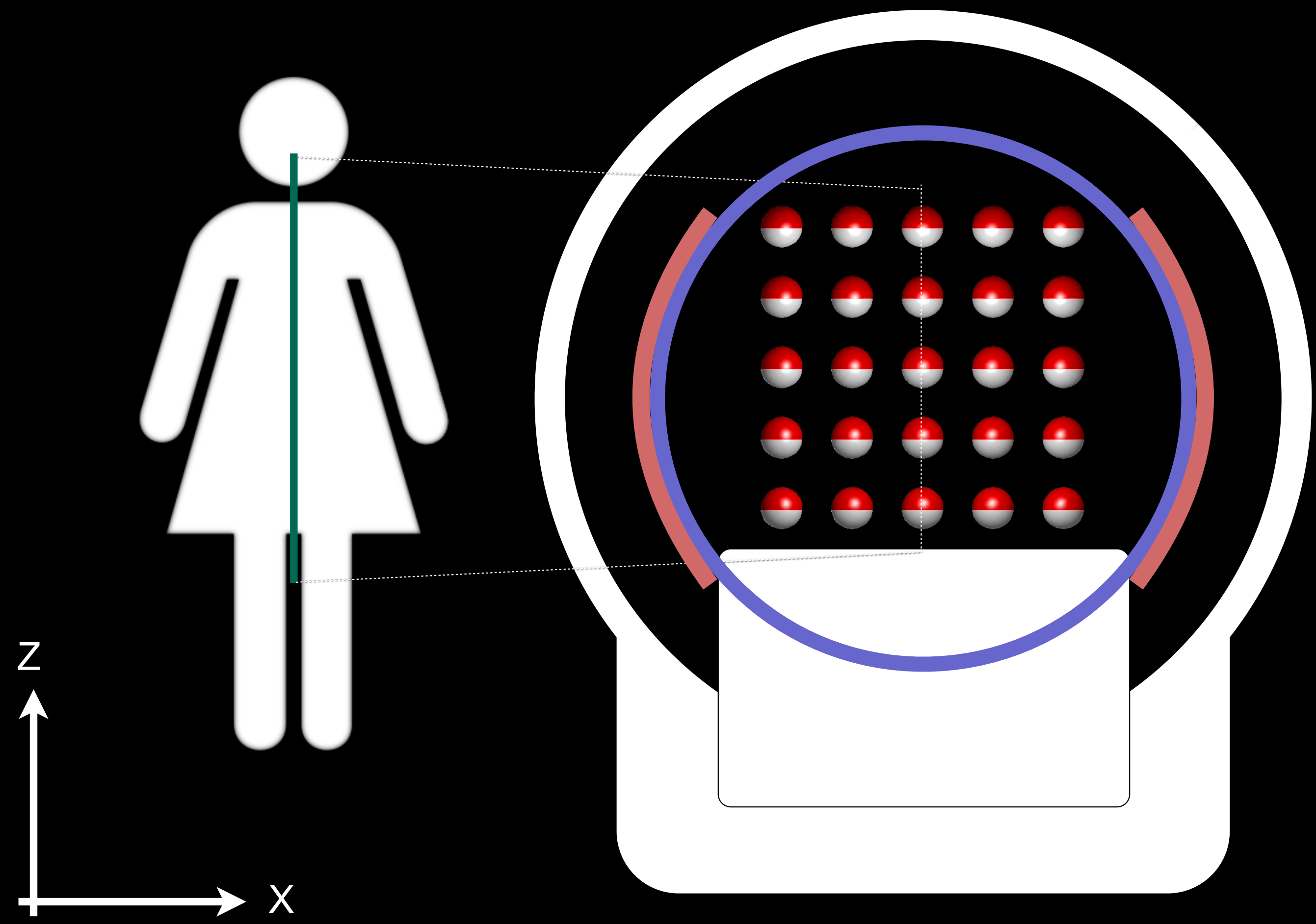


X- and Y-Gradients

- Typically “saddle coils”; Golay Coil Pairs
- Linear B-field variation along *x*- or *y*-
- Highly uniform in *yz*- or *xz*-plane
- Innermost turns produce gradient field



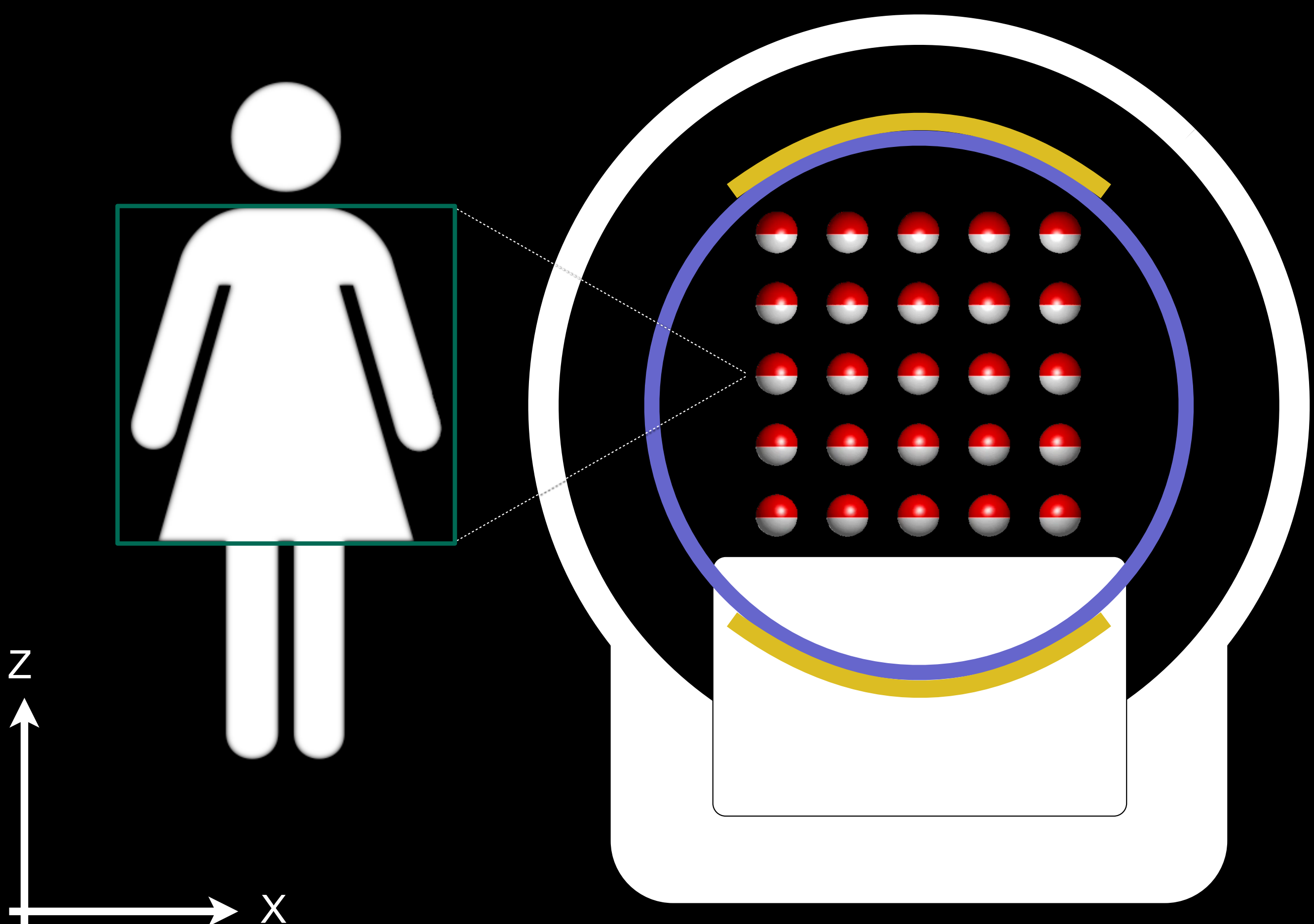
Spins and X-Gradients



Which coordinate frame is used to display the spins?

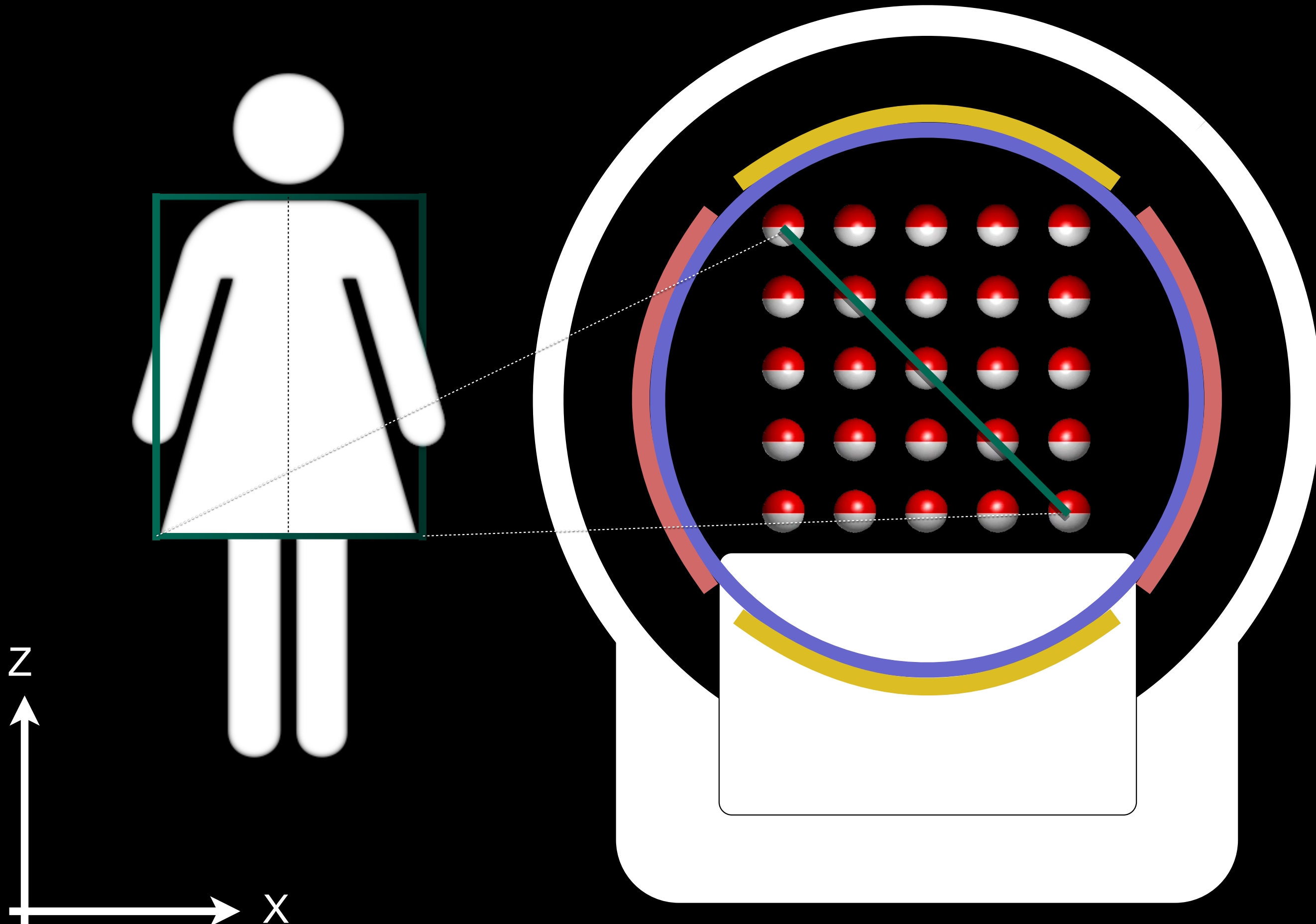
Gradients give rise to isochromats (planes of common frequency).

Spins and Y-Gradients



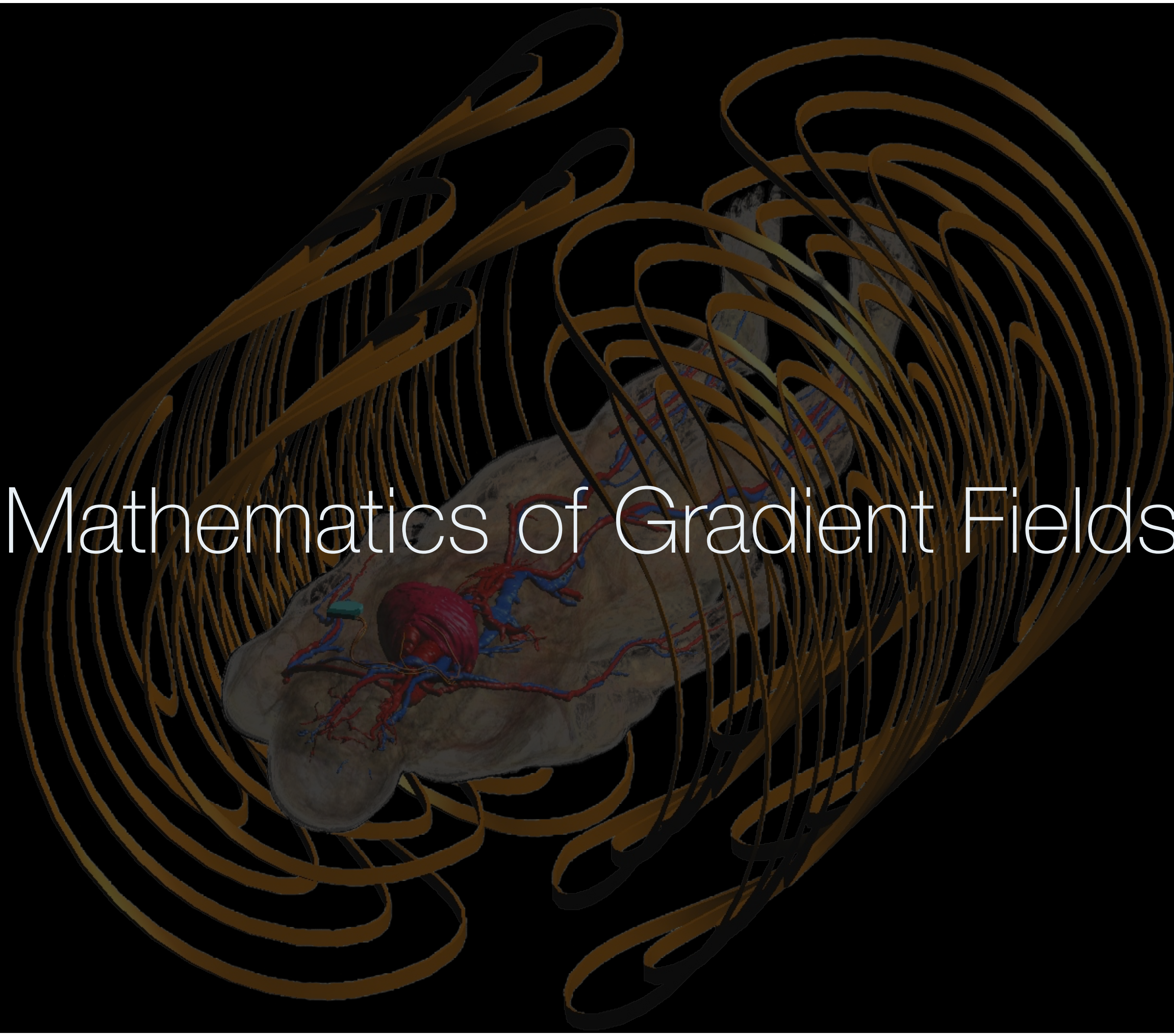
Gradients add/subtract to B_0 along a specific direction.

Spins and X- & Y-Gradients



Simultaneous gradients create an arbitrary isochromat plane.

Mathematics of Gradient Fields



Gradients and B-Fields

- Gradients are a special kind of inhomogeneous B-field whose *z*-component varies linearly along a specific direction called the gradient direction.

$$\underbrace{B_{G}}_{\text{B-field from a gradient}}{}_{,\underbrace{z}_{\text{Points along the z-direction}}}\underbrace{(x)}_{\text{Varies with the x-direction}} = \underbrace{G_x}_{\text{x-gradient amplitude}}\underbrace{x}_{x\text{-position relative to isocenter}}$$

Gradients and B-Fields

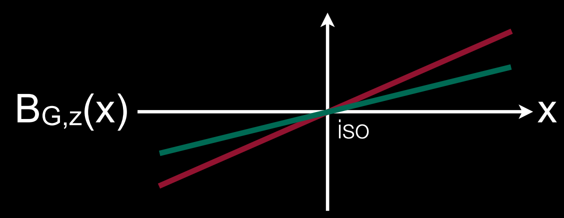
$$B_{G,z}(x) = G_x x$$ x-gradient

$$B_{G,z}(y) = G_y y$$ y-gradient

$$B_{G,z}(z) = G_z z$$ z-gradient

Pulse Sequence Timing Diagram

Magnetic Field Spatial Diagram



Each gradient coil can be activated independently and simultaneously.

Gradients and B-Fields

$$\vec{B}_{G,z}(\vec{r},t) = (G_x(t)\cdot x + G_y(t)\cdot y + G_z(t)\cdot z)\,\hat{k}'$$
$$= \left(\vec{G}(t)\cdot\vec{r}\right)\hat{k}'$$

Each gradient coil can be activated independently and simultaneously

$$\vec{B}(\vec{r},t) = (B_0 + B_{G,z})\,\hat{k}'$$
$$= \left(B_0 + \vec{G}(t)\cdot\vec{r}\right)\hat{k}'$$

Gradients contribute to the net B-field, but only along the z-direction

The magnetic field at a position depends on the magnitude of the applied gradient.

Comparison of B-fields Assumptions

- B_0-field is:
  - Perfectly uniform over space.
    - "B_0 homogeneity"
  - Perfectly stable with time.

$$\vec{B} = B_0 \hat{k}$$

- B_1-field is:
  - Perfectly uniform over space.
    - "B_1 homogeneity"
  - Temporally modulated exactly as specified.

$$\vec{B}_1(t) = B_1^e(t) \left[\cos(\omega_{RF} t)\hat{i}' - \sin(\omega_{RF} t)\hat{j}' \right]$$

- Gradient fields are:
  - Perfectly linear over space.
    - "Gradient linearity"
  - Temporally modulated exactly as specified.

$$\vec{B}_{G,z}(\vec{r}, t) = \left(\vec{G}(t) \cdot \vec{r} \right) \hat{k}'$$

Pay particular attention to the direction of each *B*-field.

Frequency from a Gradient

$$\vec{\omega} = -\gamma \vec{B}$$

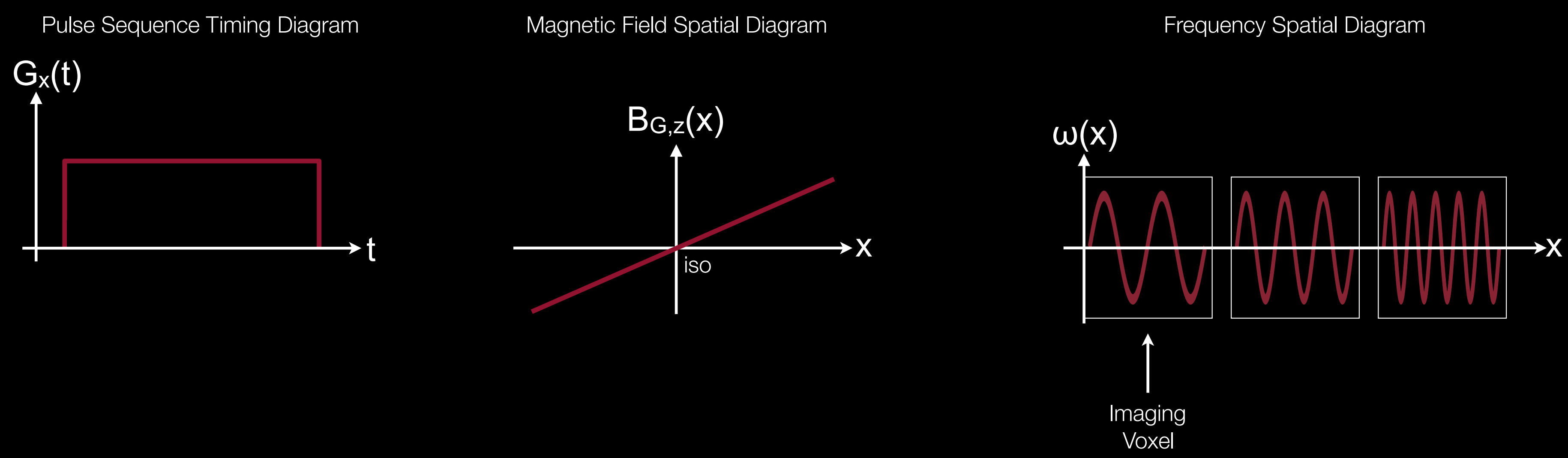
Larmor equation arises from free precession and the equation of motion.

$$-\gamma \vec{G}(t) \cdot \vec{r}$$

Gradients contribute and additional B-field that is space and time dependent.

The ability to spatially manipulate the B-field enables spatial encoding with frequency and phase.

Frequency from a Gradient



What happens when the gradient is turned off?

Active control of the magnetic field gradients spatially manipulates the B-field and local Larmor frequency.

Phase from a Gradient

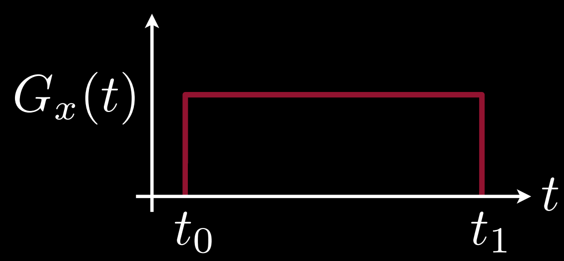
$$\phi_G(\vec{r},t) = -\gamma \int_0^t \vec{G}(\tau) \cdot \vec{r}(\tau) d\tau$$

Phase from a gradient.

$$\phi_G(x,t) = -\gamma G_x \cdot x \cdot t$$

Phase from a 1-D rectangular gradient waveform.

Pulse Sequence Timing Diagram

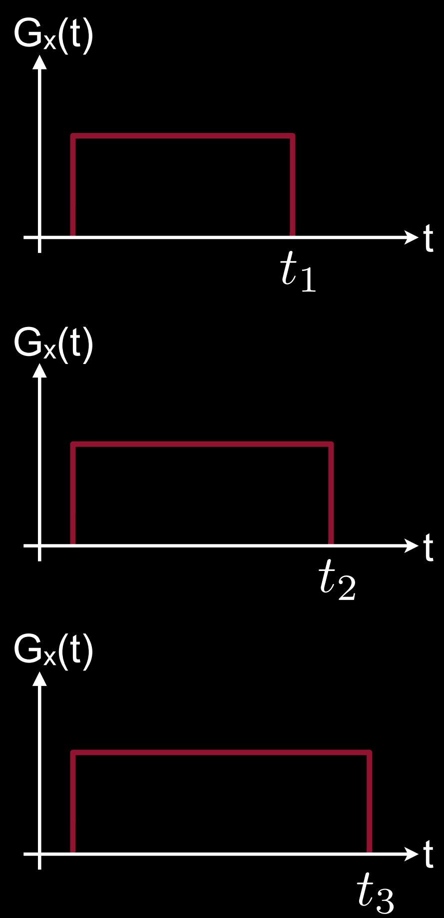


- Longer times confer more phase.
- Stronger gradients impart phase more quickly.

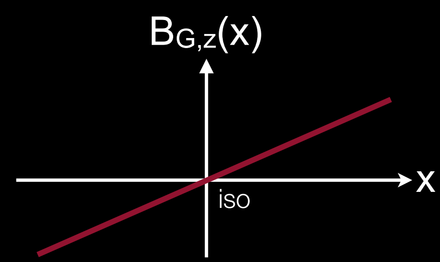
The application of a gradient for a finite period of time “labels” the spins with a spatially dependent amount of phase.

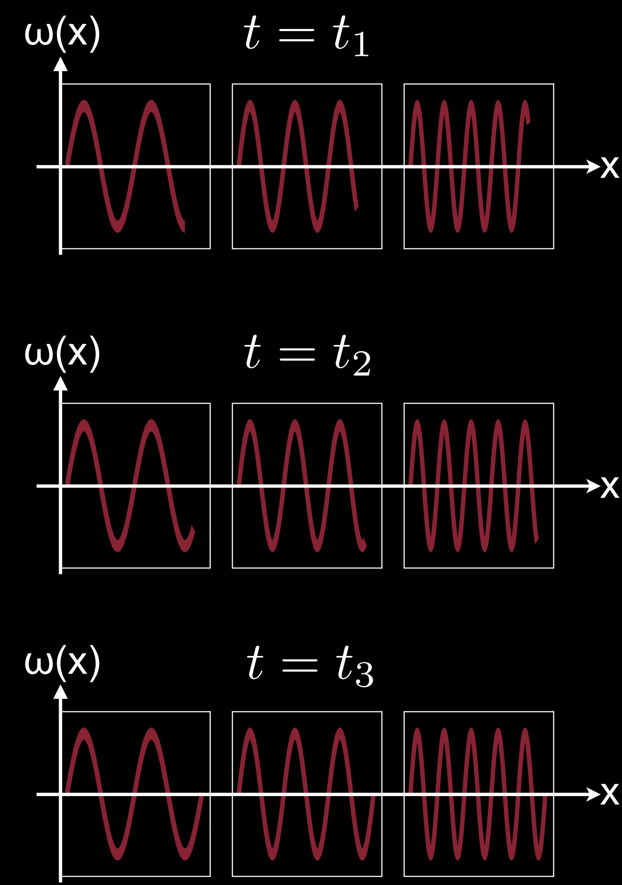
Phase from a Gradient

Pulse Sequence Timing Diagram



Magnetic Field Spatial Diagram





Active control of the magnetic field gradients spatially manipulates the B-field and local Larmor frequency.

How are gradients used for spatial localization?

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