Rad229 – MRI Signals and Sequences

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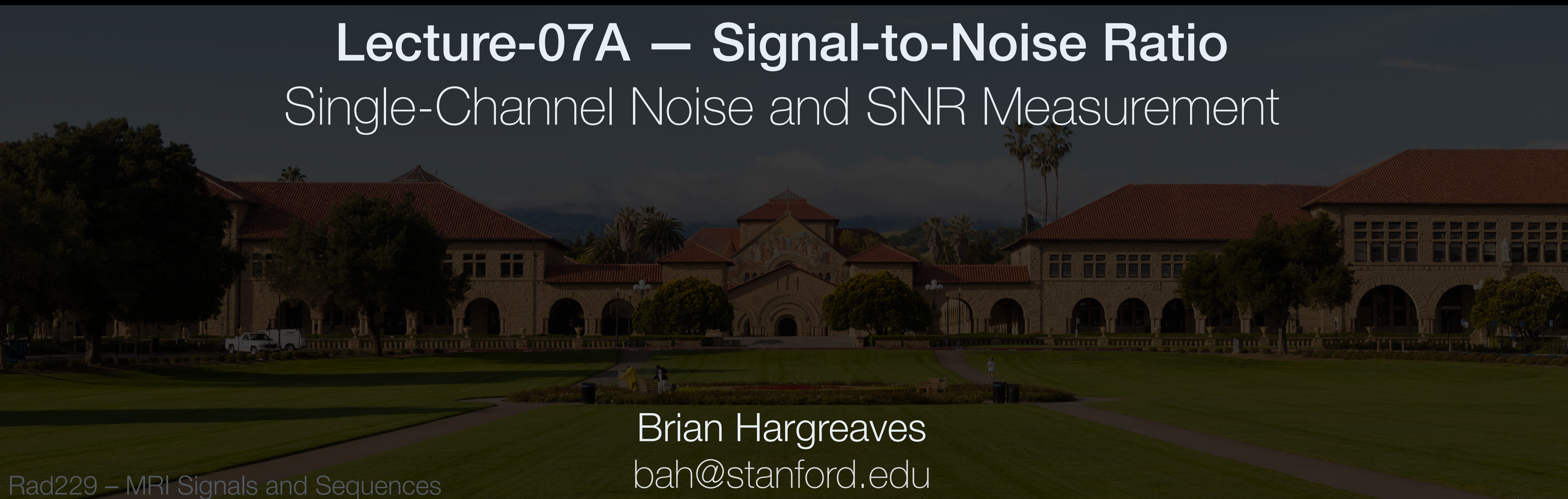
Lecture-07A — Signal-to-Noise Ratio

Single-Channel Noise and SNR Measurement



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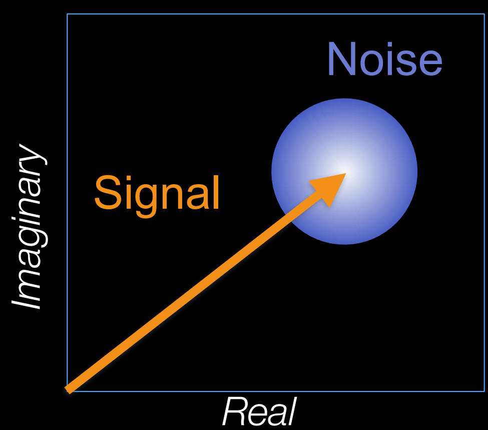
Learning Objectives

- Understand noise statistics in k-space, complex images, and magnitude images
- Explain when the distribution is Gaussian, Rayleigh and Rician
- Know basic methods to measure SNR

SNR: Signal-to-Noise Ratio

- Signal (desired) vs Noise (interference)
  - Thermal noise, depends on coil, patient size



Low SNR High SNR

Origins of SNR

- Body noise dominance
- SNR = C f(Ob) (Im)
  - C = constants
  - f(Ob) = function of object
  - Im = Imaging parameters

ω_0 = frequency

V_{voxel} = voxel volume

T_{acq} = A/D time

$$SNR = \left[\frac{2\chi\sqrt{\rho}}{\gamma\mu_0\sqrt{KT\pi}}\right]\left[\frac{1}{r_0^2\sqrt{l}}\right]\left[\omega_0 V_{voxel}\sqrt{T_{acq}}\right]$$

Macovaki A, Noise in MRI. Magn Reson Med 1996; 36:494-497

Noise and the Signal Equation

- Noise is zero-mean, complex, additive
- Recall signal equation from lecture 2:

$$S(\vec{k}) = \int_{object} M(\vec{r}) e^{-i2\pi \vec{k}\cdot\vec{r}} d\vec{r} + n_c(0,\sigma)$$

- Additive complex, gaussian noise: $n_c = n_r + i * n_i$
- Probability density function $P(n_r, n_i)$ is a simple product of gaussian distributions for real and imaginary:

$$P(n_r, n_i) = \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{n_r^2}{2\sigma^2}}\right)\left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{n_i^2}{2\sigma^2}}\right)$$

Noise is complex, gaussian and additive, with zero mean and variance σ^2 (both real and imaginary)

Gaussian Noise in (Discrete) Image Space and k-Space

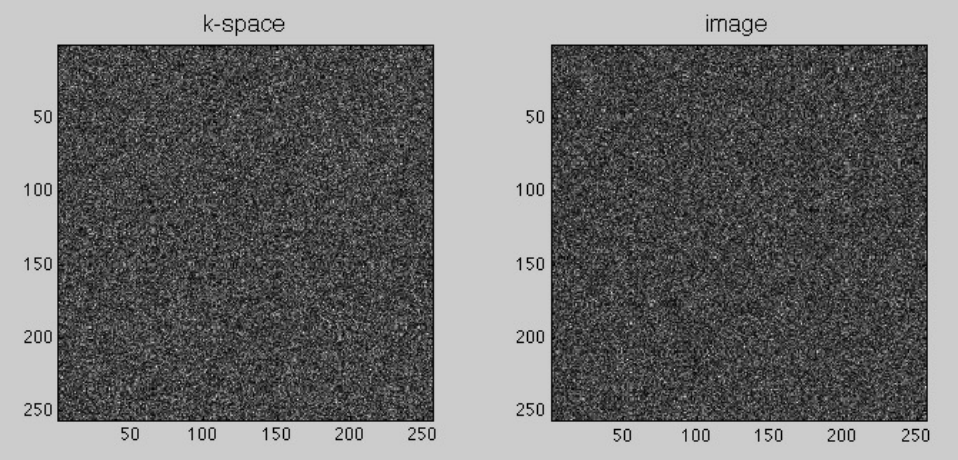
- Assume equally scaled FFT / iFFT:
  - Equations are 1D!

$$S_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} M_n e^{-i2\pi kn/N}$$

$$M_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S e^{i2\pi kn/N}$$

- Noise statistics preserved (M_n <=> S_k)
  - additive complex noise
  - gaussian, zero-mean

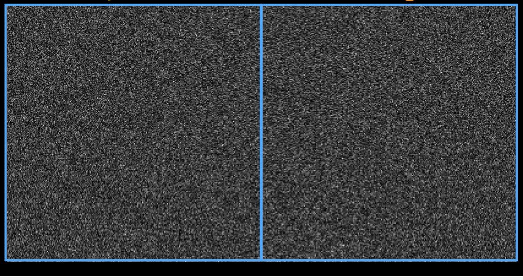
$$\sigma = \sigma_r = \sigma_i$$



Complex, zero-mean gaussian noise in k-space transforms to complex gaussian noise in image space, both same variance σ^2

Example: Verifying Noise Properties

- Make k-space noise, Fourier transform, calculate statistics for varying signal:



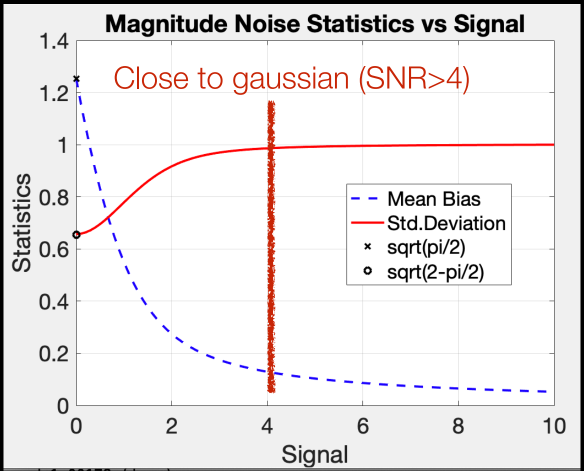

```
nsig = 1;        % Noise sigma parameter (real and imaginary)
N = 256;         % Image/k-space size.

kr = randn([N,N])*nsig;         % Generate gaussian noise (real part)
ki = randn([N,N])*nsig;         % Generate gaussian noise (imag part, same)
k = kr+i*ki;                    % Combine

im = N * ift(k);        % iFFT with scaling of sqrt(N*N)=N
[std(real(im(:))),std(imag(im(:)))]

% -- Calculate noise as a function of SNR, with magnitude images

s=[0:0.1:10];           % s = Signal, so same as SNR if nsig==1
for p=1:length(s)
  mn(p) = mean(abs(im(:)+s(p)));        % Mean of magnitude signal
  sd(p) = std(abs(im(:)+s(p)));         % Std.deviation of magnitude
end;
```



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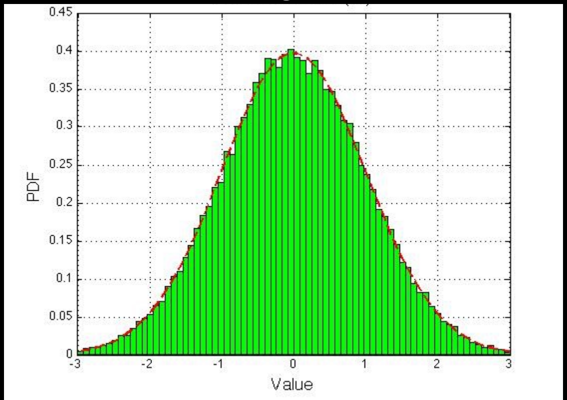
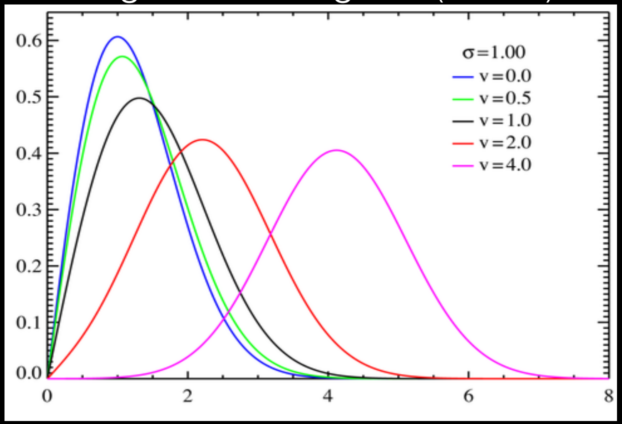
Question 1

Basic Noise Distributions (Single-Channel)

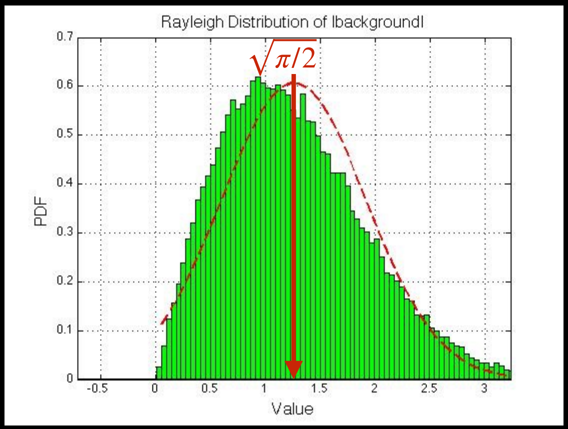
Real-part of Signal (Gaussian)



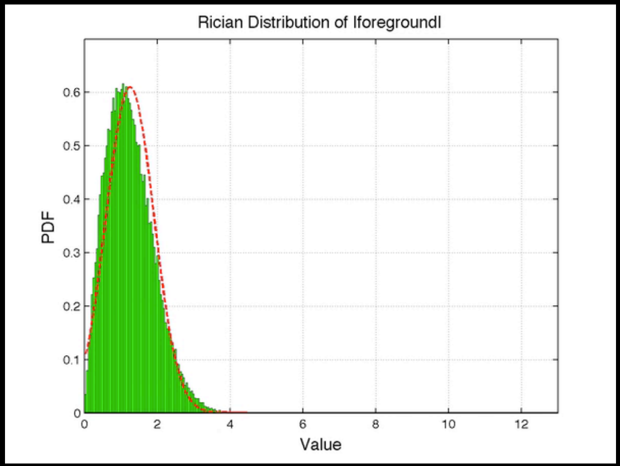
Magnitude of Signals (Rician)



Magnitude of Background (Rayleigh)



Magnitude of Signal (Varying v)

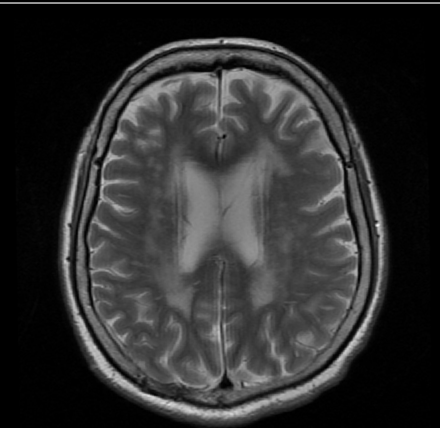


Magnitude Noise is Rayleigh for zero-mean, then Rician (Looks Gaussian above SNR ~ 4)

Question 2

Basic SNR Measurement (1 coil)

- Measure mean in signal area ROI
- Measure std-deviation in magnitude background ROI
- Correct for Rayleigh distribution in background



$mean_{Rayleigh} = 1.26$
$\sigma_{Rayleigh} = 0.65$

$\sigma_{gaussian} = mean_{Rayleigh} / sqrt(\pi/2) = 1.008$
$\sigma_{gaussian} = \sigma_{Rayleigh} / sqrt(2-\pi/2) = 0.997$

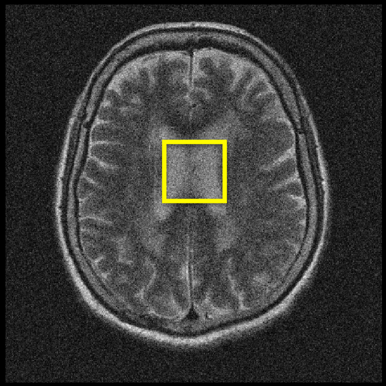
For a single channel coil, noise can be measured from the background magnitude mean or standard deviation

Difference Method SNR

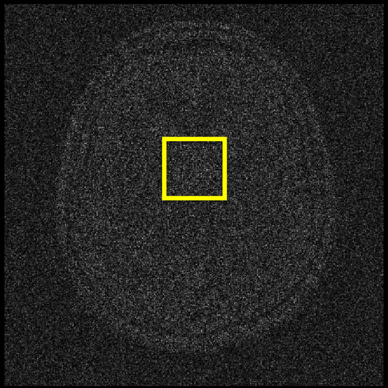
- In theory, N measurements should give you a population, and at each pixel you get a (roughly gaussian) distribution
- With 2 measurements you can still estimate mean and standard deviation (Reeder et al)
- Still want SNR > 4, or underestimate noise

$\sigma_{Mag\text{-}Diff} = 1.394$

$\sigma_{gaussian} = \sigma_{Mag\text{-}Diff} / \mathrm{sqrt}(2) \sim 1.0$



Sum



Difference of Magnitude Images

The difference method is a crude measure of the variation at a particular voxel

Question 3

Summary

- Noise in k-space and complex images is complex and Gaussian
- Magnitude image noise has:
  - Rician distribution (non-zero signal or mean)
  - Rayleigh in the background)
- SNR can be measured by foreground/background or the difference method (with corrections as appropriate)

How do different setup factors and sequence parameters affect SNR?

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