# Rad229 – MRI Signals and Sequences

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# Lecture-05A — MRI Imperfections, Part I

## Maxwell Fields

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# Learning Objectives

- Appreciate the Maxwell fields are unavoidable.
- Understand the spatial variation and direction of Maxwell fields.
- Describe the MRI hardware specifications that impact Maxwell field amplitudes.
- List two or more methods to correct Maxwell fields.

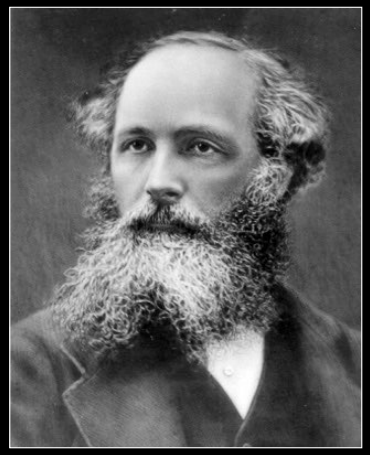
# B-Field Assumptions in MRI

- $B_0$-field is:
  - Perfectly uniform over space.
    - "$B_0$ homogeneity"
  - Perfectly stable with time.
- $B_1$-field is:
  - Perfectly uniform over space.
    - "$B_1$ homogeneity"
  - Temporally modulated exactly as specified.
- Gradient Fields are:
  - Perfectly linear over space.
    - "Gradient linearity"
  - Temporally modulated exactly as specified.
  - Do not induce Maxwell fields (!).

Gradient fields are not, in general, generated *exactly* as designed.

# Maxwell Fields

- When gradients fields are applied, other fields are unavoidably produced as described by <u>Maxwell's Equations</u>.
  - Generated concurrently with the applied gradient
  - Produce magnetic field components $\perp B_0\hat{k}$.
  - Introduces higher-order spatial dependence
- Maxwell fields also known as *concomitant fields*



**James Clerk Maxwell**
(13 June 1831 – 5 November 1879)

$$\nabla \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

Gauss's law for magnetism – There are no magnetic monopoles; magnetic flux through a closed surface is zero.

$$\nabla \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = 0$$

Ampère's circuital law - Magnetic fields can be generated by changing electric fields.

# Maxwell Fields

$$\nabla \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$\downarrow$$

$$G_z$$

$$\nabla \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = 0$$

$$\frac{\partial B_y}{\partial z} = \frac{\partial B_z}{\partial y} \qquad \frac{\partial B_z}{\partial x} = \frac{\partial B_x}{\partial z} \qquad \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x}$$

$$\downarrow \qquad \downarrow$$

$$G_y \qquad G_x$$

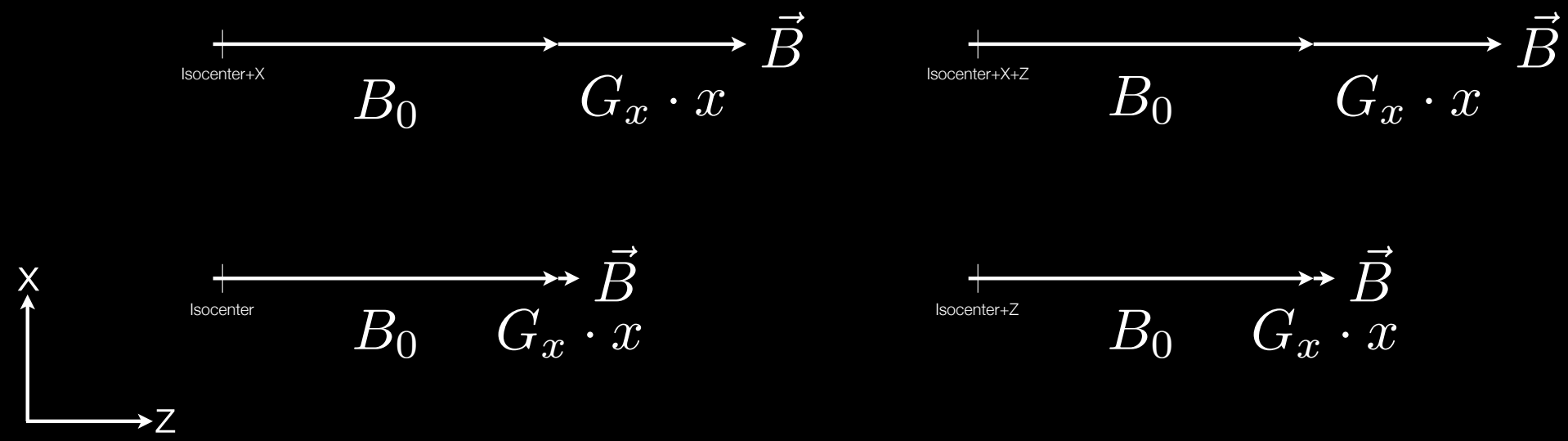
The active generation of the gradients ***necessarily*** generates additional B-field gradients.

# $B_0$ and Gradients Fields

$$\frac{\partial B_z}{\partial x} = \frac{\partial B_x}{\partial z}$$

$$\downarrow$$

$$G_x$$



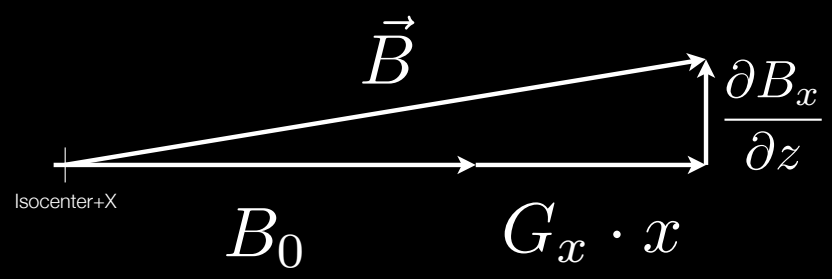

Ideally the applied gradients only generate their respective linear B-field contribution.

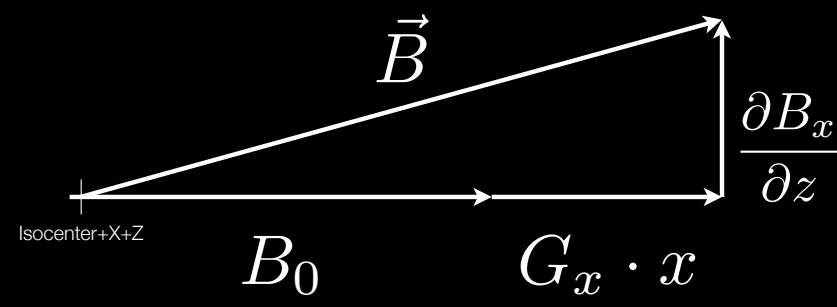
# $B_0$, Gradient, and Concomitant Fields

$$\frac{\partial B_z}{\partial x} = \frac{\partial B_x}{\partial z}$$

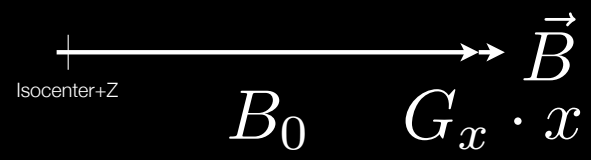
$$\downarrow$$

$$G_x$$








Concomitant fields contribute a B-field component perpendicular to the $B_0$ and gradient fields.

# Concomitant Fields

- Concomitant fields are:

A. Highest at isocenter.

B. Lowest at isocenter.

C. Only parallel to $B_0$.

D. Point in all directions.

# Concomitant Fields ($B_c$)

- Conventional MRI scanners use symmetric gradients
  - Each gradient's isocenter coincides
  - Then concomitant fields reduce to the following, which approximates a second-order spatial dependence:

$$B_c(x,y,z) = B_{c,2nd}(x,y,z)$$

Inversely proportional to $B_0$!

$$= \frac{1}{2B_0}\left[\left(G_x z - \frac{G_z x}{4}\right)^2 + \left(G_y z - \frac{G_z y}{4}\right)^2\right]$$

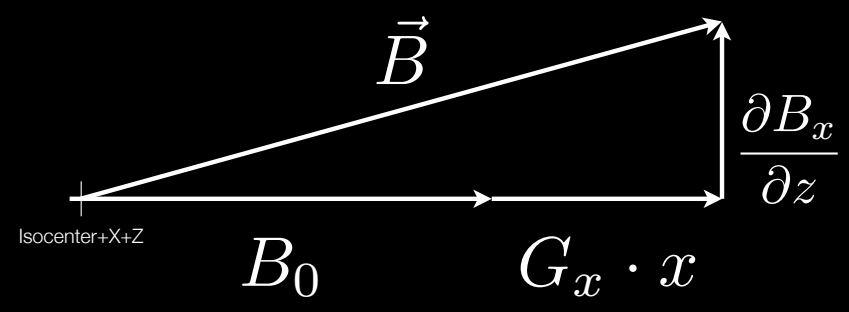
Dependent On $G^2$!

Always great than zero!

Concomitant B-fields have quadratic dependence on G and are inversely proportional to $B_0$. [See Bernstein's book for more.]

# Concomitant Fields ($B_c$)

- Net $\overrightarrow{B}$ deviates from $\hat{k}$.
  - Negligible effect on:
    - Polarization (too small)
    - Excitation (still $\perp$ to $\overrightarrow{B}_1$)
- Additional field components
  - Contribute to spin phase and frequency
    - Space and time dependent
    - Geometric and encoding distortion



# Concomitant Field Gradients ($G_c$)

$$\mathbf{G}_c(t) \approx \frac{1}{4B_0}\begin{bmatrix} G_z^2(t) & 0 & -2G_x(t)G_z(t) \\ 0 & G_z^2(t) & -2G_y(t)G_z(t) \\ -2G_x(t)G_z(t) & -2G_y(t)G_z(t) & 4G_x^2(t)+4G_y^2(t) \end{bmatrix}$$

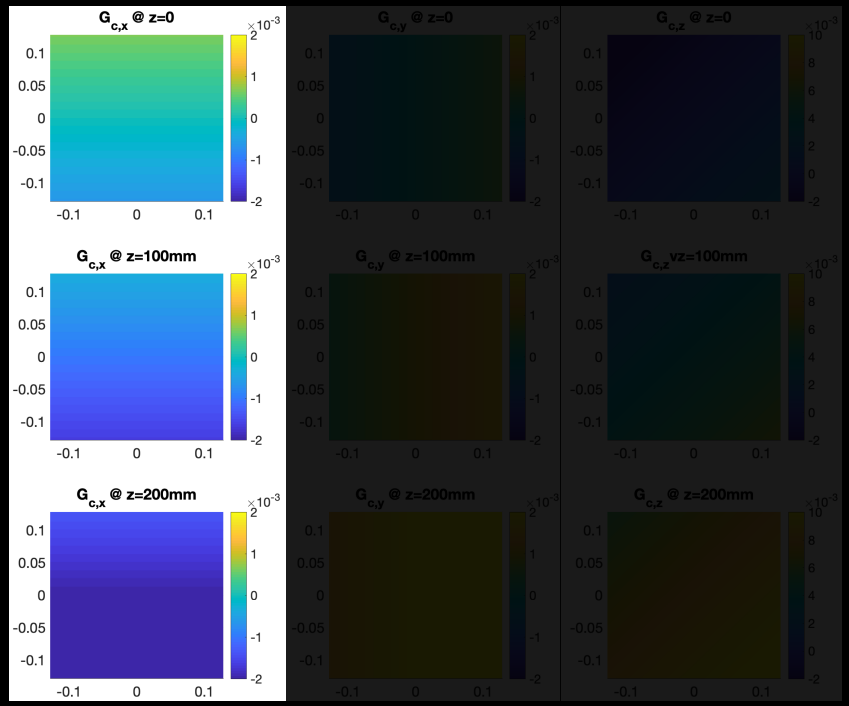
Concomitant Gradient Tensor

$$\vec{G}_c(\vec{r},t) = \mathbf{G}_c \cdot \vec{r}$$

$$\vec{G}(\vec{r},t) = \vec{G}_{target}(t) + \vec{G}_c(\vec{r},t)$$

Additional phase and frequency encoding.

```
fov=0.256;                  % Field-of-view    [m]
B0=0.5;                     % Field strength   [T]
gMax=100e-3;                % Gradient maximum [T/m]
G_dir=[1 -1 1];             % Gradient direction vector
pos=[-fov/2 fov/2 fov/2];   % Distance from isocenter [m]
```




See Szczepankiewicz et al. *MRM* 2019.

# Concomitant Fields

- Concomitant fields are least important for:

A. High-field, high $G_{max}$ systems.

B. Low-field, high $G_{max}$ systems.

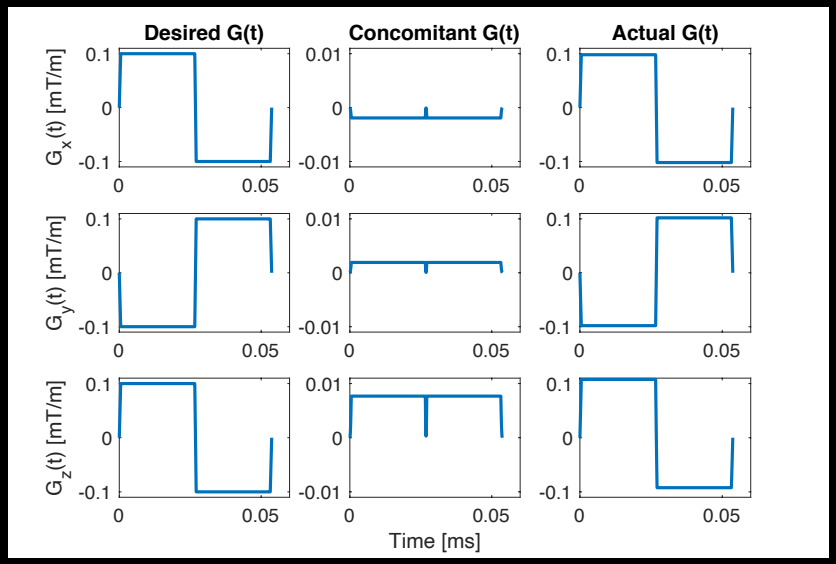
C. High-field, low $G_{max}$ systems.

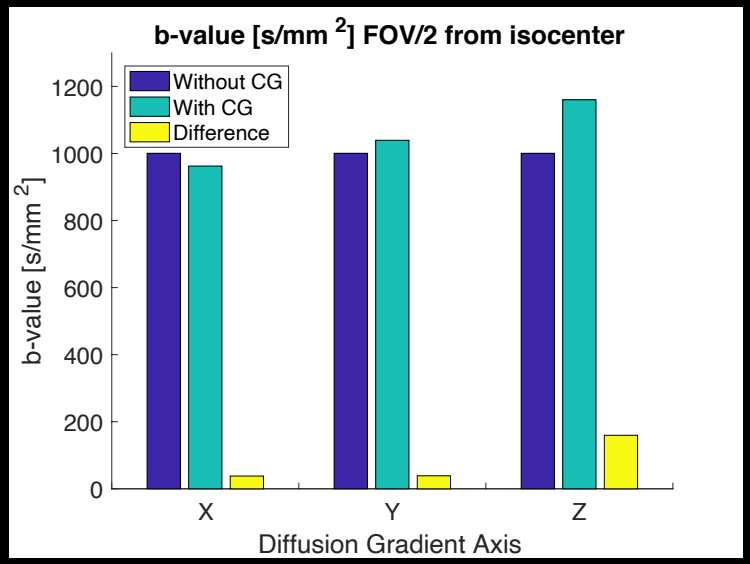
D. Low-field, low $G_{max}$ systems.

# Concomitant Fields in Diffusion Imaging

```
fov=0.256;                  % Field-of-view    [m]
B0=0.5;                     % Field strength   [T]
gMax=100e-3;                % Gradient maximum [T/m]
G_dir=[1 -1 1];             % Gradient direction vector
pos=[-fov/2 fov/2 fov/2];   % Distance from isocenter [m]

% Calculate the b-value (target~1000s/mm^2)
gamma=267.52e6;      % Gyromagnetic ratio [rad/s/T]
deltaDiff=12.8e-3;           % Diffusion gradient lobe duration [s]
G_dir=[1 -1 1];              % Gradient direction vector
b_val=2*gamma^2.*(gMax.*G_dir).^2.*((deltaDiff)^3/3)*1e-6; % [s/mm^2]
```

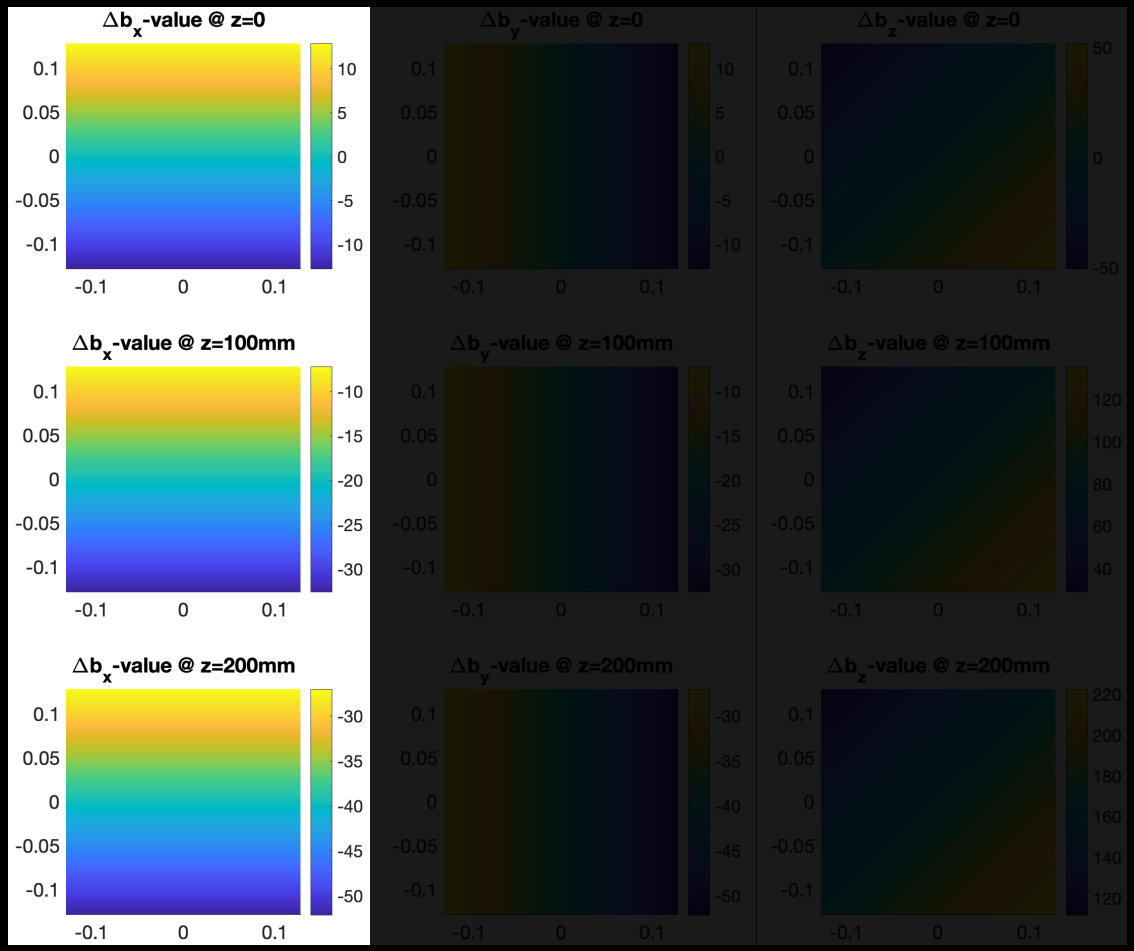



Concomitant fields produce b-value errors that are direction dependent.

# Concomitant Fields in Diffusion Imaging

Conditions:
- 0.5T and 100mT/m
- b-value=1000s/mm$^2$
- Axial slice @ isocenter
- Z-position=[0, 100, 200mm]

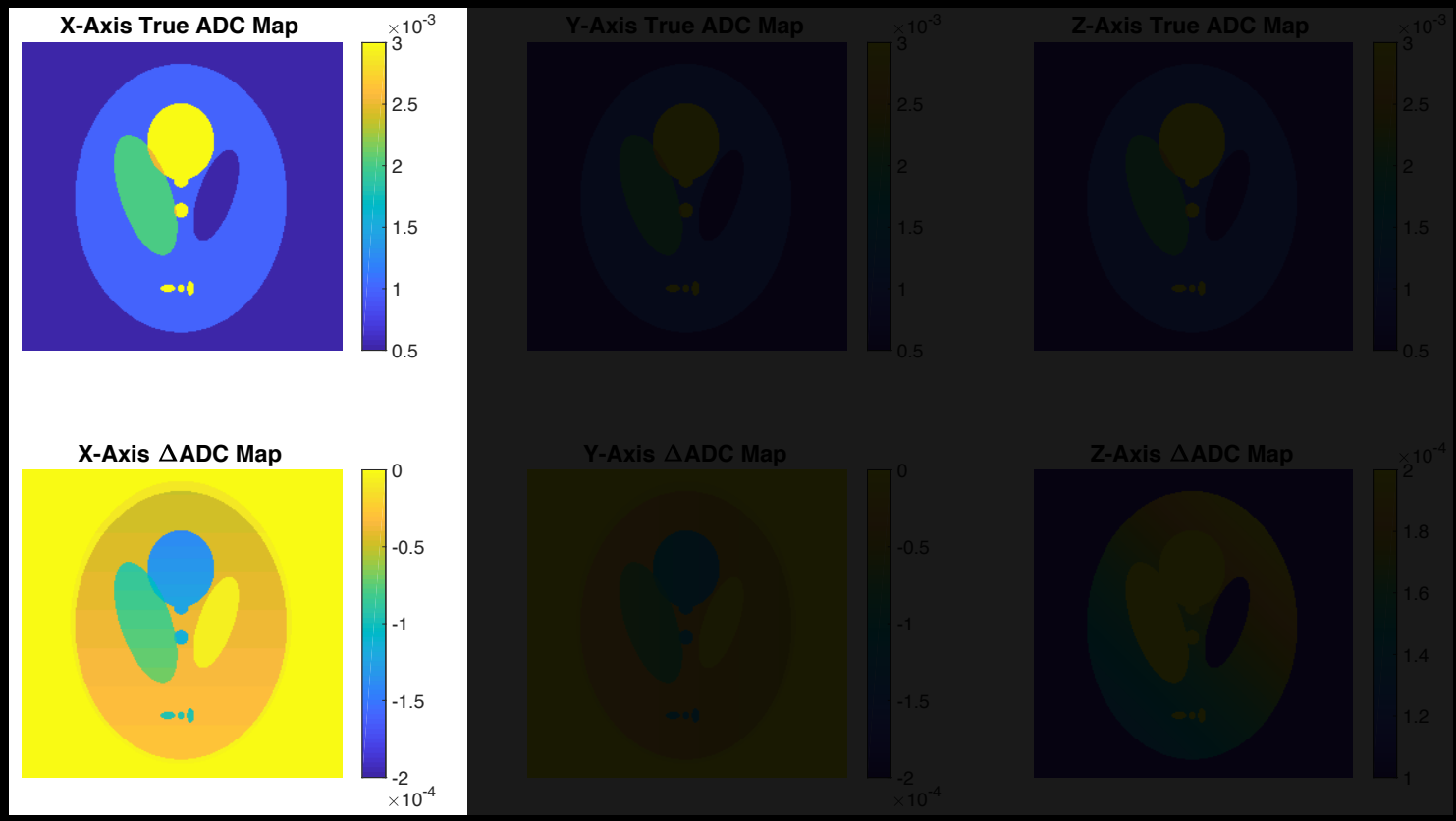



Concomitant fields create spatial gradients in b-value.

# Concomitant Fields in Diffusion Imaging

Conditions:
- 0.5T and 100mT/m
- b-value=1000s/mm$^2$
- Axial slice @ isocenter
- Z-position=[200mm]

- ADC_fat=0.2e-3 mm$^2$/s
- ADC_WM=1.0e-3 mm$^2$/s
- ADC_GM=2.0e-3 mm$^2$/s
- ADC_CSF=3e-3 mm$^2$/s




ΔADC maps are ADC(Concomitant)-ADC(True)

Concomitant fields create offsets and spatial gradients in estimated ADC if not corrected.

# Concomitant Field Correction Methods

- Waveform symmetry about 180° refocusing pulse
  - $B_C$ phase will be refocused
- Phase subtraction
  - Repeated acquisition with gradients off or toggled
- Waveform design
  - Shape the gradients to limit effect

# Further Reading…

## Artifacts Induced by Concomitant Magnetic Field in Fast Spin-Echo Imaging

Xiaohong Joe Zhou, Steve G. Tan, Matt A. Bernstein

**It has been observed that fast spin-echo (FSE) images with a large field of view (>40 cm) in certain directions exhibit unusual ghosting artifacts that cannot be eliminated with existing ghost removal methods. These artifacts have been related to a higher-order magnetic field perturbation (known as the concomitant field, or Maxwell field) concomitant to the linear imaging gradient, in accordance with the Maxwell equations $\nabla \cdot B = 0$ and $\nabla \times B$ oped to eliminate or magnetic field by rec slice-selection direc symmetrical about t ble. Surrounding th cannot be achieve which is often combi shown how crusher phase due to the co direction, the gradie prolonged. Artifacts nated by reshaping area fixed. In all the are adjusted so th methods have been typically reduce the images by a factor c Key words: concomi artifacts; phase erro**

### INTRODUCTION

Fast spin echo (F netic resonance im duce high-quality among the echo si circumstances, the eddy currents ind Several techniques inconsistent phase quently remove the ing the pulse seque correcting the erro has been found tha imated by a spatial linear component. ally limited to the

Recently, as we used stronger gradient systems (maximal gradient = 22 mT/m) to acquire sagittal FSE images over a large field of view (FOV ~48 cm), severe ghosting artifacts were observed at locations distant from the magnet isocenter. These artifacts, which cannot be removed by the conventional phase correction methods [4], in-

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From the Applied Scienc Wisconsin.
Address correspondence Laboratory, GE Medical 53201.
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0740-3194/98 $3.00


Magnetic Resonance in Medicine 68:1190–1201 (2012)

## The Effect of Concomitant Gradient Fields on Diffusion Tensor Imaging

C. A. Baron, R. M. Lebel, A. H. Wilman, and C. Beaulieu*

**Concomitant gradient fields are transverse magnetic field components that are necessarily present to satisfy Maxwell's equations when magnetic field gradients are utilized in magnetic resonance imaging. They can have deleterious effects that are more prominent at lower static fields and/or higher gradient strengths. In diffusion tensor imaging schemes that employ large gradients that are not symmetric about a refocusing radiofrequency pulse (unlike Stejskal–Tanner, which is symmetric), concomitant fields may cause phase accrual that could corrupt the diffusion measurement. Theory predicting the error from this dephasing is described and experimentally validated for both Reese twice-refocused and spilt gradient single spin-echo diffusion gradient schemes. Bias in apparent diffusion coefficient values was experimentally found to worsen with distance from isocenter and with increasing duration of gradient asymmetry in both a phantom and in the brain. The amount of error from concomitant gradient fields depends on many variables, including the diffusion gradient pattern, pulse sequence timing, maximum effective gradient amplitude, static magnetic field strength, voxel size, slice distance from isocenter, and partial Fourier fraction. A prospective correction scheme that can reduce concomitant gradient errors is proposed and verified for diffusion imaging. Magn Reson Med 68:1190–1201, 2012. © 2012 Wiley Periodicals, Inc.**
**Key words: DTI; concomitant fields; diffusion; gradient**

In MRI, magnetic field gradients are used for spatial encoding and to provide contrast for particular sequences. While the applied gradient fields are typically assumed to be linear and along the principle magnetic field direction, Maxwell's equations dictate that small nonlinear components are also present in the transverse direction. The magnetic field amplitude offset caused by these concomitant gradient fields scales inversely with the static magnetic field strength and have historically been a concern in low-field applications (1–3). While concomitant gradient fields can usually be ignored for common pulse sequences at most field strengths, there are some pulse sequences that are susceptible to their effects, such as echo-planar imaging (EPI) (4–6), fast spin-echo (7), spiral trajectories (8,9), phase contrast MRI (10), balanced steady-state free precession (11), and combined gradient and spin-echo acquisition (12). Generally, concomitant fields introduce additional phase variation. Depending on the $k$-space sampling scheme and slice orientation, this can cause misregistration of $k$-space data, which may result in signal loss or image shifting, warping, and/or blurring. Depending on the pulse sequence, corrections can sometimes be performed using preparation gradients before a 180° refocusing radiofrequency (RF) pulse to pre-warp the image phase (5), or by taking the misregistration into account during reconstruction (4,9,10,13). The amplitude of concomitant gradient fields scales with the square of gradient strength; therefore, with the advent of increasingly high amplitude gradient hardware, the effects of concomitant gradient fields will likely become increasingly relevant.

In diffusion-weighted imaging and diffusion tensor imaging (DTI), there is considerable motivation to utilize stronger gradients to achieve higher diffusion-sensitizing factors (i.e., $b$-value) and to reduce the echo time for a given $b$-value, thereby increasing the signal per shot and reducing scan time via a shorter repetition time (TR) (given the reduced echo time (TE) for each slice). The effect of concomitant gradient fields on diffusion imaging has been partly discussed in the context of asymmetric gradient coils by Meier et al. (14), where concomitant gradient effects can be more severe than for symmetric gradient coils. It was shown that a diffusion preparation containing bipolar gradients can yield concomitant gradient fields that create a linear phase ramp along the phase-encode direction, which can potentially shift the center of $k$-space out of the acquisition window, particularly when using partial Fourier undersampling. Note that diffusion pulse sequences containing bipolar gradients, as opposed to the pair of monopolar gradients applied on either side of the 180° refocusing RF pulse as in the Stejskal–Tanner (ST) method (15), minimize eddy current effects (16,17). Meier et al. also introduced a prospective correction that mitigates this specific source of error. They did not consider, however, errors in quantification of the apparent diffusion coefficient (ADC) that can arise from nonlinear phase variations or through-plane phase variations imparted by concomitant gradient fields. The directionality and nonlinearity of the phase accrual generated by concomitant gradient fields has a dependence on net gradient direction, which represents a considerable concern in DTI, where numerous gradient directions (six or greater) are acquired with simultaneous gradients that are often at maximum strength. This directionally dependent signal decay beyond diffusion will adversely impact the tensor and higher order models used for the determination of anisotropy and the orientation of white matter tracts.

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## Concomitant Field Terms for Asymmetric Gradient Coils: Consequences for Diffusion, Flow, and Echo-Planar Imaging

C. Meier,[1*] M. Zwanger,[2] T. Feiweier,[2] and D. Porter[2]

**As a consequence of the Maxwell equations, linear field gradients are accompanied by additional spatially dependent field components. A de sented which expli gradient coil. It is that, in contrast to ates concomitant Artifacts induced pronounced for a ones. For the stro MR systems the ef the evolution of m ation in a variety o ism is used expe served in three di standard echo pla weighted EPI, and nique based on p 60:128–134, 2008. Key words: conco metric gradient co ing; flow quantific**

In MRI, linear m spatial informati coils produce a However, this ro as it contradicts t expressed by Ma they require a z rotation (rot **B** = linear magnetic f terms, which are comitant fields. Concomitant f were first describ recognized in ro tions, improvem it possible to sig applied magneti studies. This has fields and has le They have subs series of publicat

ematical description of the concomitant fields and an understanding of some of the associated image artifacts

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FULL PAPER
Magnetic Resonance in Medicine

## Maxwell-compensated design of asymmetric gradient waveforms for tensor-valued diffusion encoding

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**Purpose:** Diffusion encoding with asymmetric gradient waveforms is appealing because the asymmetry provides superior efficiency. However, concomitant gradients may cause a residual gradient moment at the end of the waveform, which can cause significant signal error and image artifacts. The purpose of this study was to develop an asymmetric waveform designs for tensor-valued diffusion encoding that is not sensitive to concomitant gradients.
**Methods:** The "Maxwell index" was proposed as a scalar invariant to capture the effect of concomitant gradients. Optimization of "Maxwell-compensated" waveforms was performed in which this index was constrained. Resulting waveforms were compared to waveforms from literature, in terms of the measured and predicted impact of concomitant gradients, by numerical analysis as well as experiments in a phantom and in a healthy human brain.
**Results:** Maxwell-compensated waveforms with Maxwell indices below 100 $(mT/m)^2$ ms showed negligible signal bias in both numerical analysis and experiments. By contrast, several waveforms from literature showed gross signal bias under the same conditions, leading to a signal bias that was large enough to markedly affect parameter maps. Experimental results were accurately predicted by theory.
**Conclusion:** Constraining the Maxwell index in the optimization of asymmetric gradient waveforms yields efficient diffusion encoding that negates the effects of concomitant fields while enabling arbitrary shapes of the b-tensor. This waveform design is especially useful in combination with strong gradients, long encoding times, thick slices, simultaneous multi-slice acquisition, and large FOVs.

**KEYWORDS**
asymmetric gradient waveforms, concomitant gradients, Maxwell terms, tensor-valued diffusion encoding

### 1 | INTRODUCTION

Diffusion-weighted MRI uses magnetic field gradients to sensitize the signal to the diffusive motion of signal-carrying spins. The gradients are assumed to be linear in space, but in practice, they are accompanied by so-called concomitant gradients, also referred to as Maxwell terms.[1] In spin-echo sequences, diffusion-weighting is most commonly achieved



## Concomitant Gradient Terms in Phase Contrast MR: Analysis and Correction

Matt A. Bernstein, Xiaohong Joe Zhou, Jason A. Polzin, Kevin F. King, Alexander Ganin, Norbert J. Pelc, Gary H. Glover

**Whenever a linear gradient is activated, concomitant magnetic fields with non-linear spatial dependence result. This is a consequence of Maxwell's equations, i.e., within the imaging volume the magnetic field must have zero divergence, and has negligible curl. The concomitant, or Maxwell field has been described in the MRI literature for over 10 years. In this paper, we theoretically and experimentally show the existence of two additional lowest-order terms in the concomitant field, which we call cross-terms. The concomitant gradient cross-terms only arise when the longitudinal gradient $G_z$ is simultaneously active with a transverse gradient ($G_x$ or $G_y$). The effect of all of the concomitant gradient terms on phase contrast imaging is examined in detail. Several methods for reducing or eliminating phase errors arising from the concomitant magnetic field are described. The feasibility of a joint pulse sequence–reconstruction method, which requires no increase in minimum TE, is demonstrated. Since the lowest-order terms of the concomitant field are proportional to $G^2/B_0$, the importance of concomitant gradie given the current interest in and/or weaker main magnet Key words: concomitant gr Maxwell terms; phase contr**

### INTRODUCTION

Phase contrast angiograph tion and volume flow rate with excellent stationary t trast images are usually a MRI data sets with differe ferent values of the first g are then individually Fou difference is calculated o ing a map where phase i component along the dir gradient. Phase maps acquired i systematic errors, e.g., th eddy currents (6). Phase facts in MR phase contra velocity measurements. I phase errors from a diffe

magnetic field (7–16). The concomitant magnetic field, or Maxwell field, was first described in the MRI literature over ten years ago. The effect of the concomitant field is small (≤2 ppm at 1.5 T with a gradient amplitude of 10 mT/m and a 20-cm distance from isocenter), and is thus largely neglected in most imaging situations. With the current interest in low-field systems and higher amplitude gradient systems, however, the importance of the concomitant magnetic field increases. In this paper we build on prior work by theoretically and experimentally demonstrating the existence of two cross-terms in the concomitant field. We then explore the effect of the concomitant gradient terms on phase contrast in greater detail. We propose several modifications to the phase contrast pulse sequence that can minimize phase errors from the concomitant field. We then demonstrate a joint pulse

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From the GE Medical Systems, J.A.P., K.F.K., A.G.), and Departm Center, Stanford University School G.H.G.).
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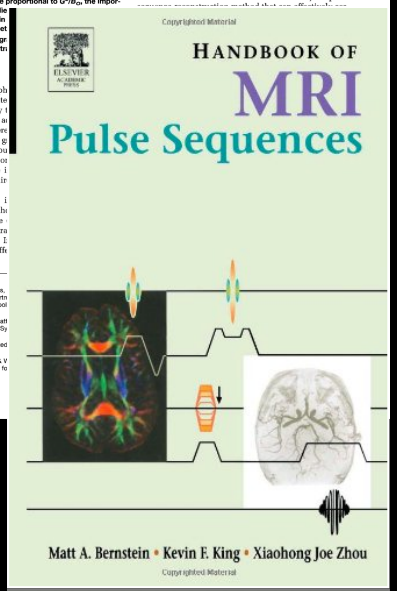

What about imperfections that arise from object motion or flow?

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