Rad229 – MRI Signals and Sequences

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Lecture-02D — *k*-space and Imaging

Phase and Frequency Encoding

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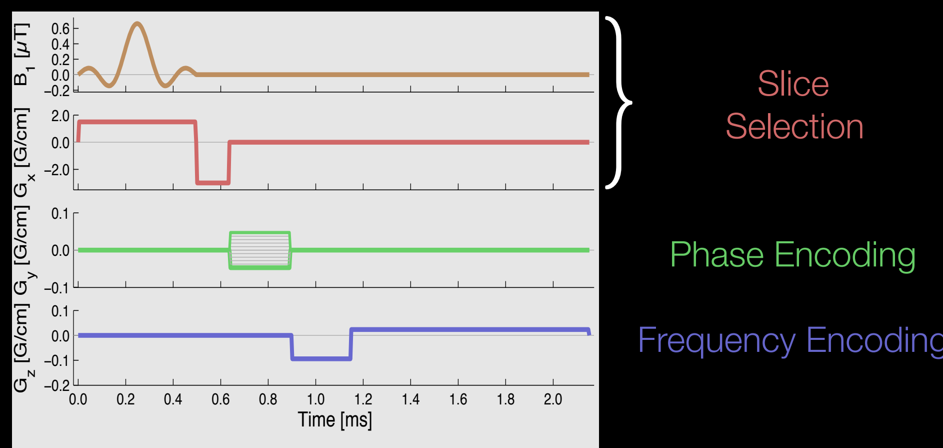
Learning Objectives

- Describe three different roles for gradients.
- Outline the steps required for spatial localization.
- Explain the role of frequency and phase encoding.

Spatial Encoding

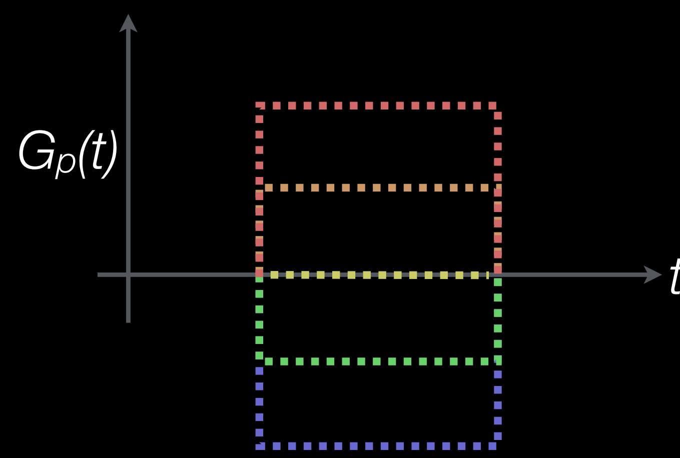
- Three key steps:
  - Slice selection
    - You have to pick slice!
  - Phase Encoding
    - You have to encode 1 of 2 dimensions within the slice.
  - Frequency Encoding (aka *readout*)
    - You have to encode the other dimension within the slice.



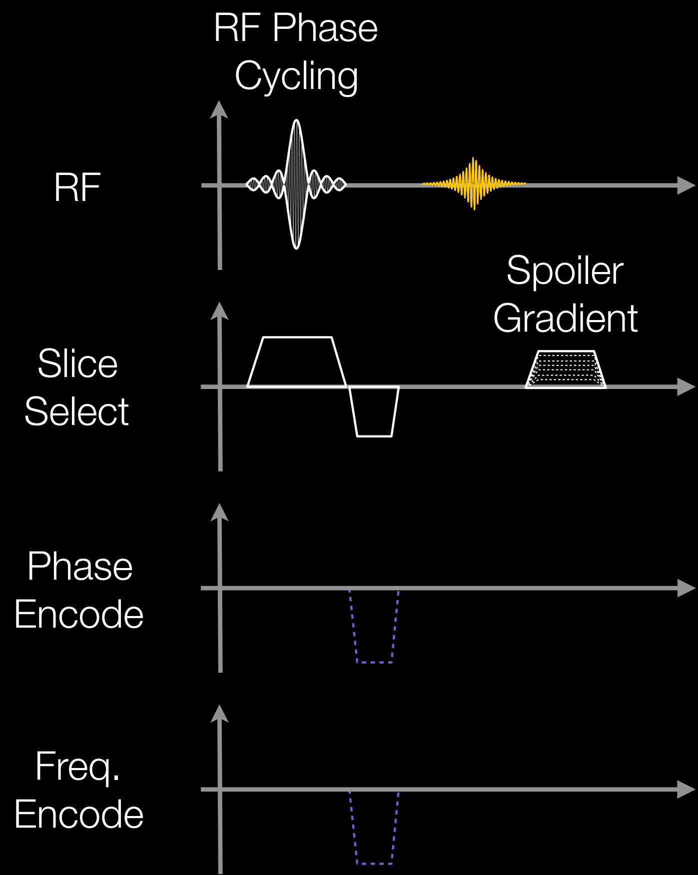
Phase Encoding

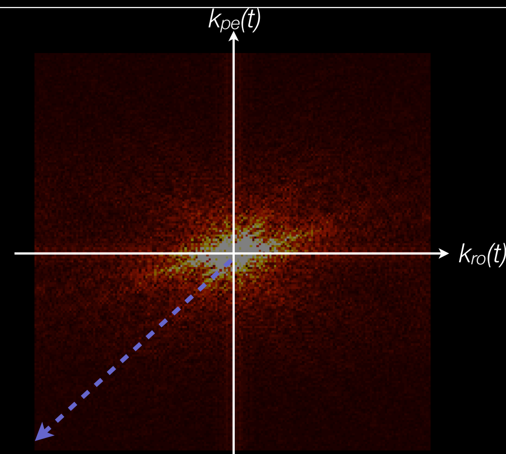
Phase Encoding

- Consists of:
  - Phase encoding gradient
    - Magnitude changes with each TR
    - Can be played with other gradients
      - Crushers, Slice-selection rephaser, readout dephasing
- Used with Cartesian imaging
- After excitation, before readout
- Adds linear spatial variation of phase
- Phase encode in
  - one direction for 2D imaging
  - two directions for 3D imaging
- Only one PE step per echo

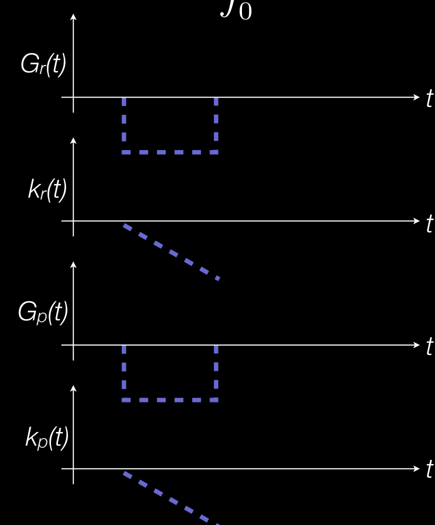


Where am I in *k*-space?



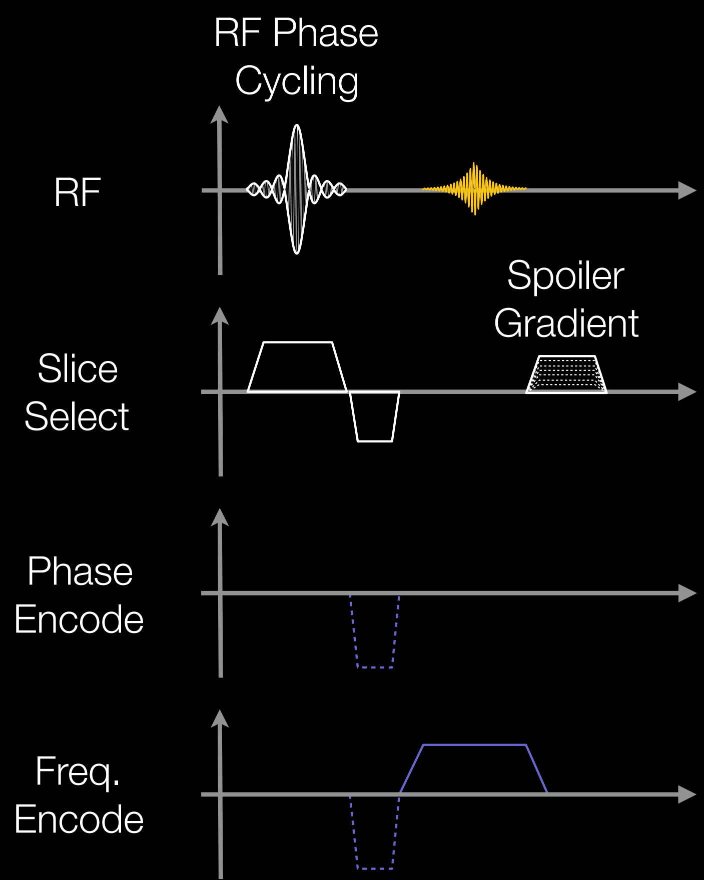


$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}(t)\, d\tau$$

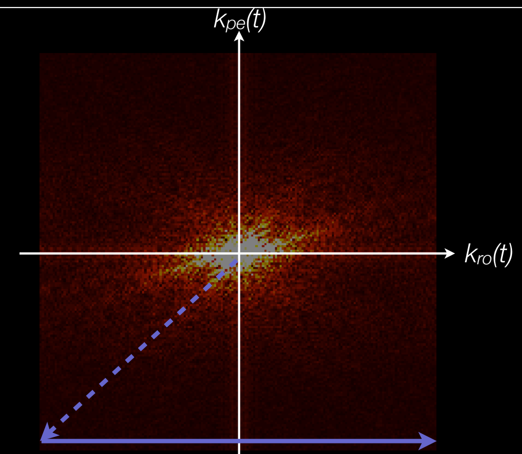


One phase encoded echo is acquired per TR.

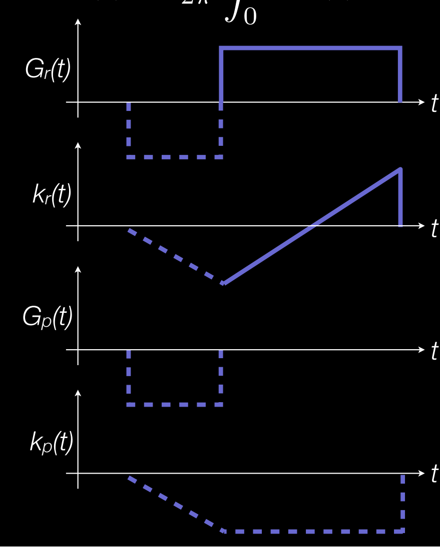
Where am I in *k*-space?



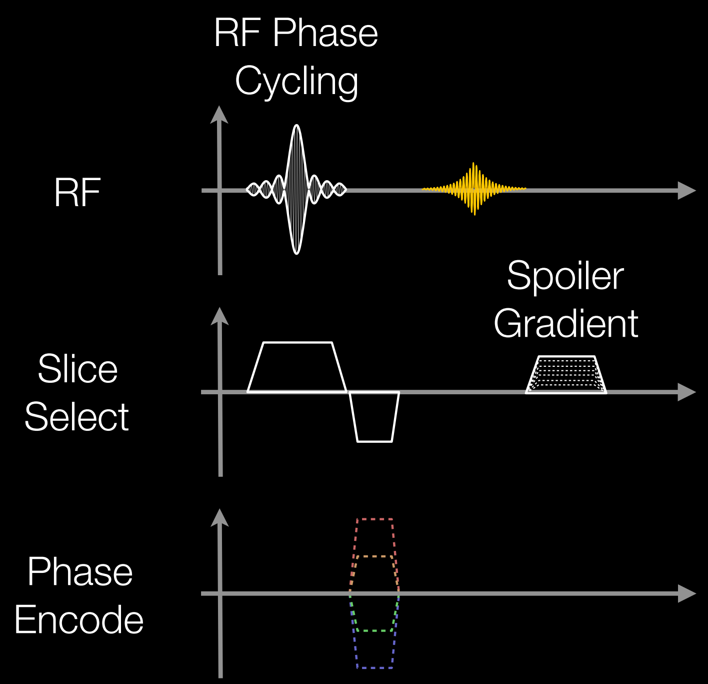
One phase encoded echo is acquired per TR.

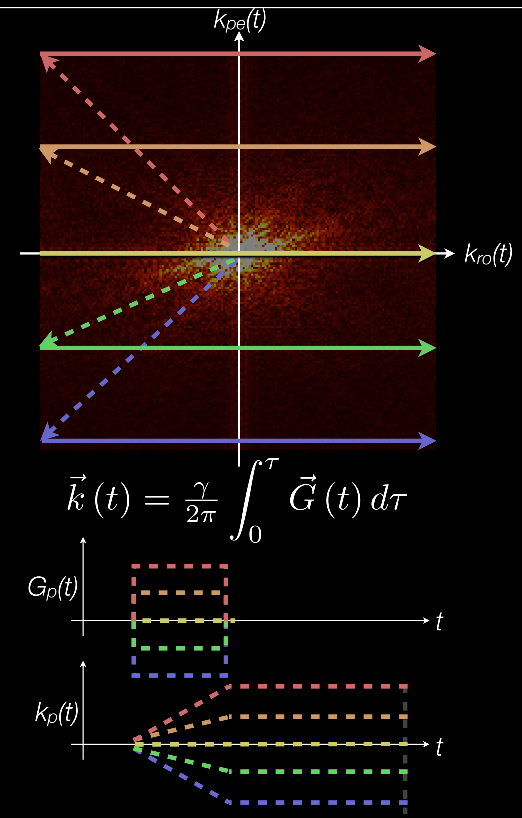


$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}(t)\, d\tau$$



Phase Encode Gradients





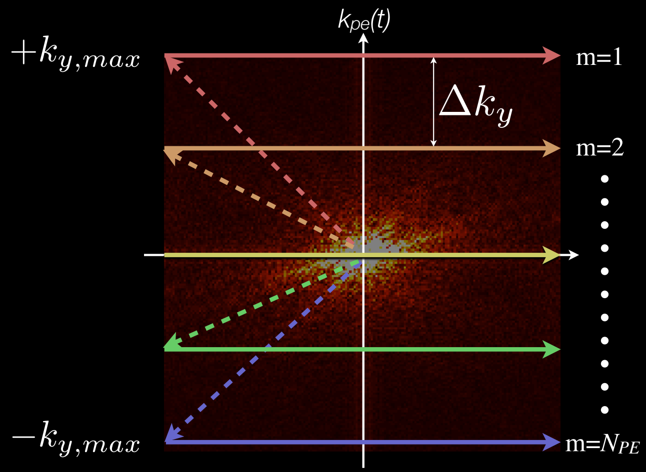
$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}(t)\, d\tau$$

For sequence efficiency the slice-select re-phasing gradient and the phase encode gradient can overlap.

Phase Encode Gradient Design

$FOV = \frac{1}{\Delta k_y}$, encoded with N_{PE} steps.

$$\Delta k_y = \frac{1}{N_{PE} \cdot \Delta y} = \frac{1}{128 \cdot 0.1\text{cm}} = 0.078\text{cm}^{-1}$$



$$k_{y,max} = \frac{1}{2}(N_{PE} - 1)\Delta k_y = \frac{1}{2}(128 - 1) \cdot 0.078\text{cm}^{-1} = 4.95\text{cm}^{-1}$$

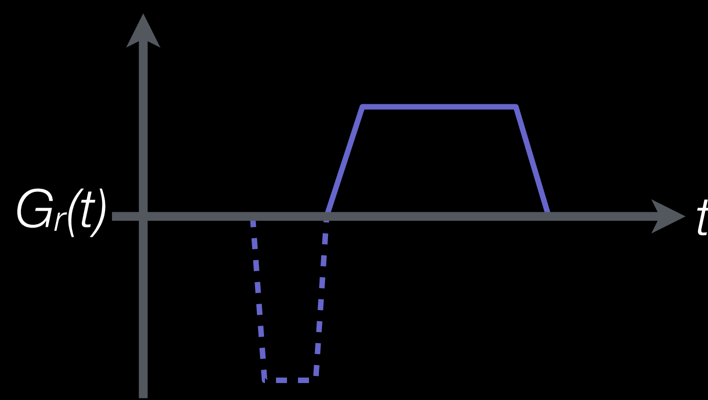
$$\tau_{PE} = \frac{2\pi k_{y,max}}{\gamma G_{max}} = \frac{4.95\text{cm}^{-1}}{4248\frac{\text{Hz}}{\text{G}} \cdot 4\frac{\text{G}}{\text{cm}}} = 0.290\text{ms}$$

In general, $k_y(m) = \left(\frac{N_{PE}-1}{2} - m\right)\Delta k_y$

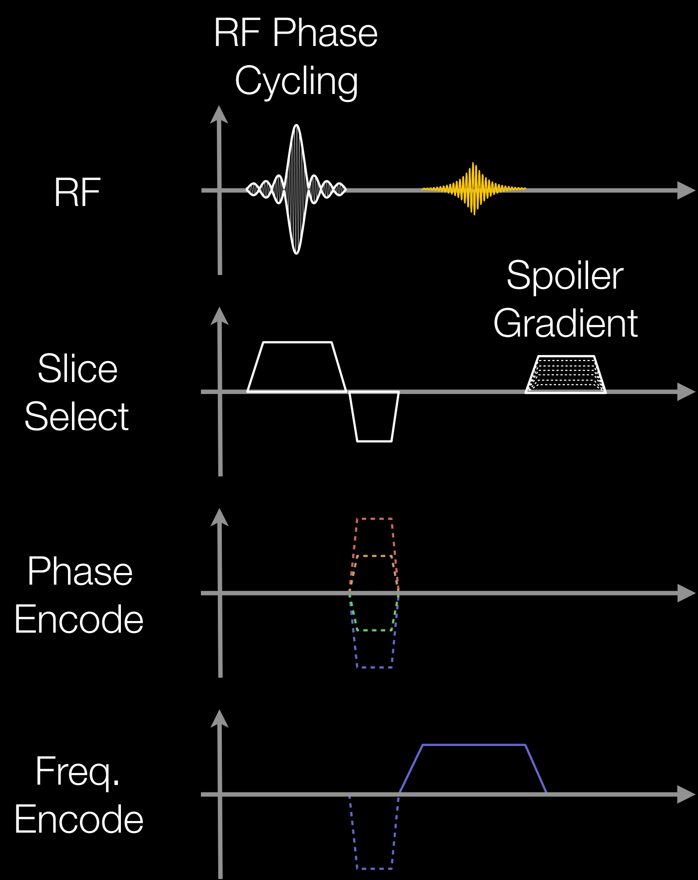
Frequency Encoding

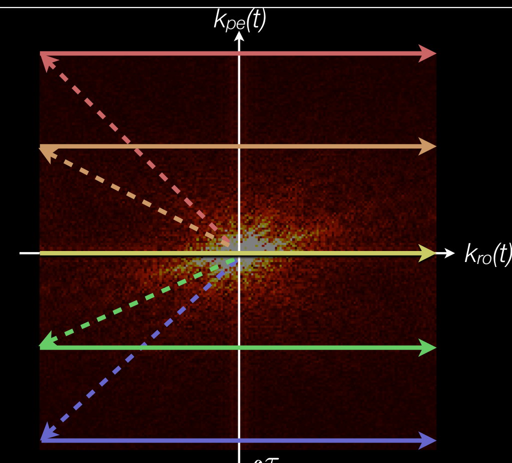
Frequency Encoding

- Frequency encoding gradient
  - Constant magnitude for Cartesian imaging
  - No simultaneous
    - RF (B1)
    - Other gradients
      - phase encoding, slice encoding, crushers
- Readout pre-phasing gradient
  - Prepares spin phase so peak echo amplitude occurs at middle of readout (TE)
  - AKA “readout de-phasing gradient”
- Adds linear spatial variation of frequency
- Helps form an echo

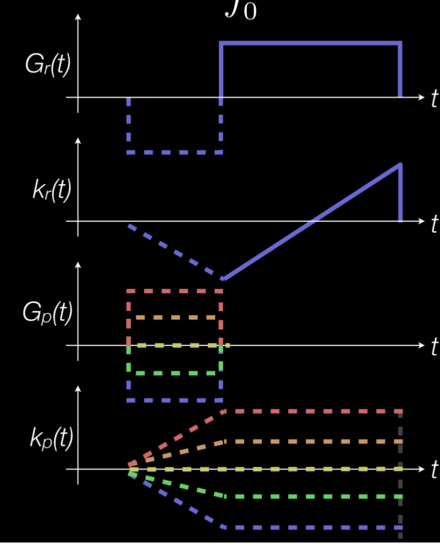


Gradient Echo Sequence



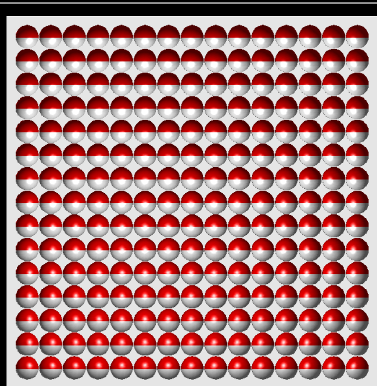


$$\vec{k}\,(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}\,(t)\,d\tau$$

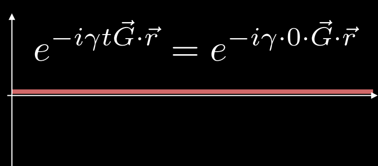


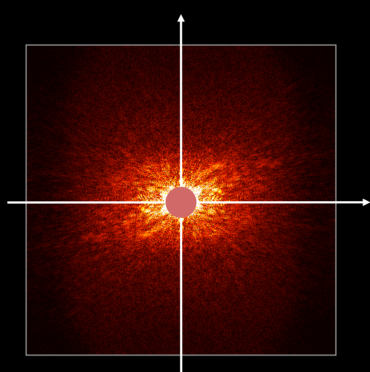
One phase encoded echo is acquired per TR.

Frequency Encoding



G_{Freq}=0



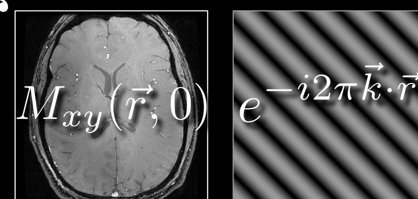


$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^{\tau}\vec{G}(t)\,d\tau$$

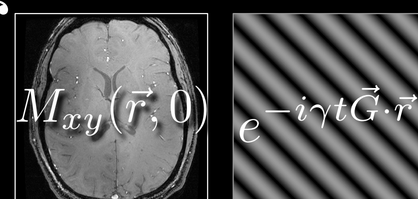
In general…

$$2\pi\vec{k}(t) = \gamma\vec{G}t$$

For a constant amplitude gradient…

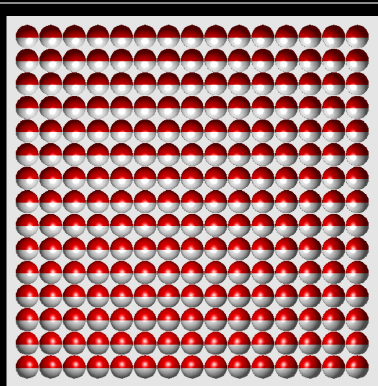


$$S(\vec{k}) = \int \underbrace{M_{xy}(\vec{r},0)}_{object}\ e^{-i2\pi\vec{k}\cdot\vec{r}}\ d\vec{r}$$

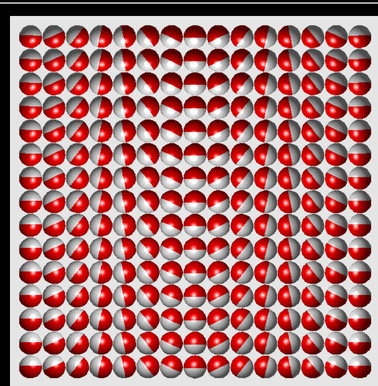


$$\int \underbrace{M_{xy}(\vec{r},0)}_{object}\ e^{-i\gamma t\vec{G}\cdot\vec{r}}\ d\vec{r}$$

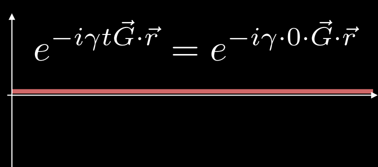
Frequency Encoding

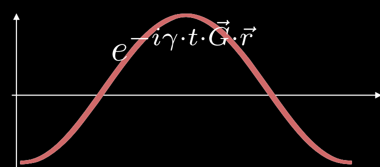


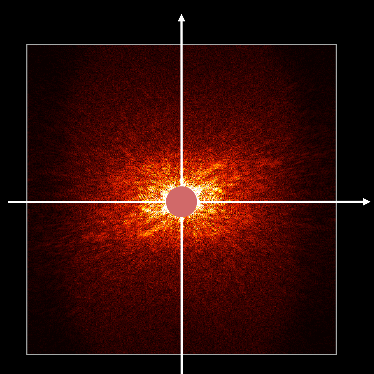
$G_{Freq}=0$

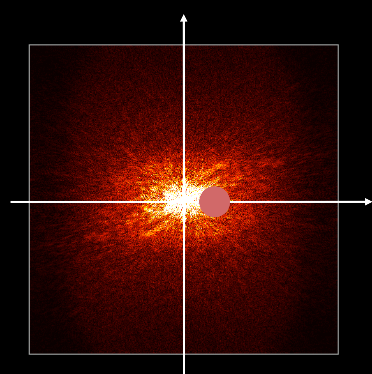


$G_{Freq}=G\bullet t$

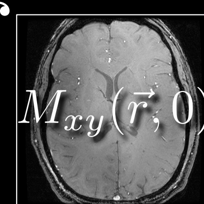


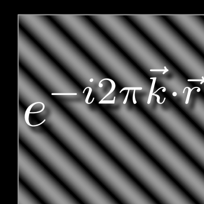






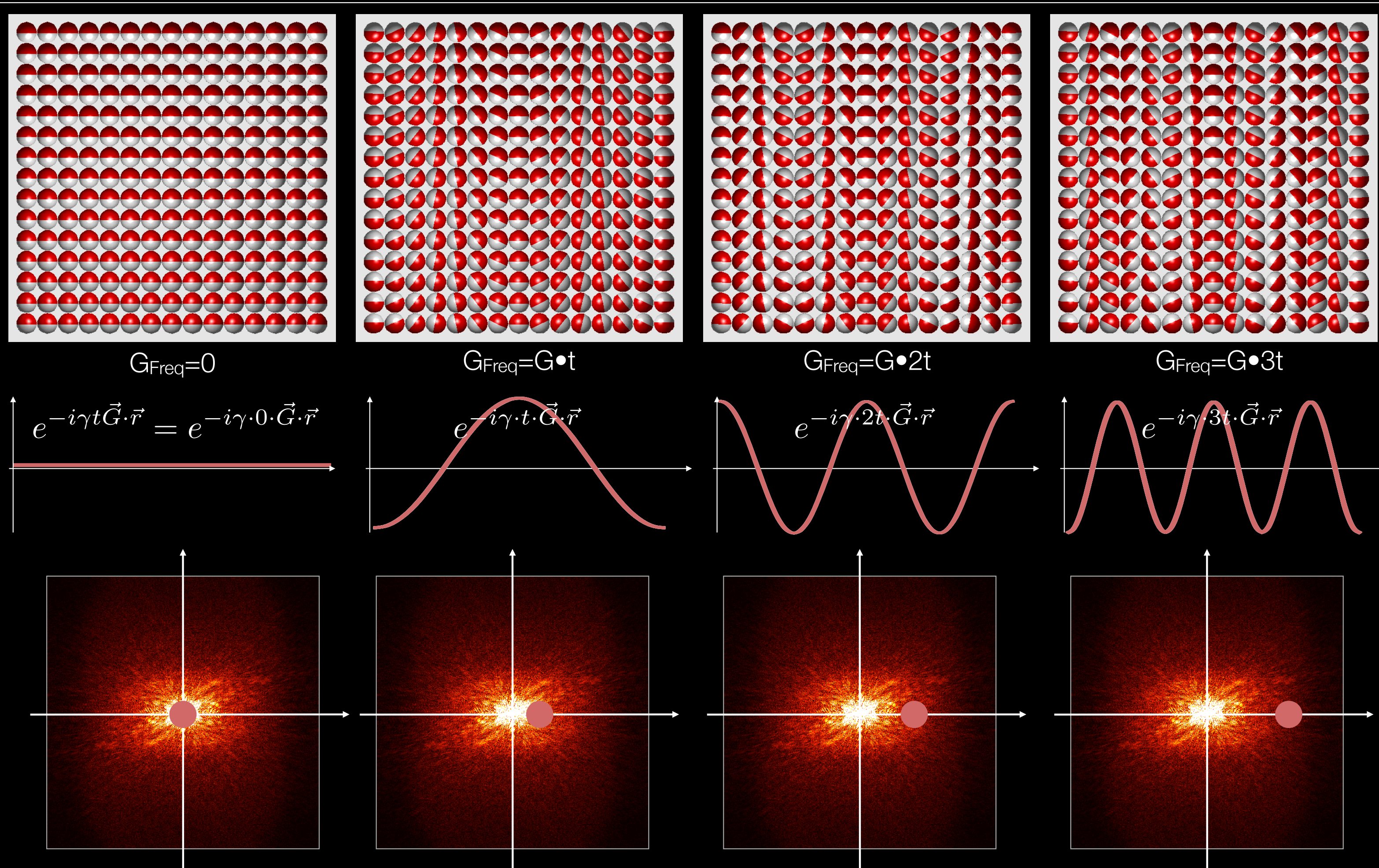
$$S(\vec{k}) = \int M_{xy}(\vec{r},0)\, e^{-i2\pi\vec{k}\cdot\vec{r}}\, d\vec{r}$$





object

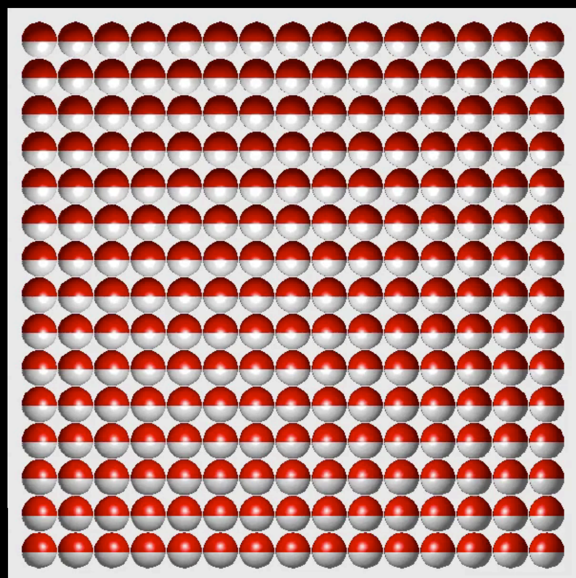
Frequency Encoding

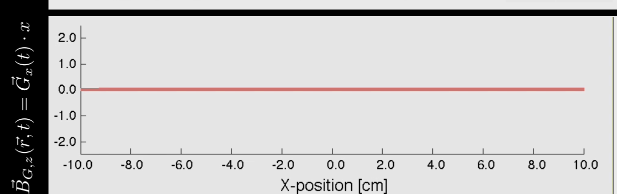


At each time point in this process the signal can be measured by many cycles of precession.

Frequency Encoding

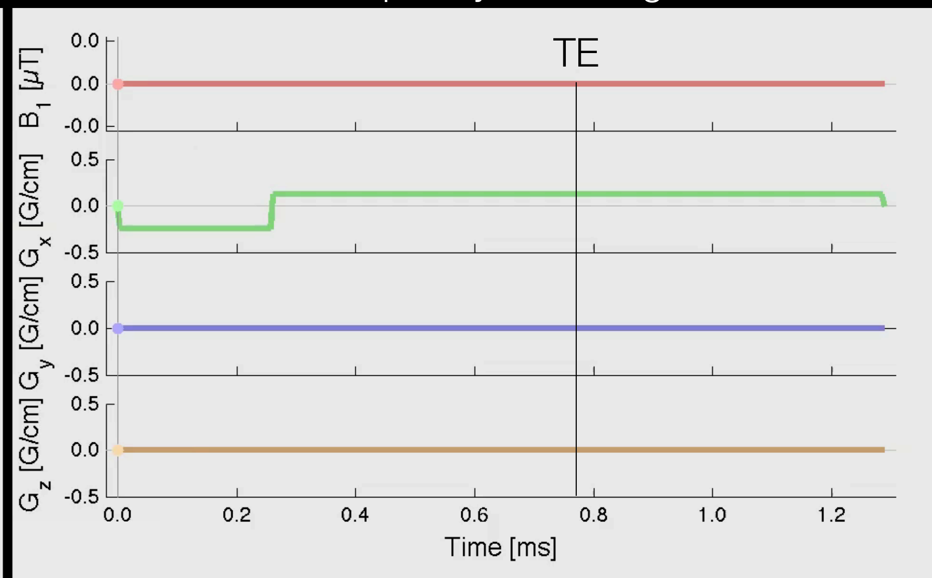
$B_0 + G_x \bullet x$ B_0 $B_0 - G_x \bullet x$





Applied Magnetic Field (B_G)

Frequency Encoding



Readout Gradient Amplitude

- High Receiver Bandwidth (RBW, Δf)
  - Stronger gradients
  - Larger range of frequencies across the FOV (or pixel)
  - Less chemical shift (larger freq. difference per pixel)
  - Lower SNR (shorter acquisition time)
  - Shorter TE (move across *k*-space faster)

$$\Delta f = \frac{1}{2}\frac{\gamma}{2\pi}G_x \cdot FOV_x$$

User can pick 2 of 3 (Δf, G_x, FOV_x)

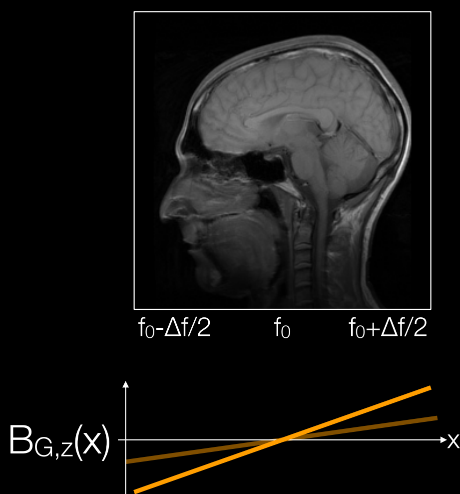
Receiver Bandwidth (e.g. 32kHz)

Field of View (e.g. 30cm)

$$G_x = \frac{2 \cdot \Delta f}{\gamma FOV_x} = \frac{2 \cdot 32000\text{Hz}}{4258\frac{\text{Hz}}{\text{G}} \cdot 30\text{cm}} = 0.501\frac{\text{G}}{\text{cm}}$$

Readout Gradient Duration

- High Receiver Bandwidth (RBW, Δf)
  - Stronger gradients
  - Larger range of frequencies across the FOV (or pixel)
  - Less chemical shift (larger freq. difference per pixel)
  - Lower SNR (shorter acquisition time)
  - Shorter TE (move across *k*-space faster)



Temporal Nyquist Sampling Requires: $\Delta t = \frac{1}{2\Delta f}$

$$
\begin{aligned}
\Delta t &= \frac{1}{2\Delta f} \\
&= \frac{1}{2 \cdot 32000\text{Hz}} \\
&= 15.625\mu\text{S}
\end{aligned}
$$

$$
\begin{aligned}
\tau_{RO} &= N_{read} \cdot \Delta t \\
&= 128 \cdot 15.625\mu\text{S} \\
&= 2000\mu\text{S}
\end{aligned}
$$

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