Rad229 – MRI Signals and Sequences

Daniel Ennis & Brian Hargreaves
dbe@stanford.edu –or– bah@stanford.edu

Lecture-04C —Extended Phase Graphs

Examples using Extended Phase Graphs

Brian Hargreaves
bah@stanford.edu

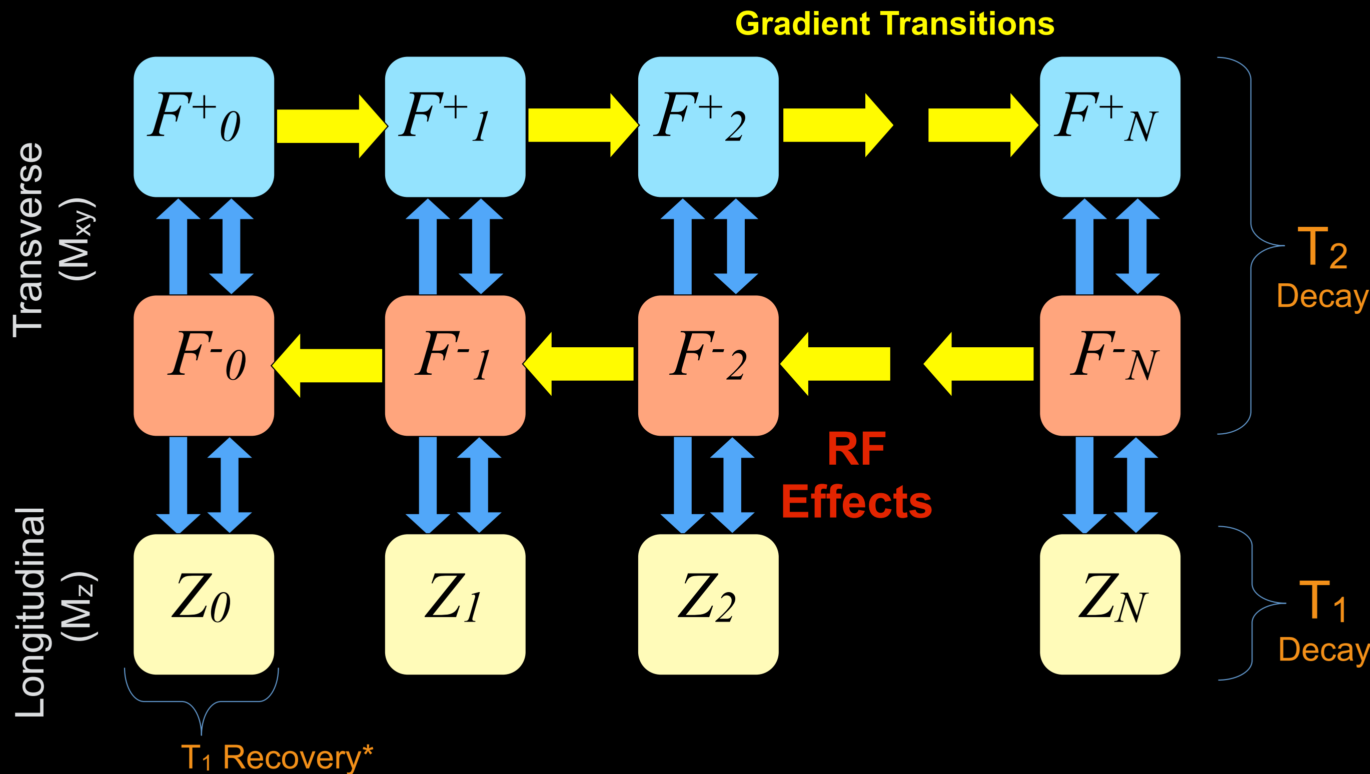
Learning Objectives

- Describe how a spin-echo forms using EPG
- Explain and draw phase coherence diagrams
- Describe how a stimulated echo forms using EPG
- Understand how to use Matlab or Python representations of EPG

Review: Phase Graph "States" (Flow Chart)

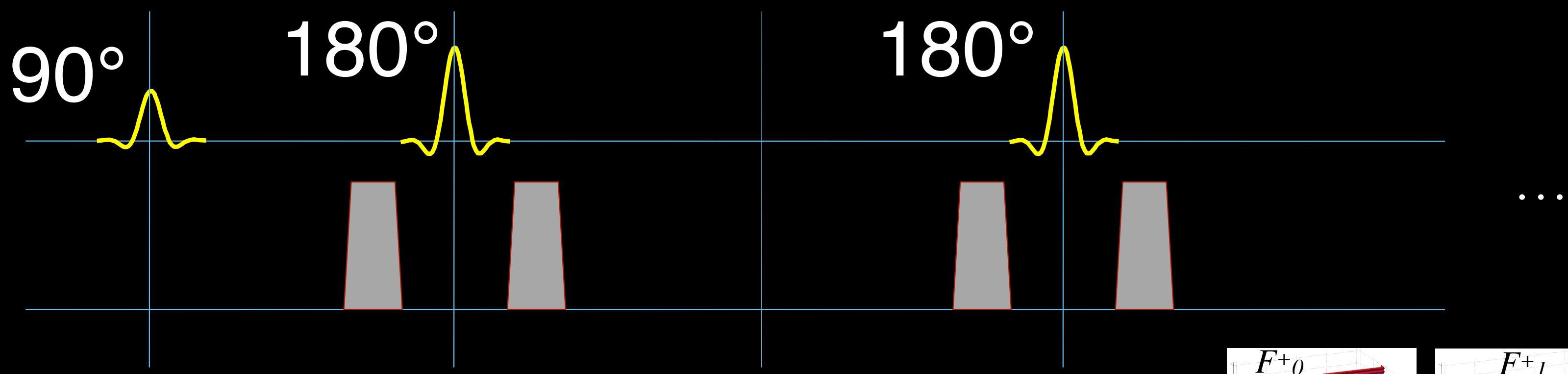
- State Matrix
- RF Mixes
- Gradients Shift
- T_1, T_2 Attenuate*
- Diffusion: Increasing Attenuation with n

$$Q = \begin{bmatrix} F_0^+ & F_1^+ & F_2^+ & \cdots & F_N^+ \\ F_0^{+*} & F_1^- & F_2^- & \cdots & F_N^- \\ Z_0 & Z_1 & Z_2 & \cdots & Z_N \end{bmatrix}$$



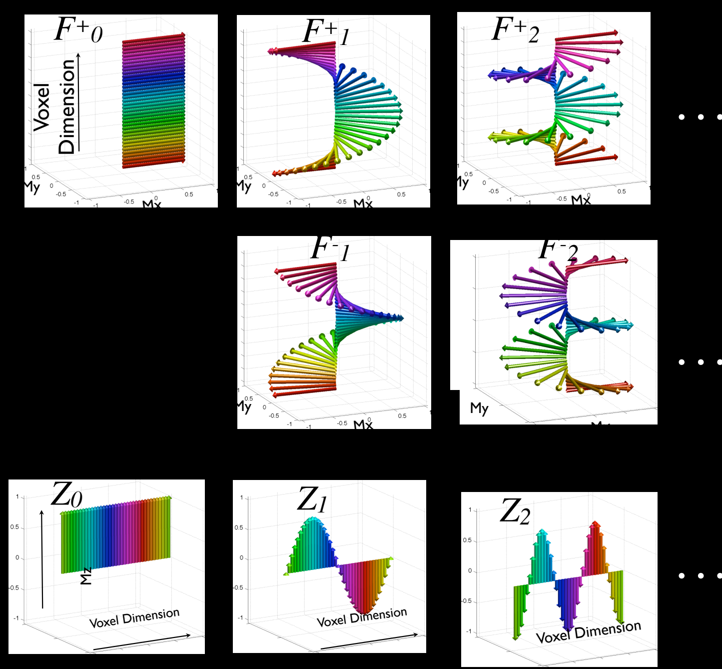
Weigel et al. J Magn Reson 2010; 205:276-285

Example 1: Ideal Spin Echo Train



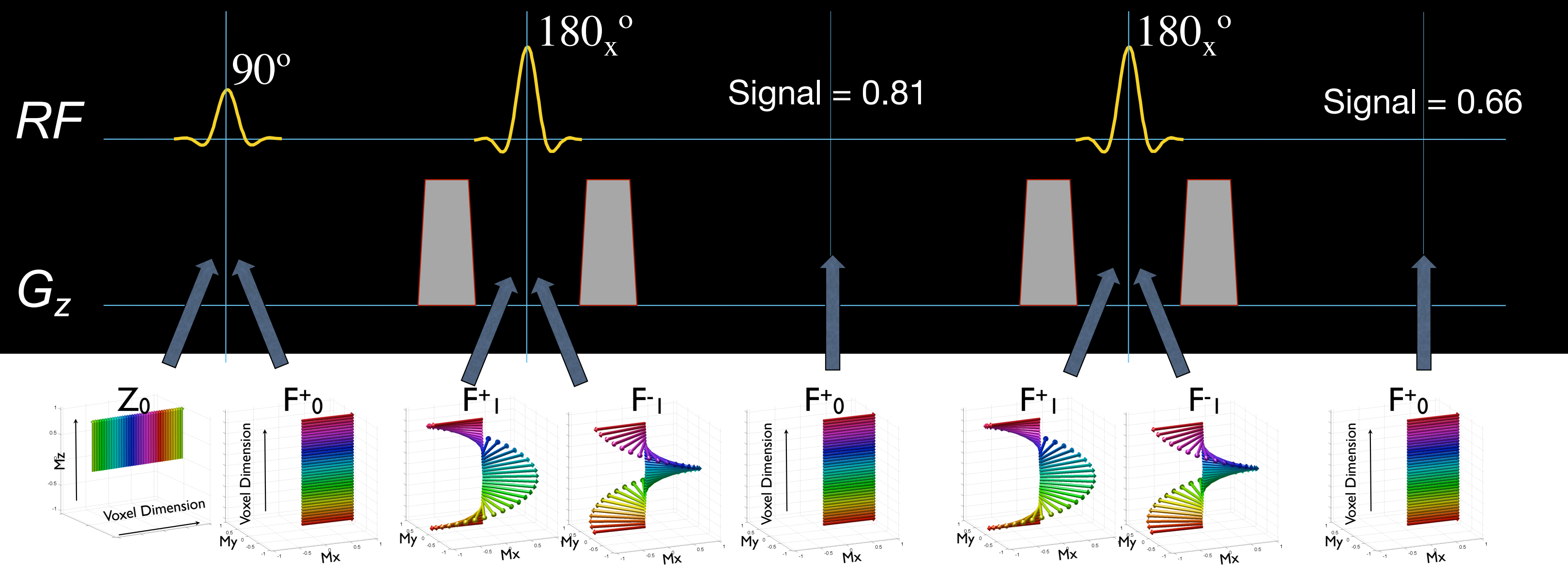
- What states will we need to represent the spins with ideal 180° pulses?

$$R_x(180^\circ) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$



Example 1: Ideal Spin Echo Train

Over TE/2:
$E_1 = 0.95$
$E_2 = 0.90$



$$\mathbf{Q} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0.90 \\ 0 & 0 \\ 0.05 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0.90 \\ -.05 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.81 & 0 \\ 0.81 & 0 \\ 0 & 0 \end{bmatrix}$$

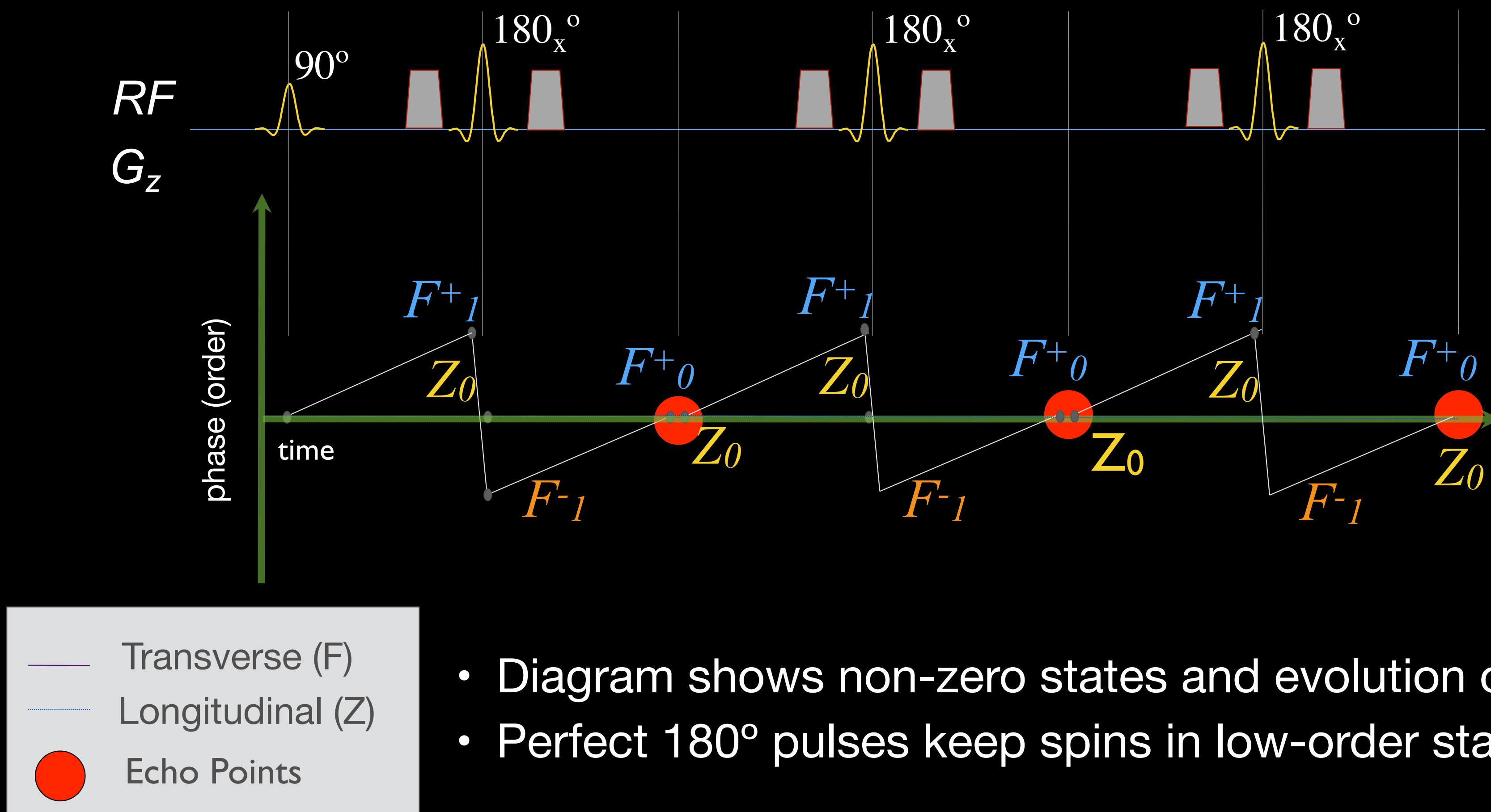
$$\begin{bmatrix} 0 & 0.73 \\ 0 & 0 \\ 0.05 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0.73 \\ -.05 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.66 & 0 \\ 0.66 & 0 \\ 0 & 0 \end{bmatrix}$$

lec4_02.m

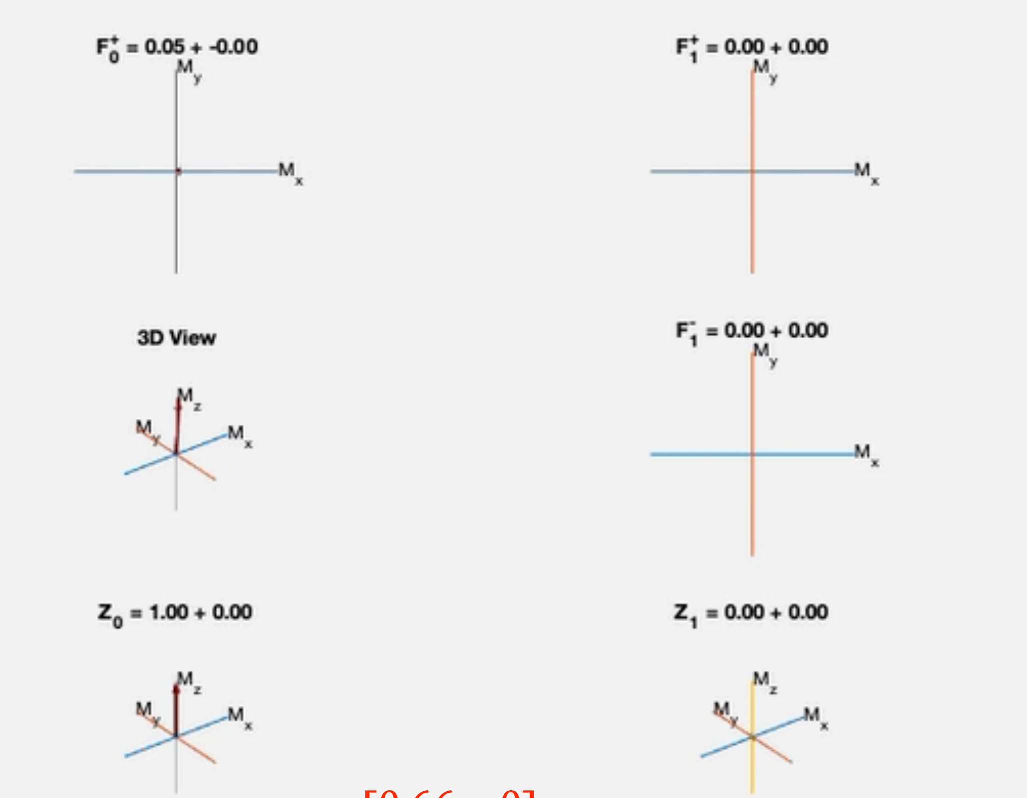
Coherence Pathways: Spin Echo



- Diagram shows non-zero states and evolution of states
- Perfect 180° pulses keep spins in low-order states

Spin Echo Example - Ideal 180° Pulses

- Never higher order than n=1
- Coefficients during gradient not really defined - just illustrating
- Relaxation results in transverse signal decay
- Z_0 has some relaxation, but flips
- Z_1 only has signal during 180° pulses
- Observed Signal is F^{+}_{0} (always!)



$$\begin{bmatrix} 0.66 & 0 \\ 0.66 & 0 \\ 0 & 0 \end{bmatrix}$$

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Example 2: Non-180° Spin Echo

- Ideal spin echo train gives simple RF rotations
- Now assume refocusing flip angle of 130°
- Compare RF rotations:

180_x^o

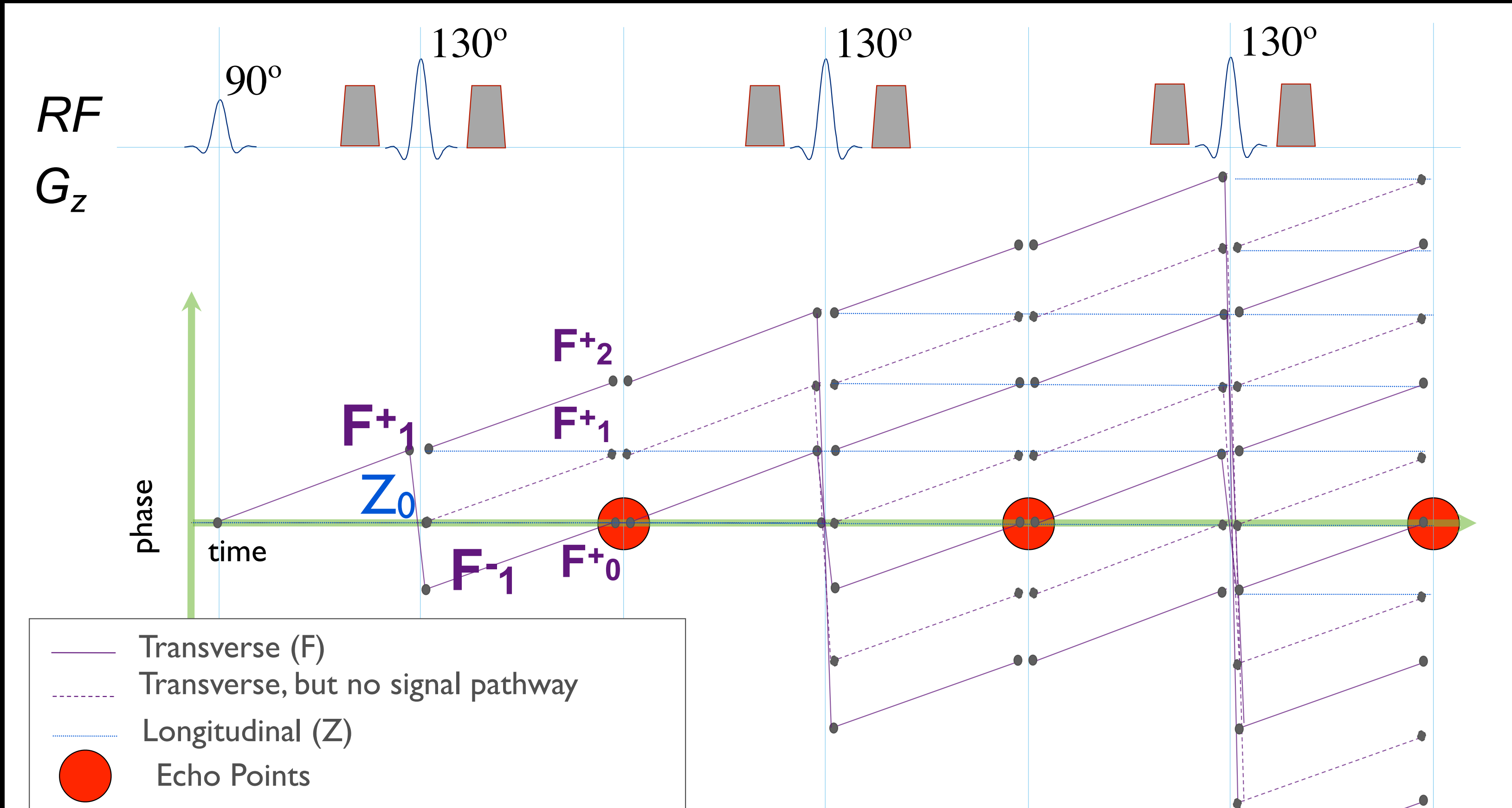
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

130_x^o

$$\begin{bmatrix} 0.18 & 0.82 & -0.77i \\ 0.82 & 0.18 & 0.77i \\ 0.38i & 0.38i & -0.64 \end{bmatrix}$$

- Positive $F^+{}_n$ states remain (from $F^+{}_n$) because magnetization is not perfectly reversed, generating higher order states
- Many more coherence pathways (see next slide…)

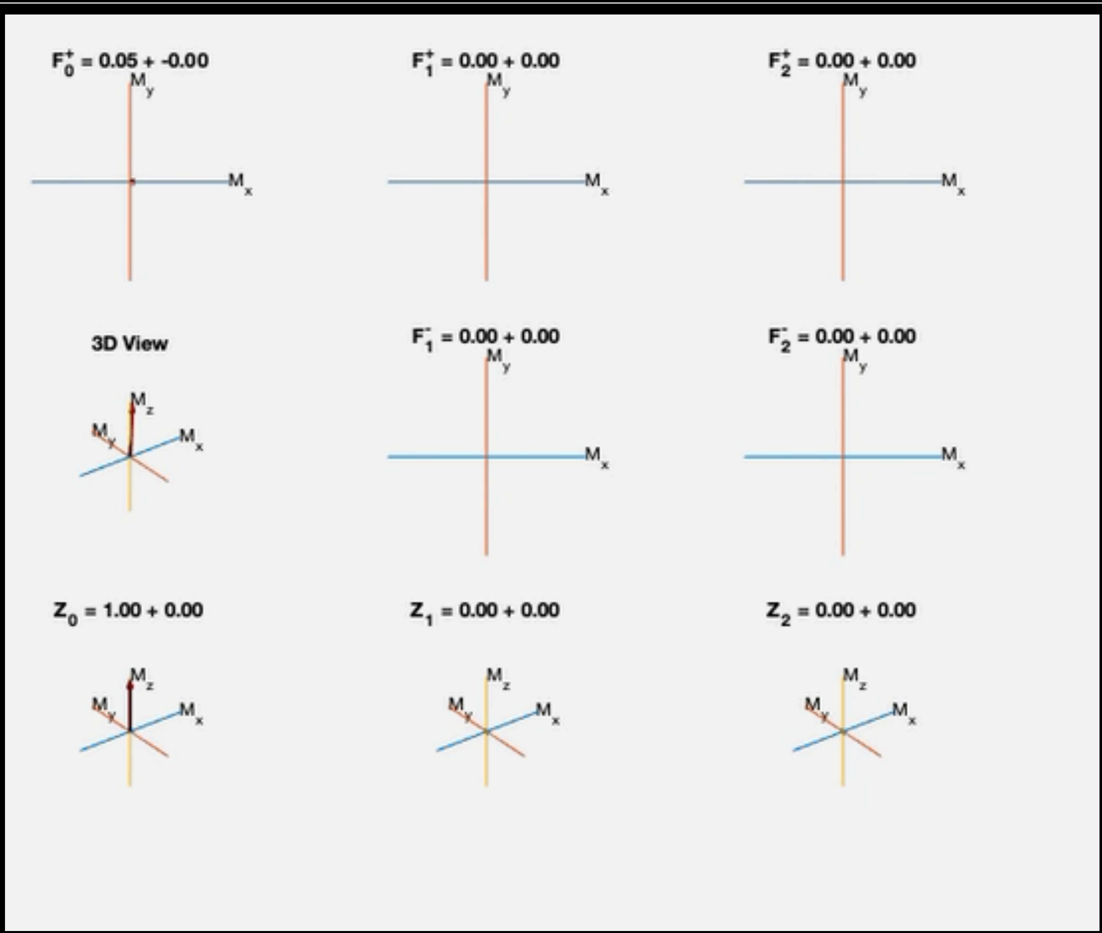
Coherences: Non-180° Spin Echo



Only F_0 produces a signal… other F_n states are perfectly dephased

Spin-Echo Example - 130° Pulses

- Magnetization builds up in higher-order states
- Does not contribute to spin-echo much



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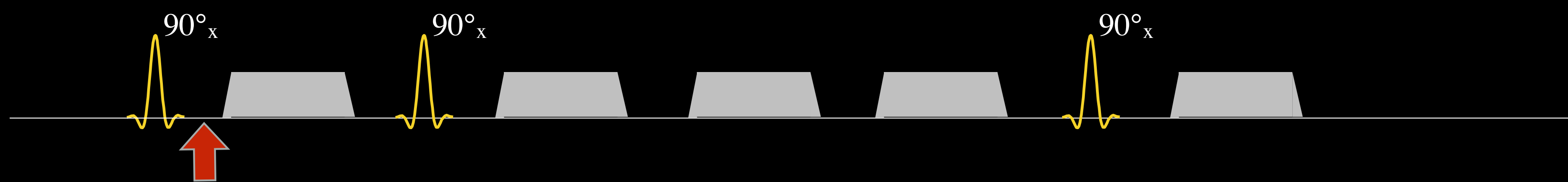
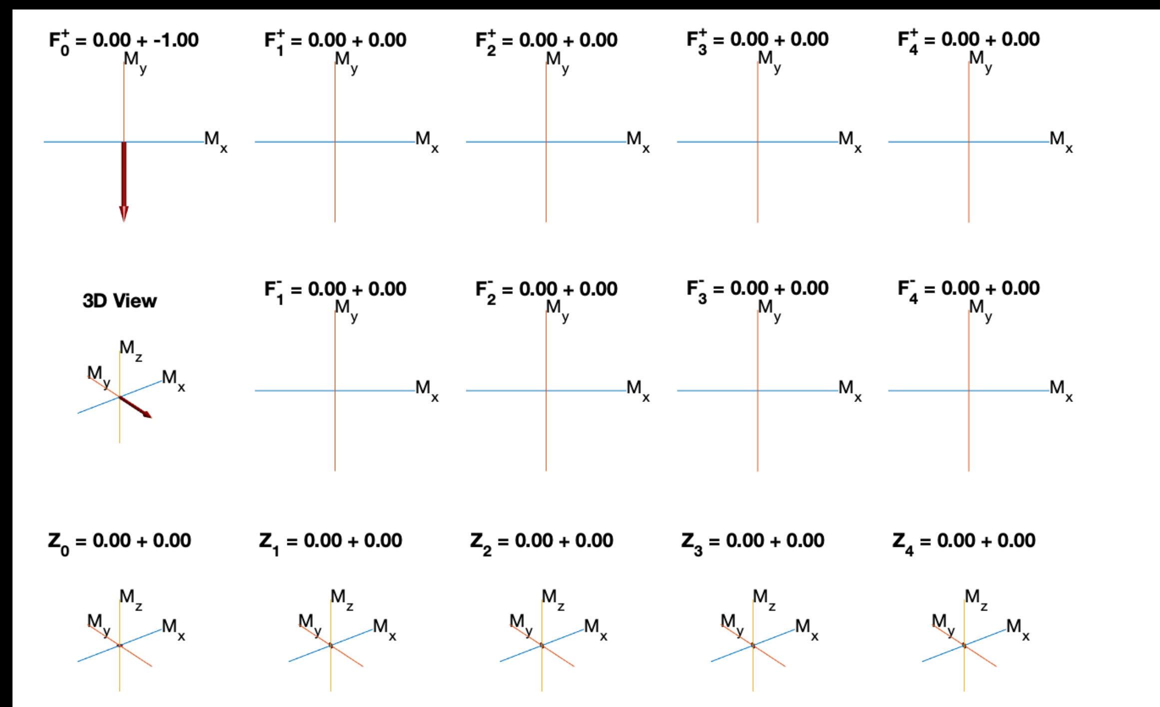
Example 3: Stimulated Echo and Spin Echo

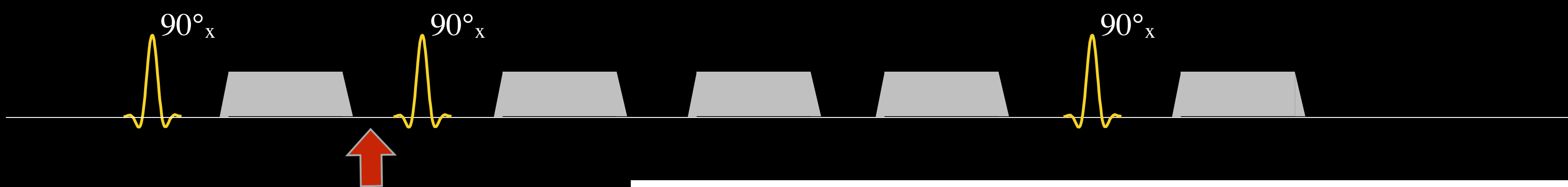


- 90° pulses
- Unit gradients
- Track using epg_show()
- Post excitation, only $F^{+}{}_{0}$



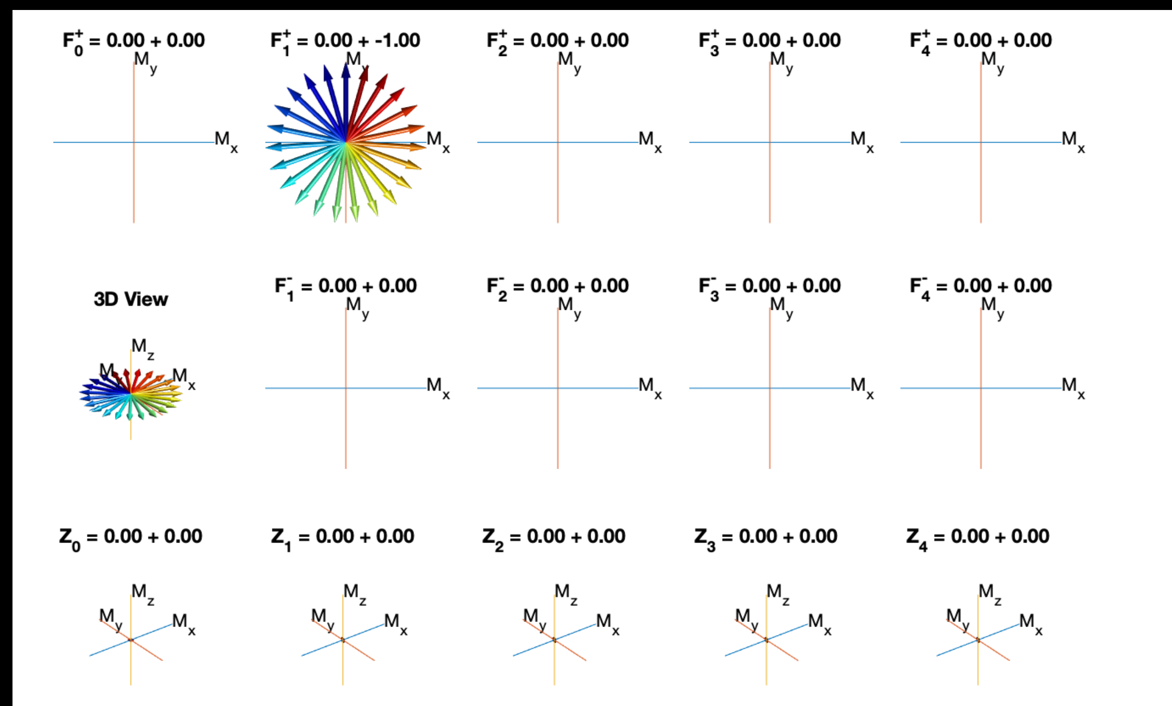
lec4_05.m

Example 3: Stimulated Echo and Spin Echo



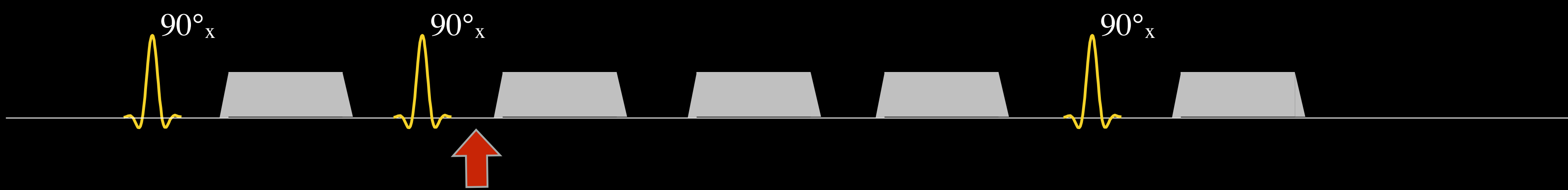
- **After gradient:**

  $F^{+}_{0} >>> F^{+}_{1}$



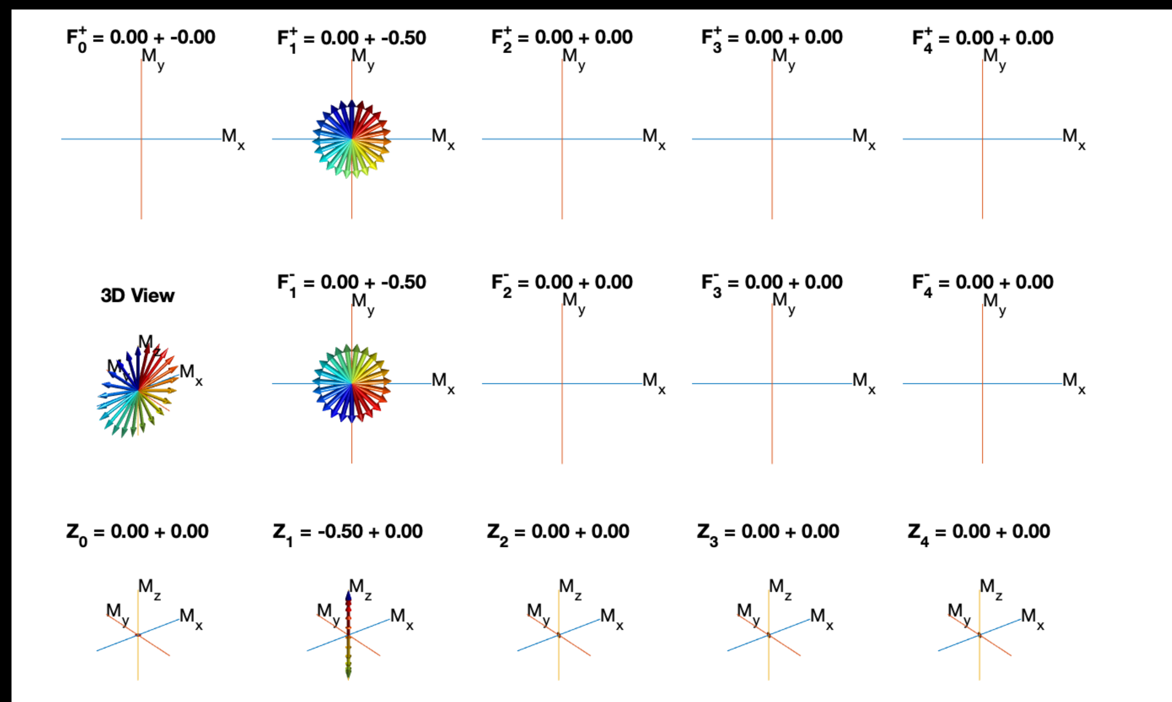
lec4_05.m

Example 3: Stimulated Echo and Spin Echo



- After 2nd RF:

  $F^{+}{}_{1} >>> F^{+}{}_{1}, F^{-}{}_{1}, Z_{1}$



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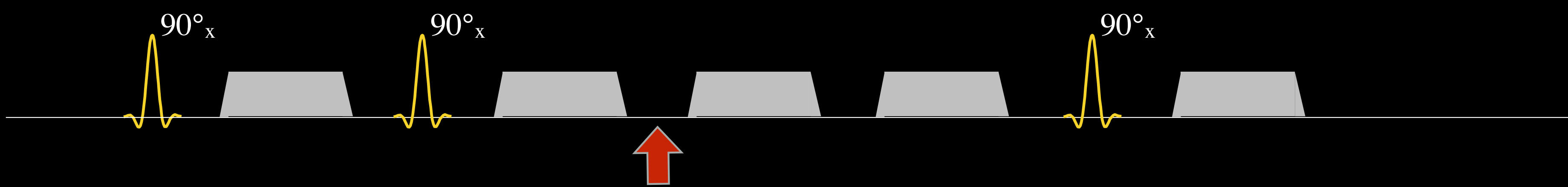
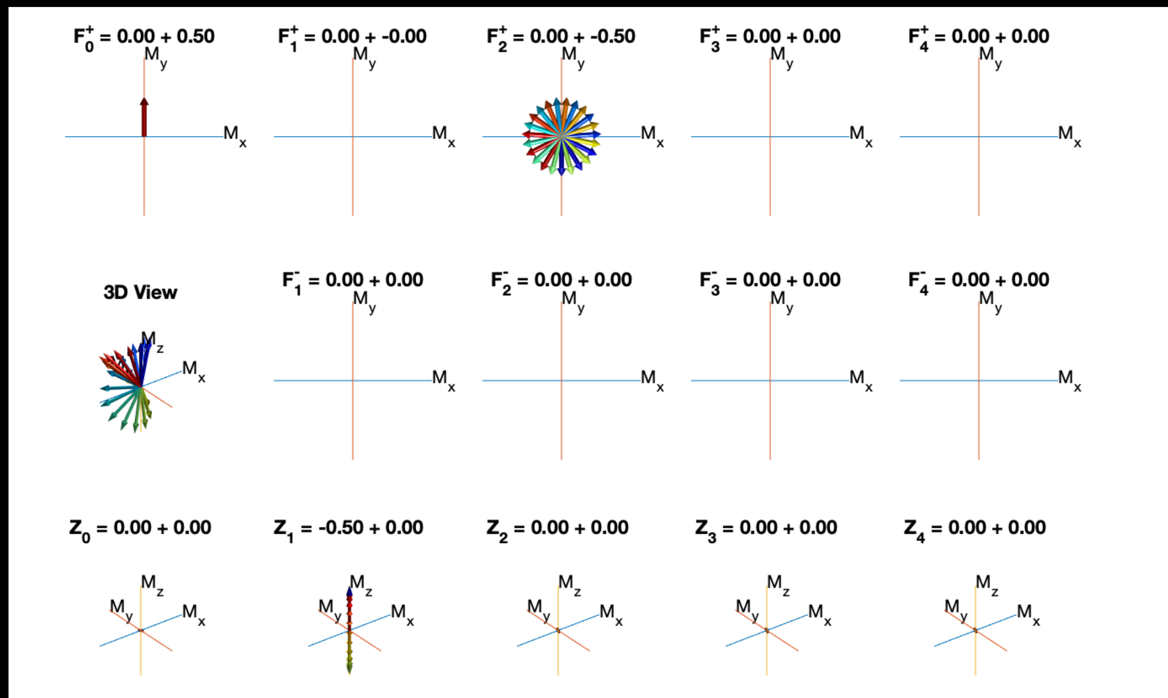
Example 3: Stimulated Echo and Spin Echo



- After gradient:
  - F^{+}_{1} >>> F^{+}_{2}
  - F^{-}_{1} >>> F^{+}_{0} (spin echo)
  - Z_{1} unchanged



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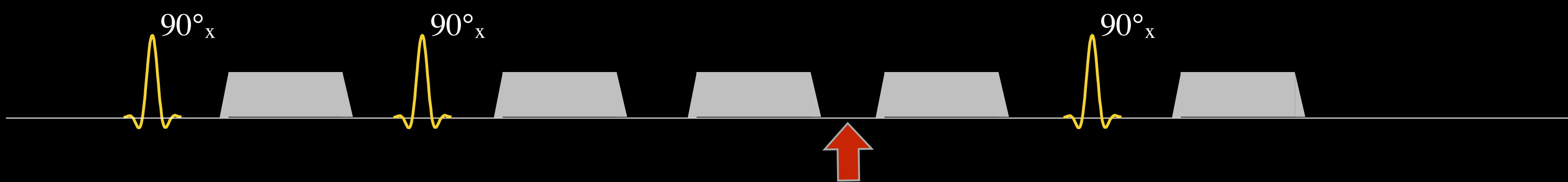
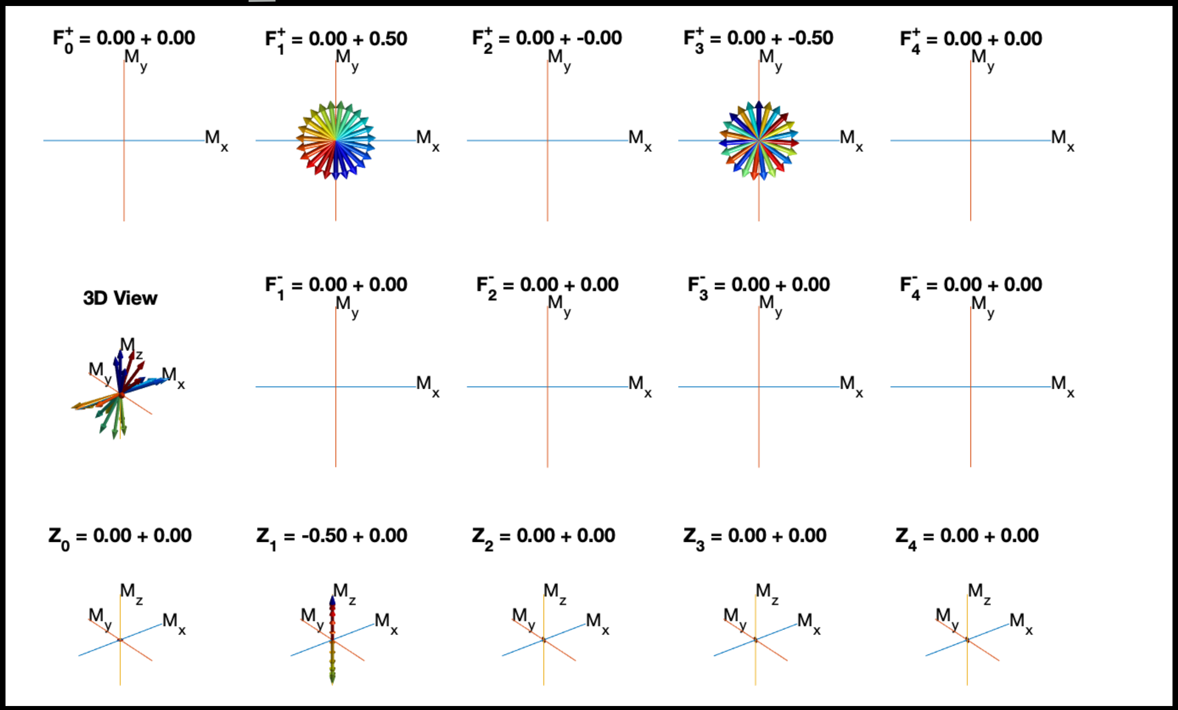
Example 3: Stimulated Echo and Spin Echo



- After another gradient:
  - $F^+{}_2$ >>> $F^+{}_3$
  - $F^+{}_0$ >>> $F^+{}_1$
  - Z_1 unchanged



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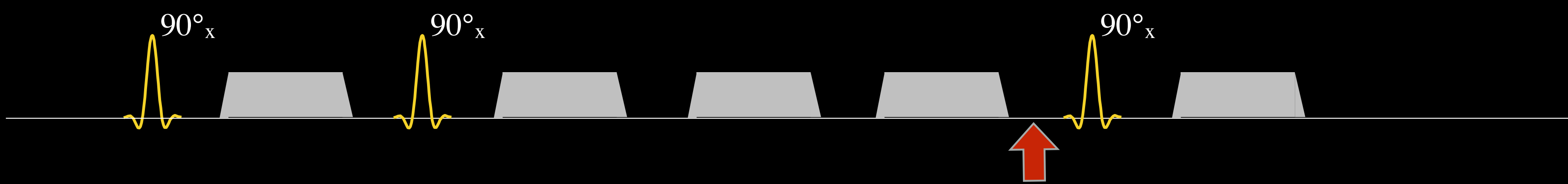
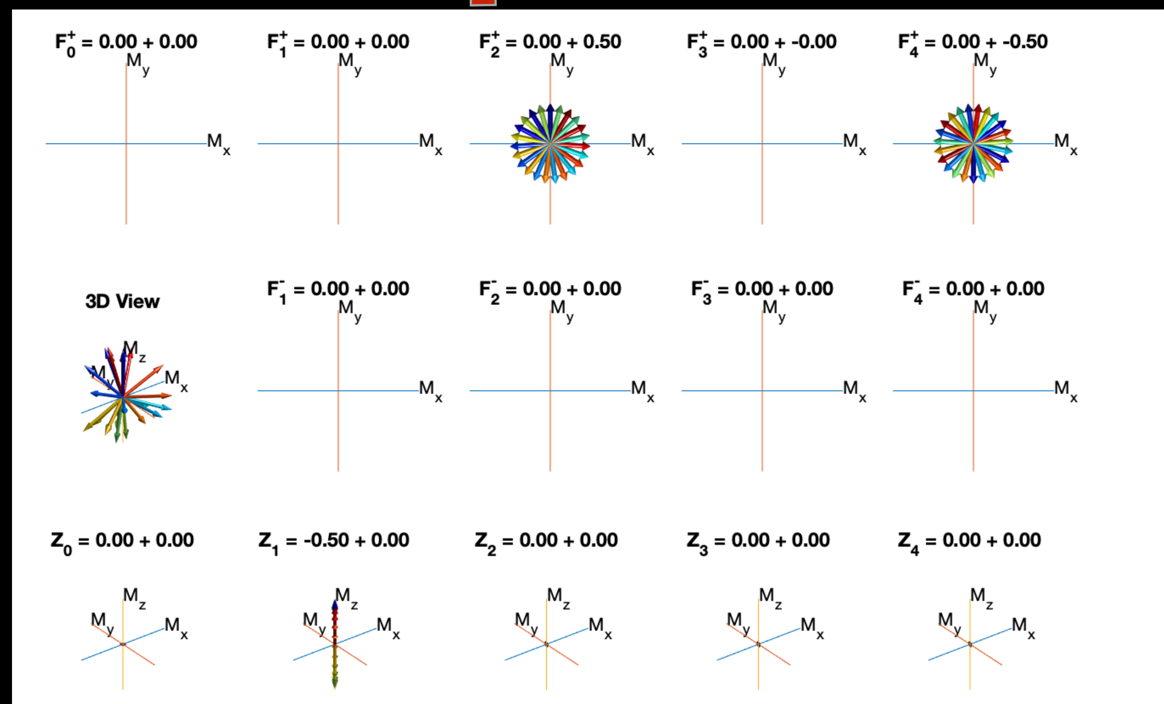
Example 3: Stimulated Echo and Spin Echo



- After another gradient:
  - $F^{+}_{3} >>> F^{+}_{4}$
  - $F^{+}_{1} >>> F^{+}_{2}$
  - Z_{1} unchanged



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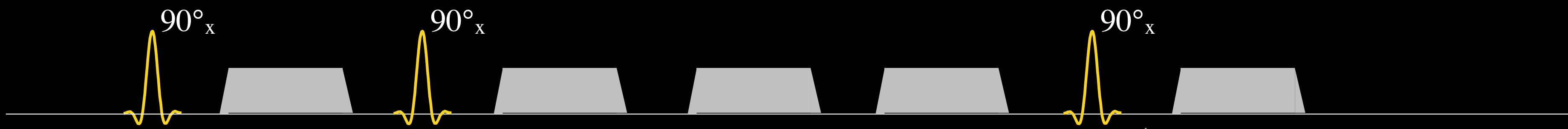
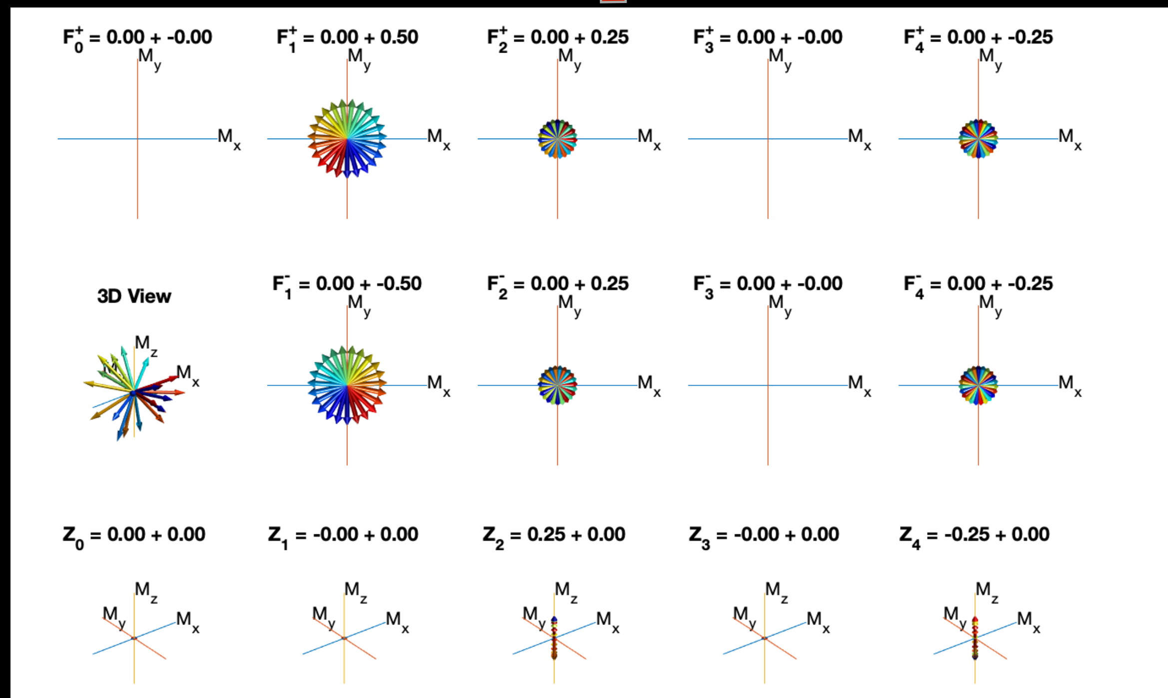
Example 3: Stimulated Echo and Spin Echo



- After another RF:

  $Z_1 >>> F^+_1, F^-_1$

  others split too



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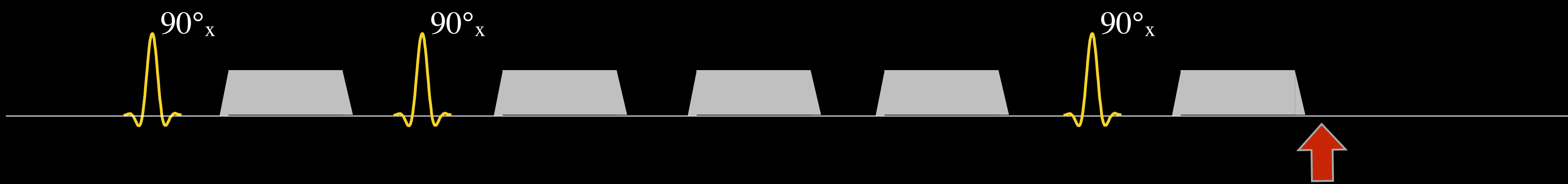
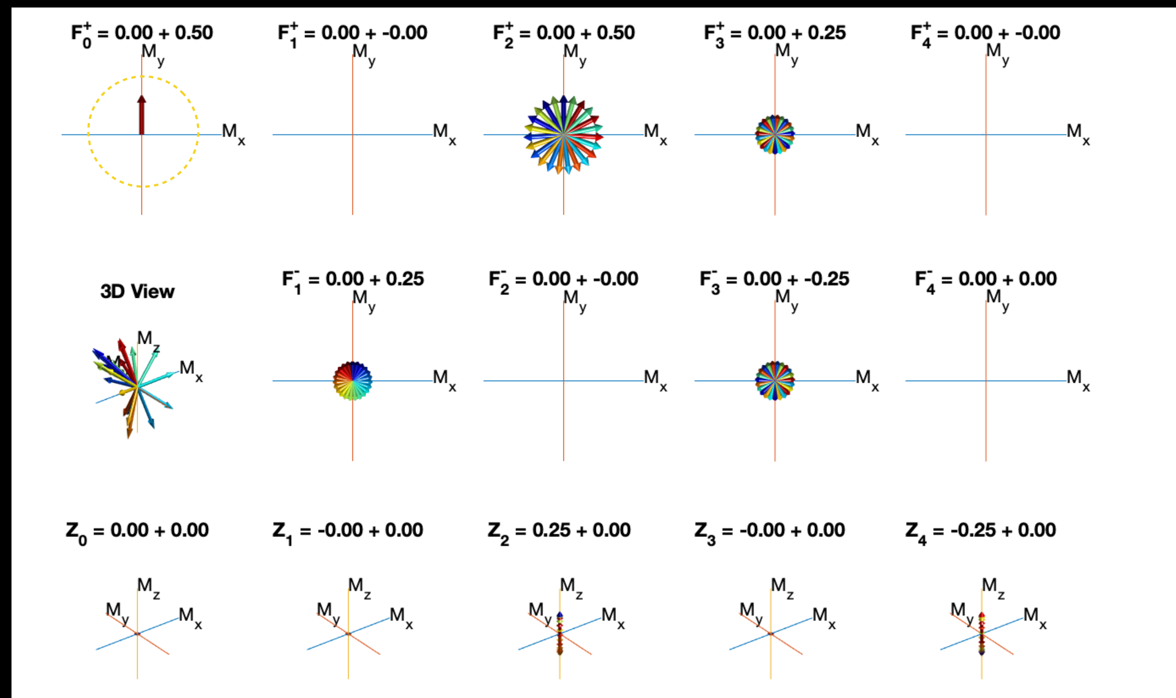
Example 3: Stimulated Echo and Spin Echo



- Another gradient:

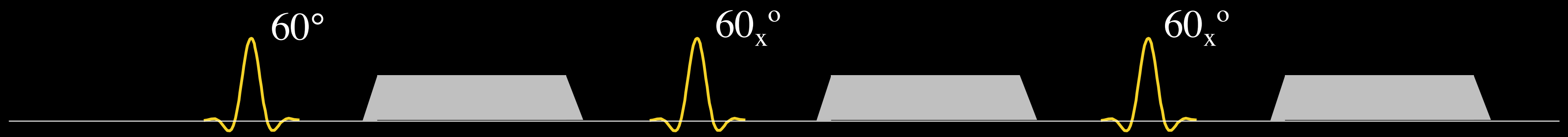
  $F^{-}_{1} >>> F^{+}_{0}$

  (Stimulated echo)



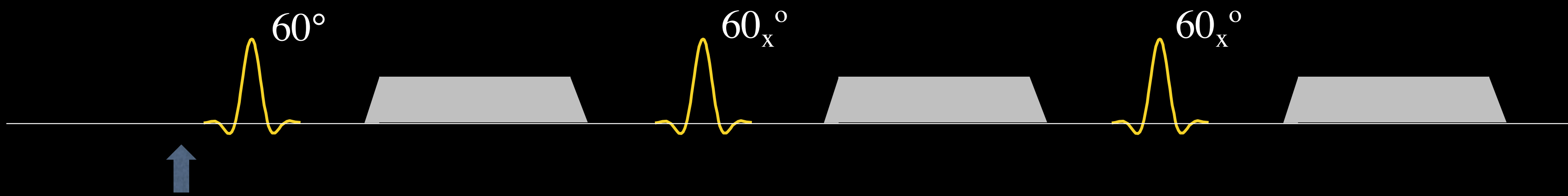
lec4_05.m

Example 4: Stimulated Echo Sequence



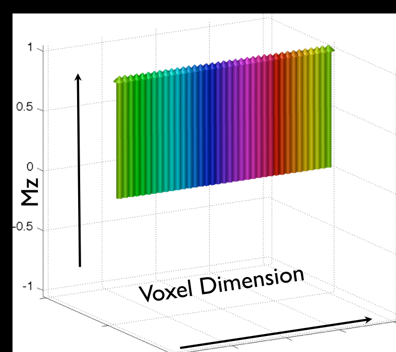
- We will follow this sequence through time
- Show which states are populated at each point

Example 4: General Stimulated Echo Sequence

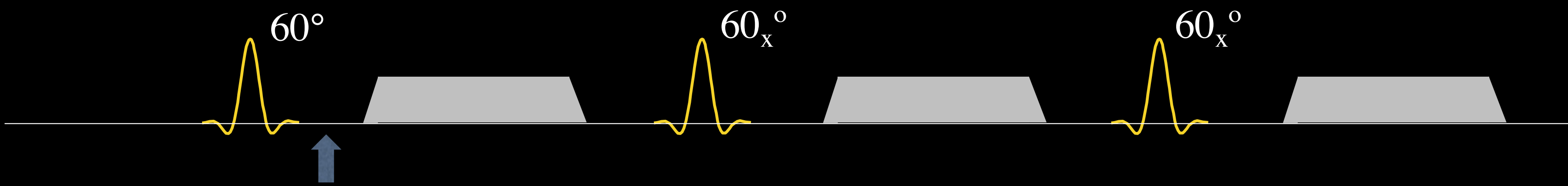


- Prior to the first pulse, all spins lie along M_z
- This is Z_0=1

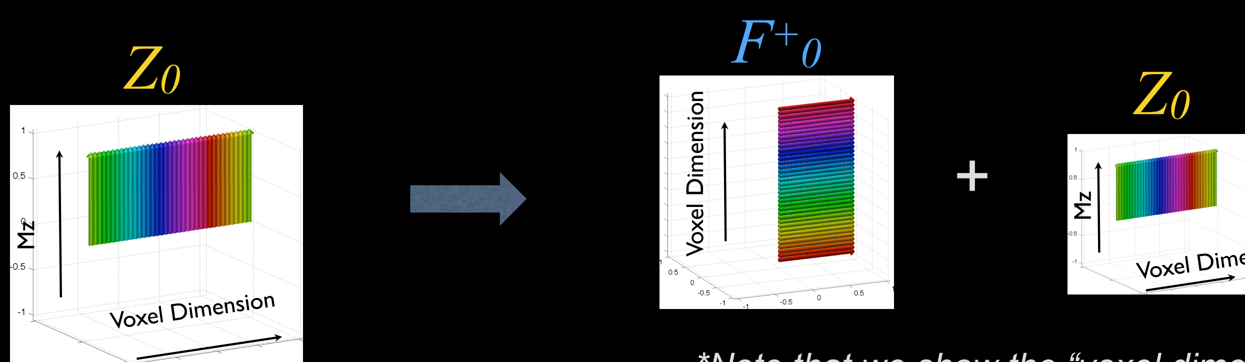
Z_0



Stimulated Echo: Excitation: F^+_0

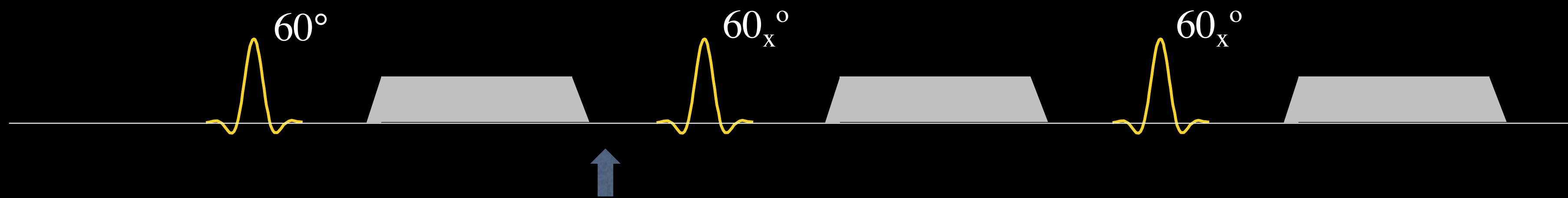


- After a 60º pulse, transverse spins are aligned along M_y (F^+_0 = sin60º)
- One half (cos60º) of the magnetization is still represented by Z_0

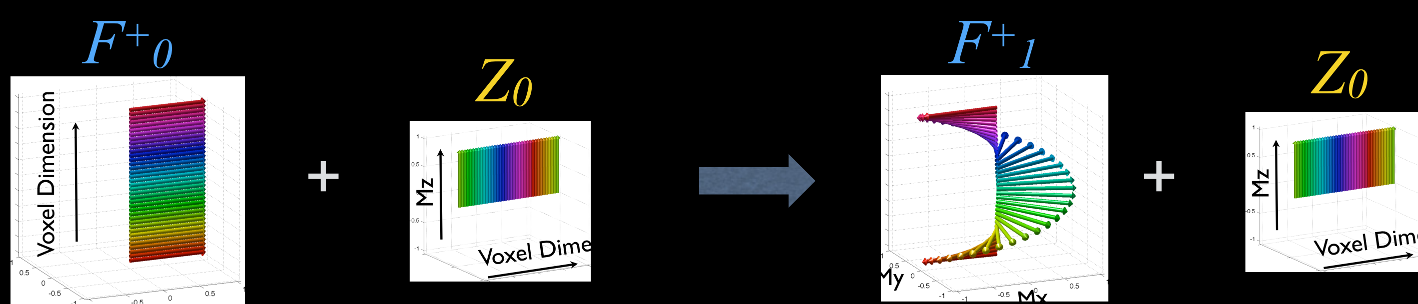


**Note that we show the "voxel dimension" along different axis for Z and F states*

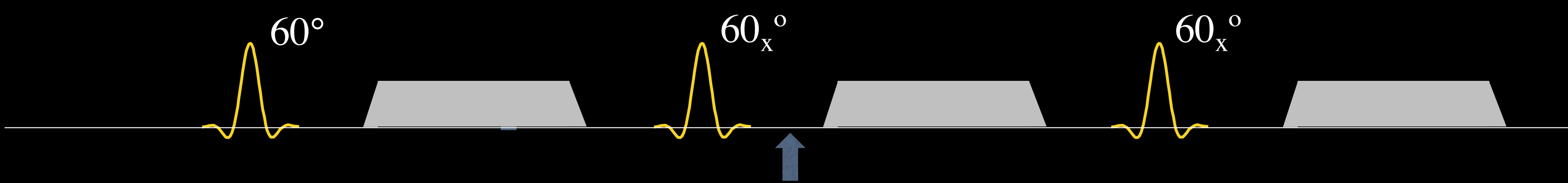
One Gradient Cycle



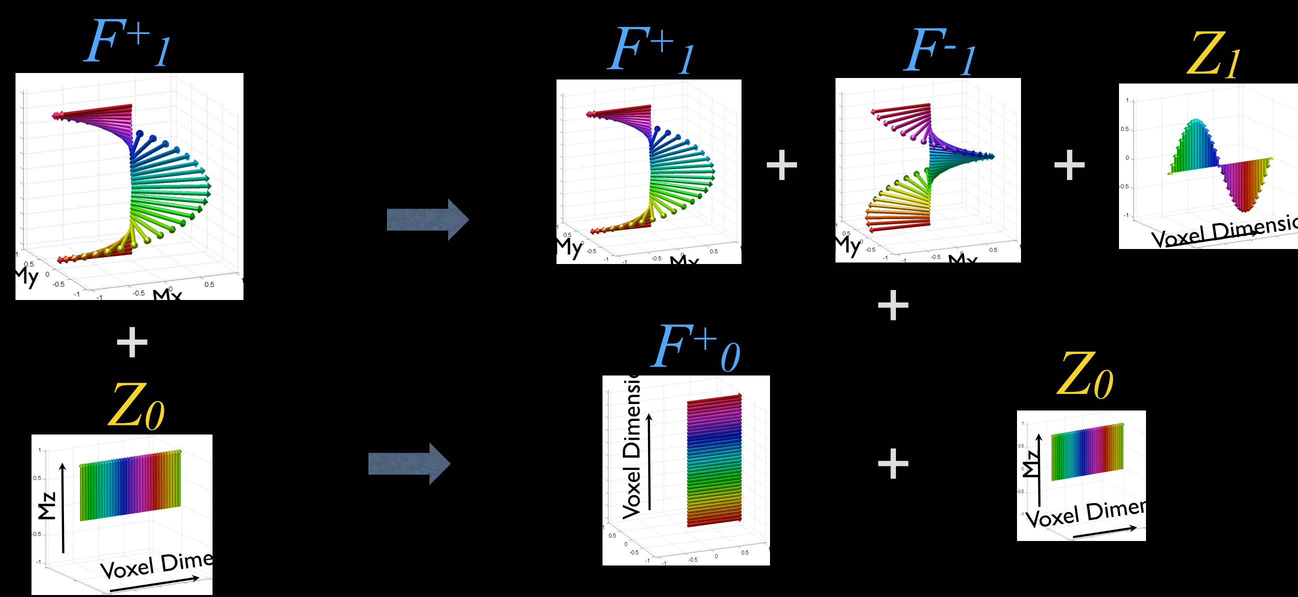
- The gradient "twists" the spins represented by F^{+}_{0}
- We call this state F^{+}_{1}, where the 1 indicates one cycle of phase (F^{+}_{1}=0.86)
- The spins represented by Z_0 are unaffected



Another Excitation (60°)

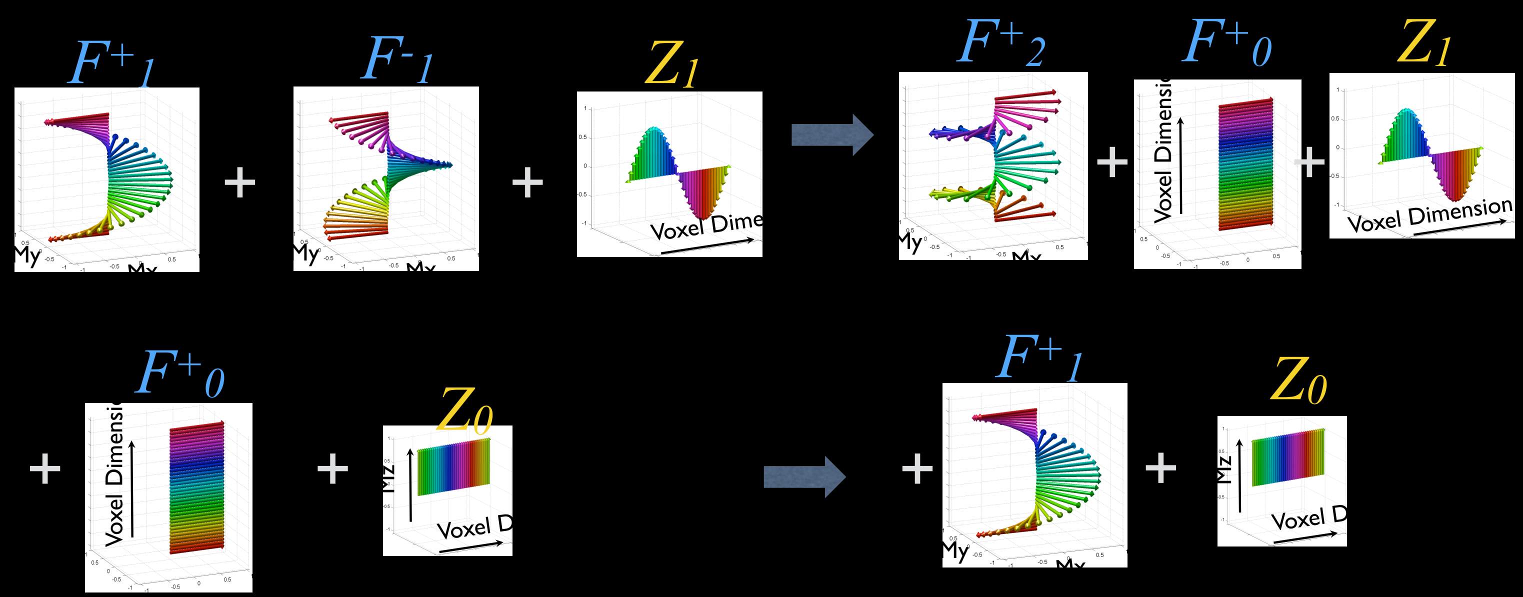


- The Z_0 magnetization is again split to F^+_0 and Z_0
- The F^+_1 magnetization is split three ways, to F^+_1, F^-_1 (reverse twisted) and Z_1



Another Gradient Cycle

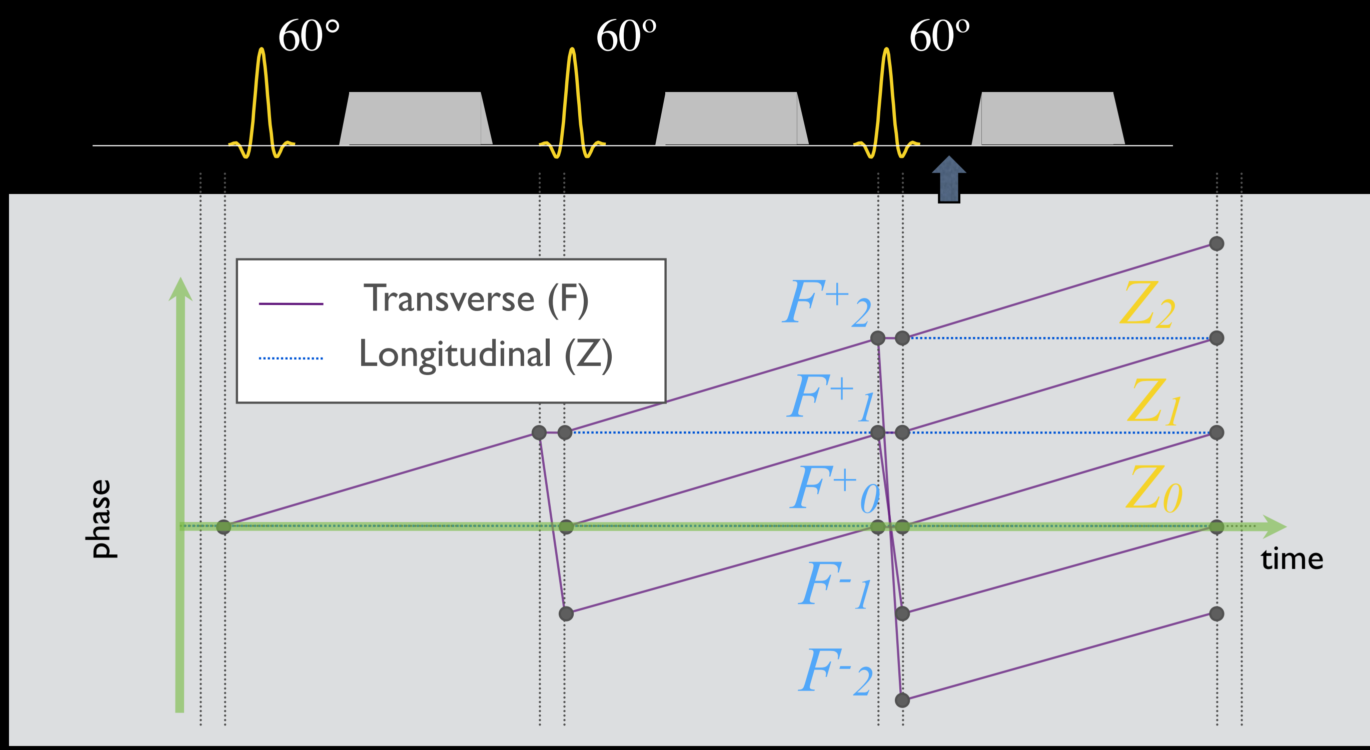
- The F^{-}_{1} state is refocused to F^{-}_{0} or F^{+}_{0}
- The F^{+}_{0} and F^{+}_{1} states become F^{+}_{1} and F^{+}_{2}
- The Z_0 states are all unaffected
- The process continues…!



Example 4: Stimulated Echo Coherence Pathways

The stimulated echo sequence coherence diagram is shown below

Compare with F and Z states on prior slide (location of arrow)



Matlab Formulations

EPG simulations can be easily built-up using modular functions - class GitHub

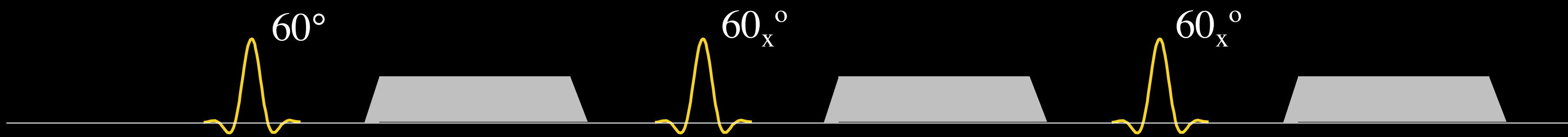
Simple Transition functions:

epg_RF.m	Applies RF to Q matrix
epg_grad.m	Applies gradient to Q matrix
epg_grelax.m	Gradient, relaxation and diffusion

Transformation to/from (M_x, M_y, M_z):

epg_spins2FZ.m	Convert M vectors to F,Z state matrix Q
epg_FZ2spins.m	Convert F,Z state matrix Q to M vectors

Stimulated-Echo Example



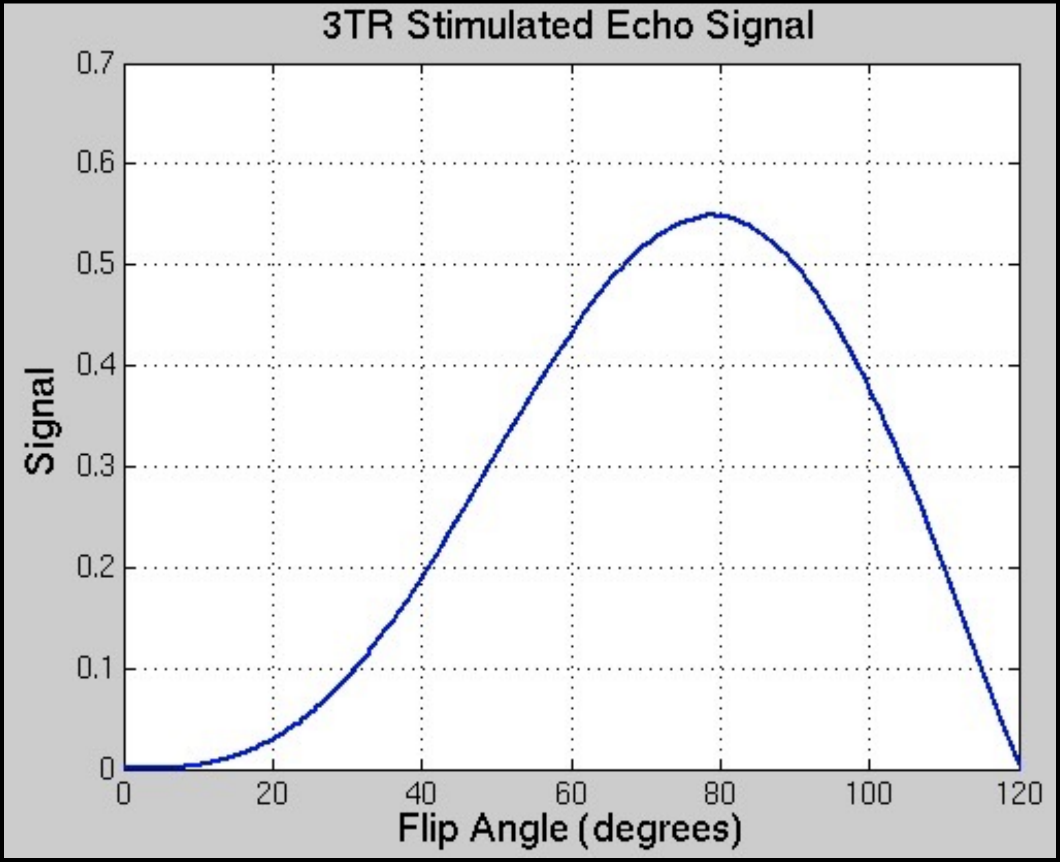
- Simulate 3 Steps: RF, gradient and relaxation
- Sample Matlab code:

(See epg_stim.m)

```
function [S,Q] = epg_stim(flips)
Q = [0 0 1]';                                   % Z0=1 (Equilibrium)
for k=1:length(flips)
  Q = epg_rf(Q,flips(k)*pi/180,pi/2);           % RF pulse
  Q = epg_grelax(Q,1,.2,0,1,0,1);               % Gradient/Relax
end;
S = Q(1,1);                                     % Signal from F0
```

Stimulated Echo Example (Cont)

Calculate signal vs flip angle: fplot(epg_stim([x x x],[0,120])



Diffusion

- Diffusion weighting can easily be applied
  - *Details in Weigel et al. J Magn Reson 2010; 205:276-285*
- Attenuation greater with n for both F_n and Z_n
- Requires physical "2π" gradient twist k (m^{-1})
- During gradient, $\Delta k = k$, otherwise $\Delta k = 0$

$$b_n(k, \Delta k) = \left[\left(nk + {}^{\Delta k}/_{2}\right)^2 + \left({}^{\Delta k^2}/_{12}\right)\right] T$$

$$F_n{}' = F_n e^{-b_n(k,\Delta k)D}$$

$$Z_n{}' = Z_n e^{-b_n(k,0)D}$$

Summary of Sequence Examples

- 90° and 180° RF pulses "swap" states
- Generally RF pulses "mix" states of order n
- Gradient pulses transition ${F^+}_n$ to ${F^+}_{n+p}$ and ${F^+}_n$ to ${F^+}_{n\text{-}p}$
- Coherence diagrams show progression through F and/or Z states to echo formation
- Signal calculations actually *quantify* the population of each state
- Matlab:
  - epg_grelax.m and epg_rf.m simulate many things
  - many epg_*.m examples and functions

How are EPGs used for common sequence calculations?

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