


A photograph of a large, multi-story building with a red-tiled roof and arched windows, likely a Stanford University building. The building is set against a dark, overcast sky. In the foreground, there is a green lawn and a paved path. The text is overlaid on the image.

Rad229 – MRI Signals and Sequences

Daniel Ennis & Brian Hargreaves

dbe@stanford.edu –or– bah@stanford.edu

A wide-angle photograph of the Stanford University Main Quad, featuring the central building with its iconic red-tiled roof and arches, surrounded by a large green lawn and trees. The image is dimmed to serve as a background for the text.

Lecture-11B — Radial and Spiral Sequences

Spiral Imaging

Brian Hargreaves
bah@stanford.edu

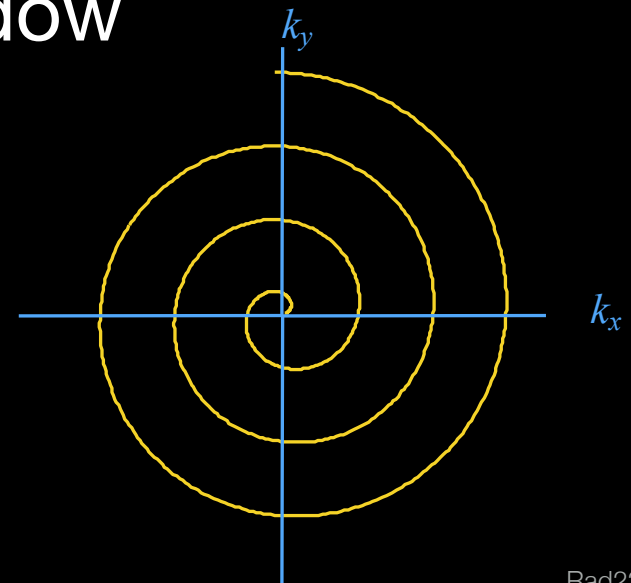
Learning Objectives

- Explain the rationale for spiral imaging
- Describe how a spiral waveform can be designed
- Explain the trade-offs between number of interleaves, trajectory duration and scan time
- Explain how off-resonance and delays impact spiral images
- Explain the steps of a gridding reconstruction



Spiral

- Flexible duration/coverage trade-off
 - Like radial, center-out, $TE \sim 0$
 - Low first-moments
- Longer readouts maximize acq window
 - Archimedean, TWIRL, WHIRL
 - Variable-density



Spiral offers more flexibility than radial imaging, but sampling a greater portion of k-space



Archimedean Spiral

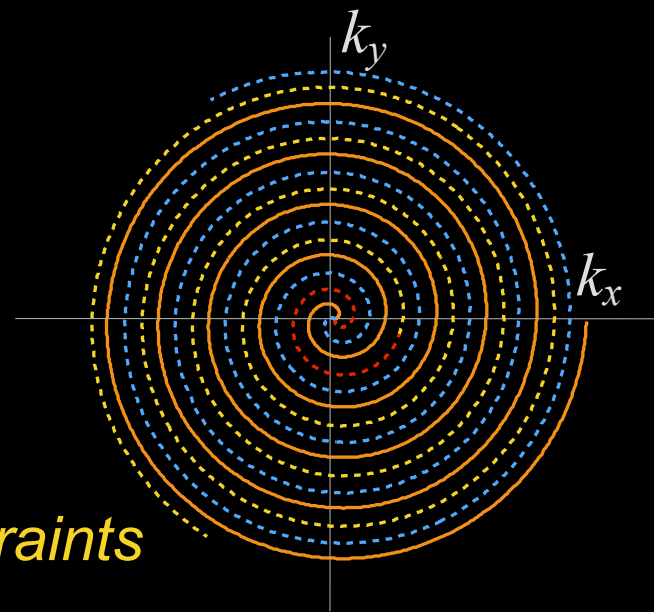
- Radius proportional to angle: $k(t) = k_x + ik_y = A \theta(t)$
- Somewhat uniform density, with N interleaves
- Extreme case: single-shot with $N=1$
- θ increases 2π per turn... what is A ?

$$k(t) = \frac{N\theta}{2\pi FOV} e^{i\theta}$$

Stopping point:

$$\theta_{max} = \frac{2\pi}{N} k_{max} FOV$$

Challenge is to design $\theta(t)$ to meet constraints



An Archimedean spiral has radius proportional to angle - leading to somewhat uniform sampling



Archimedean Spiral Design

- Begin with spiral equation:
- Differentiate to obtain dk/dt and d^2k/dt^2
- Amplitude limit: $dk/dt < \gamma/2\pi G_{max}$
- Slew limit: $d^2k/dt^2 < \gamma/2\pi S_{max}$

$$k(t) = \frac{N\theta}{2\pi FOV} e^{i\theta}$$

1. Approximations for $\theta(t)$ (Glover 1999)

- Consider slew-limited and amplitude-limited regions

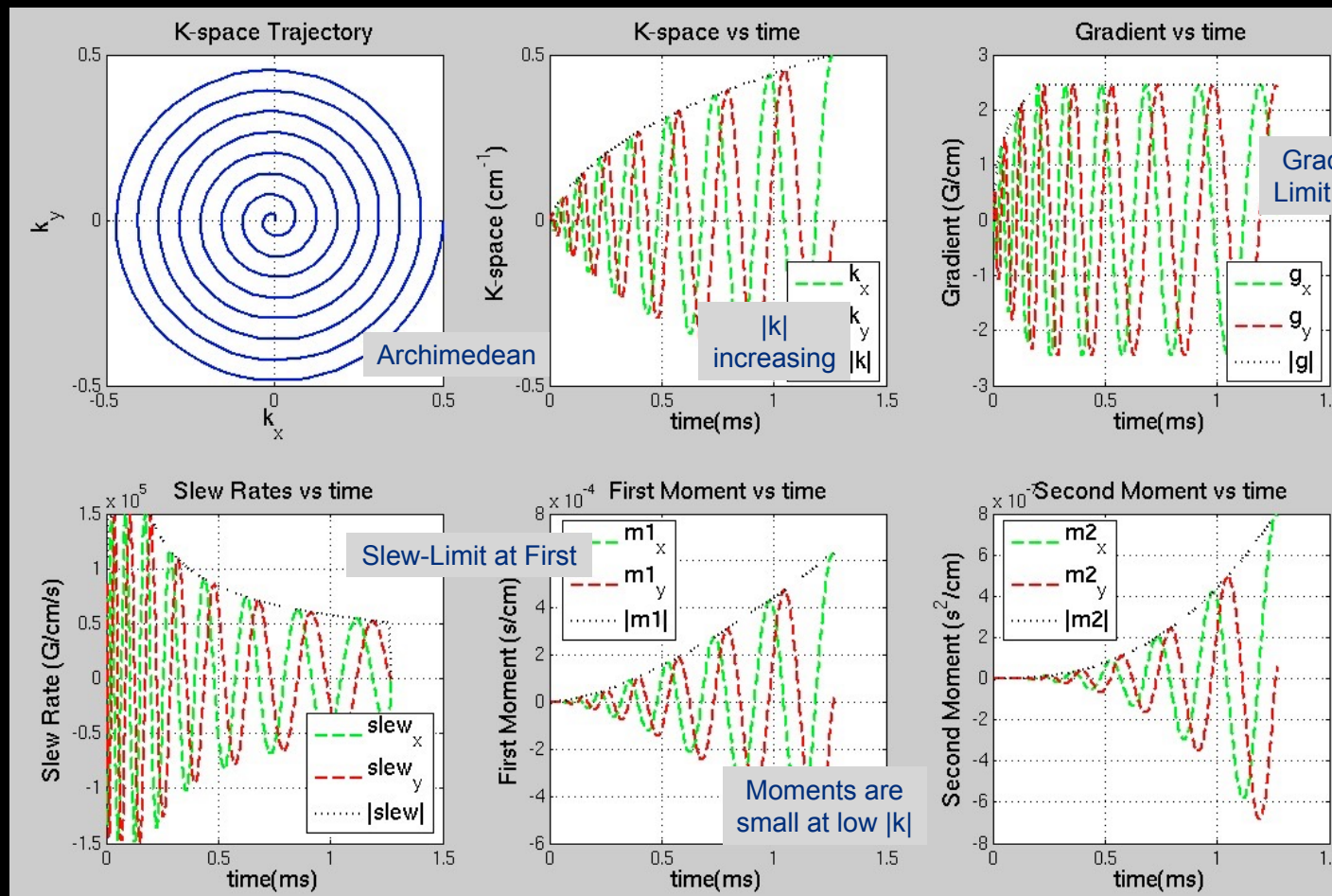
2. Solve numerically at each point

- Find all limits, use active limit
- Can include circuit model easily

(Meyer, King methods ~ 1995)



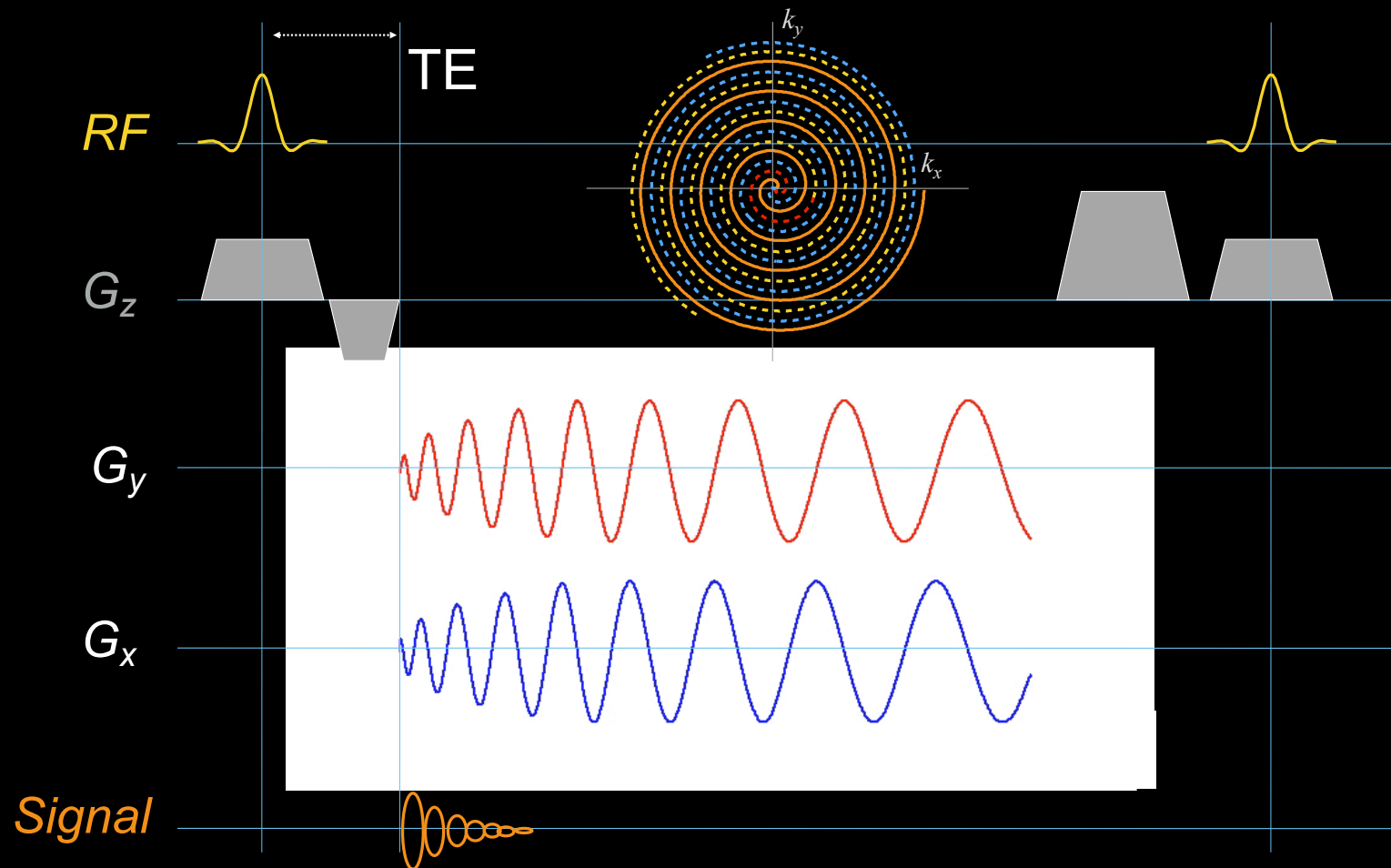
Example Spiral Waveforms



Spiral waveforms follow a chirp-like pattern, mostly limited by gradient amplitude or slew-rate



Spiral Imaging Sequence



The spiral sequence replaces the G_x and G_y readout with oscillating waveforms



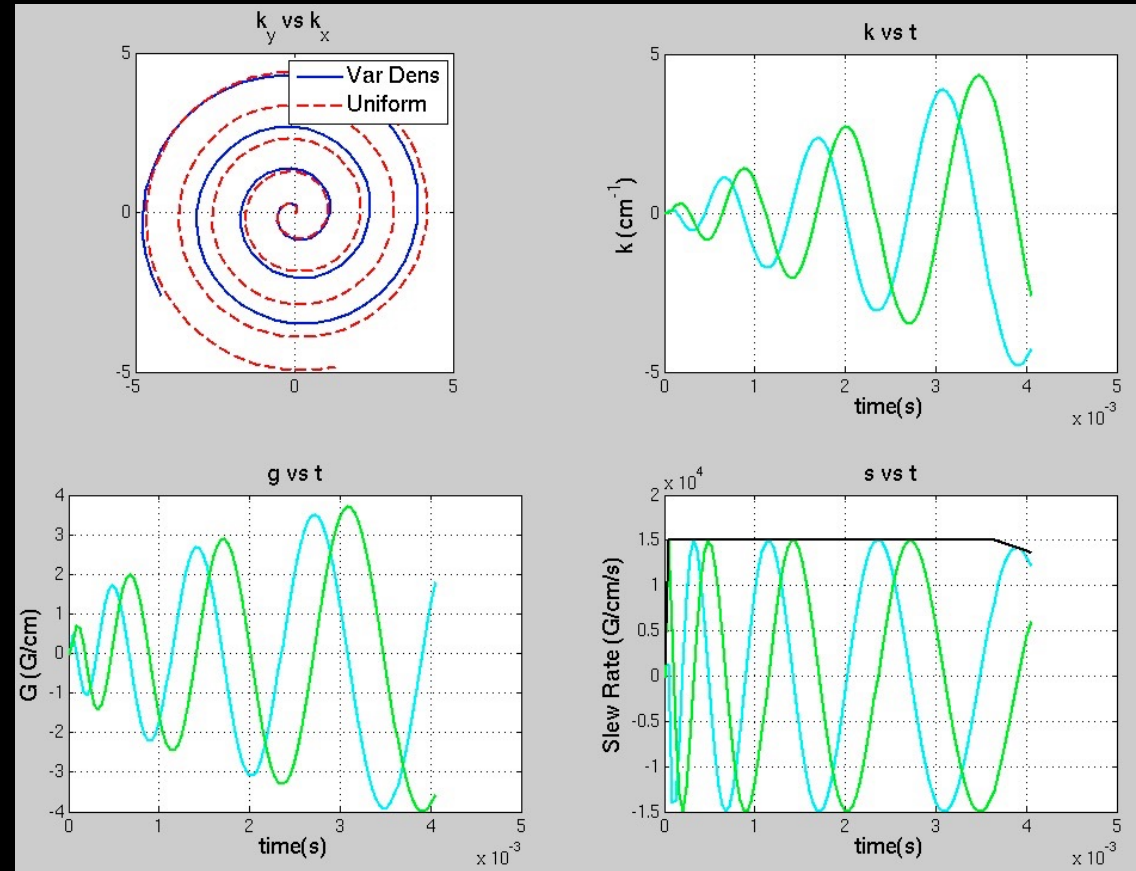
Question 1: Spiral Characteristics



Variable-Density Spiral

- Undersample outer k-space
- Vary spacing (1/FOV) as function of $|k/k_{max}|$ or θ
- Increase spacing along trajectory

$$k(t) = \frac{N\theta}{2\pi \text{FOV}(\theta)} e^{i\theta}$$



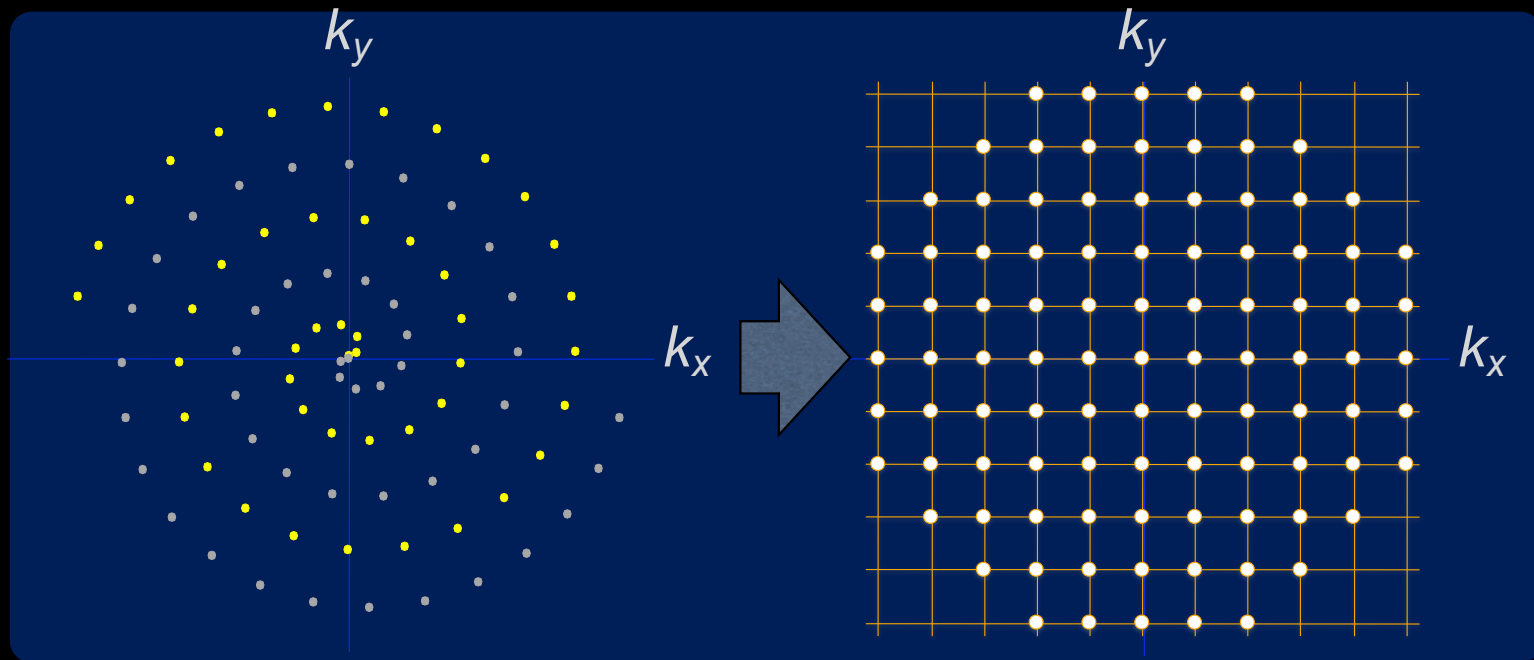
Variable-density spiral offers greater flexibility for faster sampling — vds.m



Non-Cartesian Sampling / Gridding

- Irregularly sampled data
- Resample to grid to perform DFT

gridmat.m

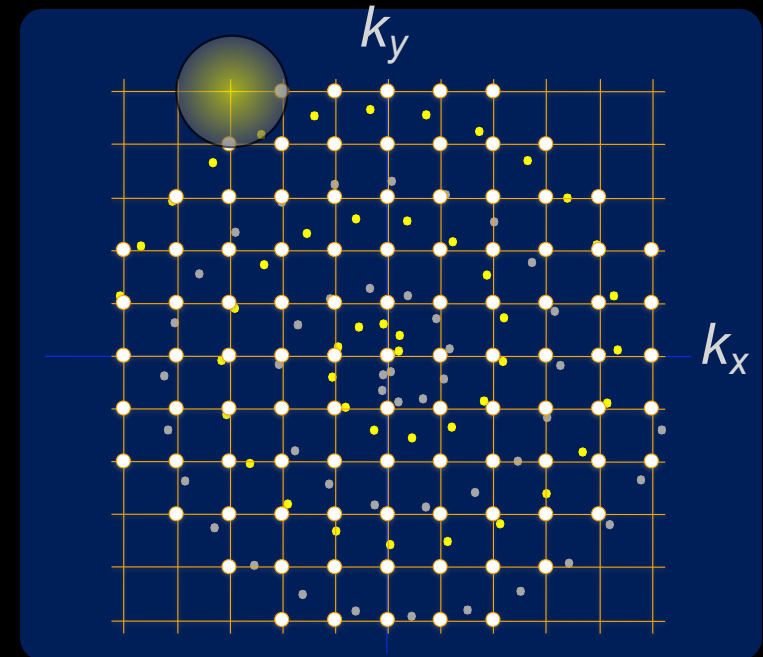


Gridding is a simple process to interpolate samples to a Cartesian grid to perform a DFT/FFT



Gridding Steps (Conceptual)

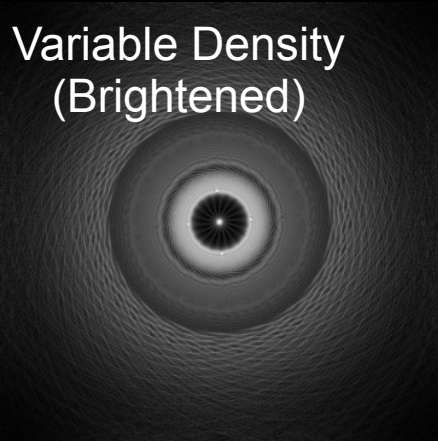
- Divide samples by density at location k
 - Want to have uniform signal if we grid 1's
- Convolve sampled k locations with kernel $c(k)$
- Resample at grid points
- FFT Reconstruction
- De-apodize to undo convolution side effects



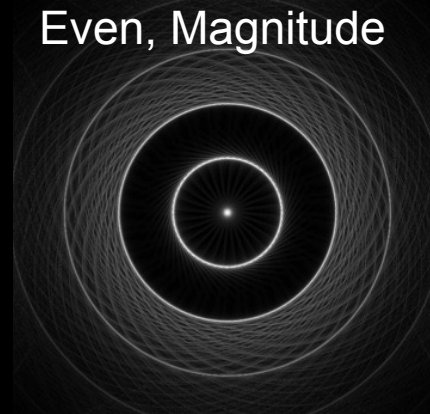
Spiral Point-Spread Functions

- “Swirl” artifacts from undersampling
- Rings of aliasing
- Odd/even changes character
- Variable Density: Less coherent

Variable Density
(Brightened)



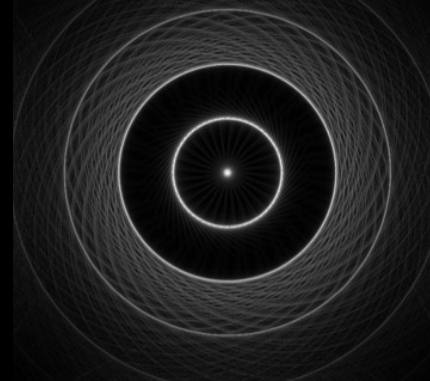
Even, Magnitude



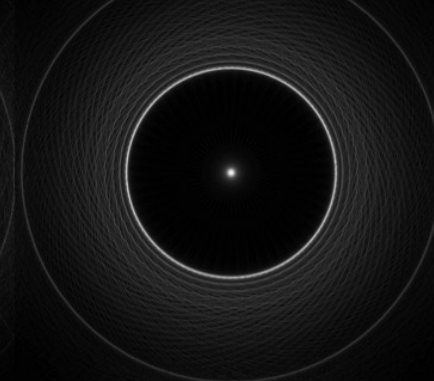
Odd, Magnitude



Even, Real Part

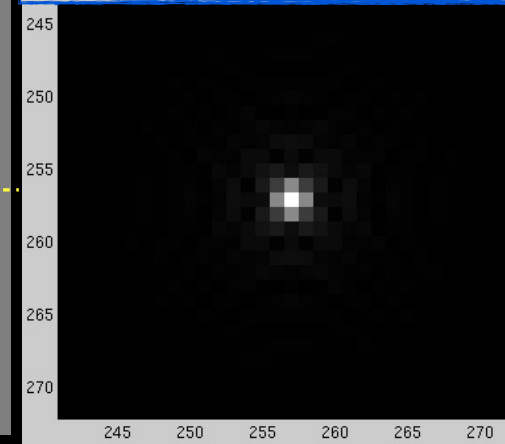
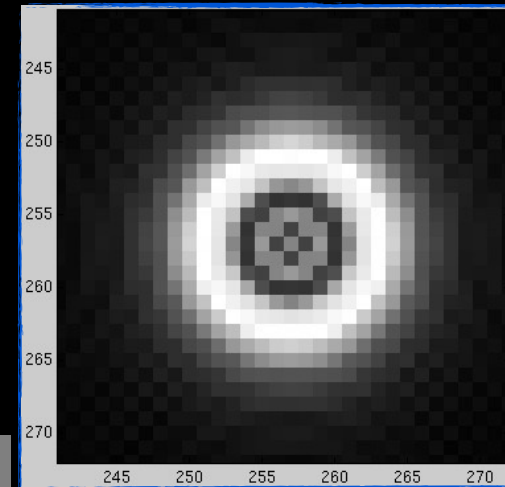
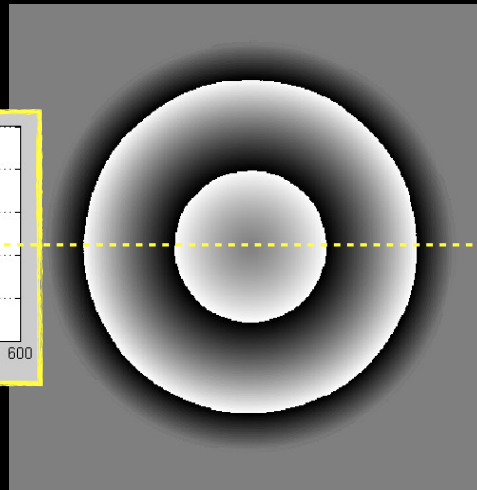
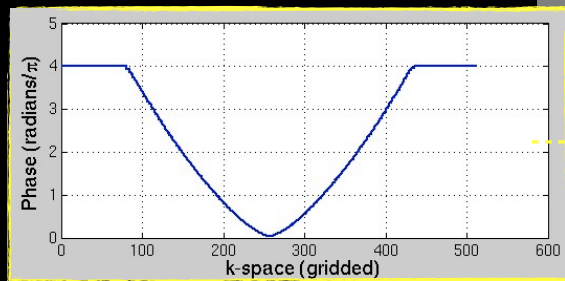


Odd, Real Part



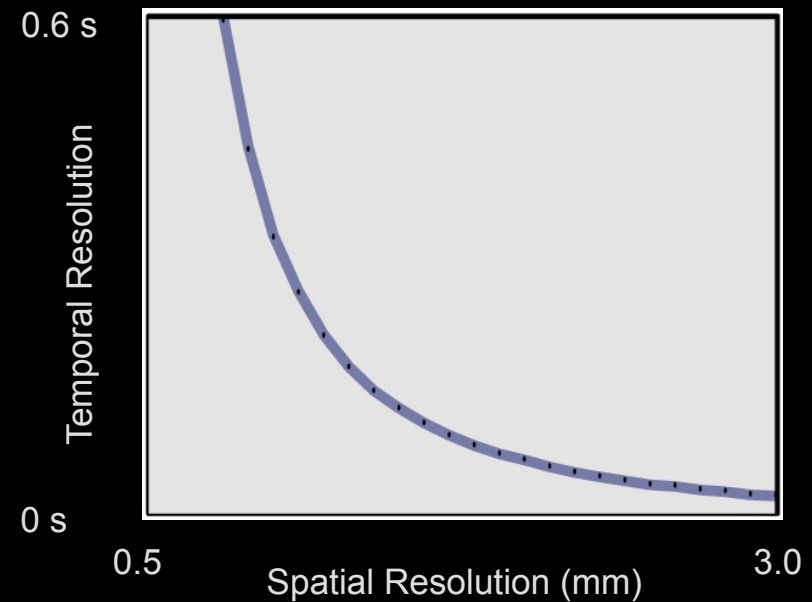
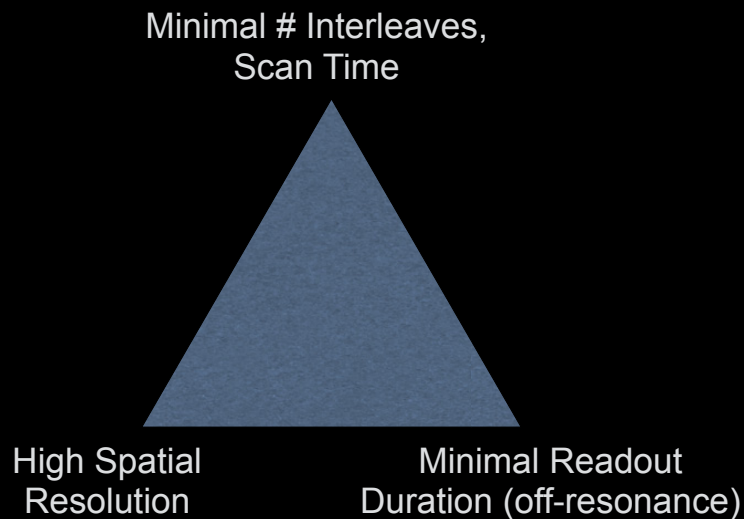
Off-Resonance Sensitivity

- Uniform-density - $\phi \sim A|k|^2$
- Spiral usually longer than radial
- PSF broadening
- Off-resonance correction in recon



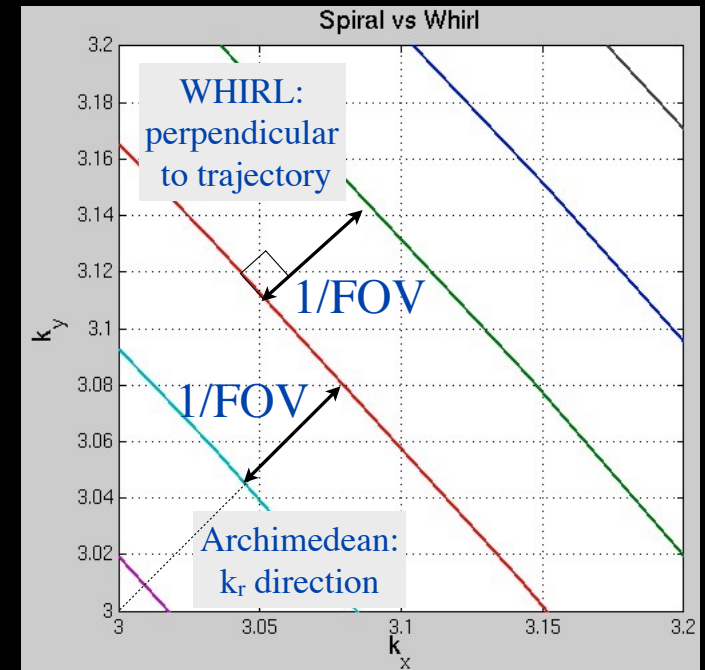
Spiral Design Trade-offs

- 1. Choose tolerable readout duration
- 2. Trade scan time (#interleaves) for spatial resolution



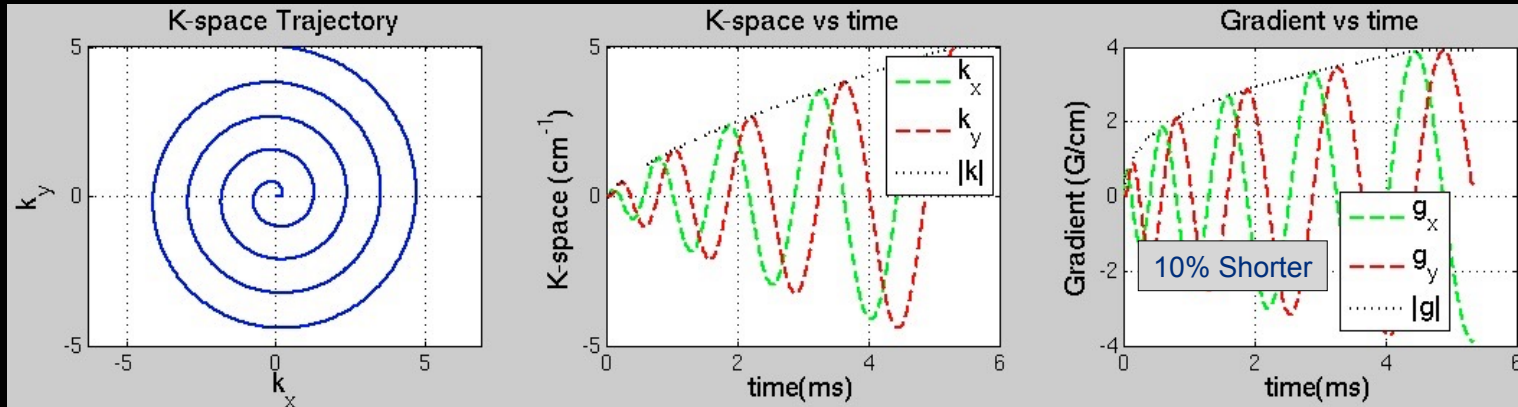
TWIRL / WHIRL

- Faster start, with radial segment
- TWIRL: *Jackson 1990*
 - Radial, then Archimedean spiral
- WHIRL: *Pipe 1999*
 - Non-archimedean spiral
 - constrained by trajectory spacing
 - *Faster spiral*, particularly for many interleaves
 - whirl.m on website

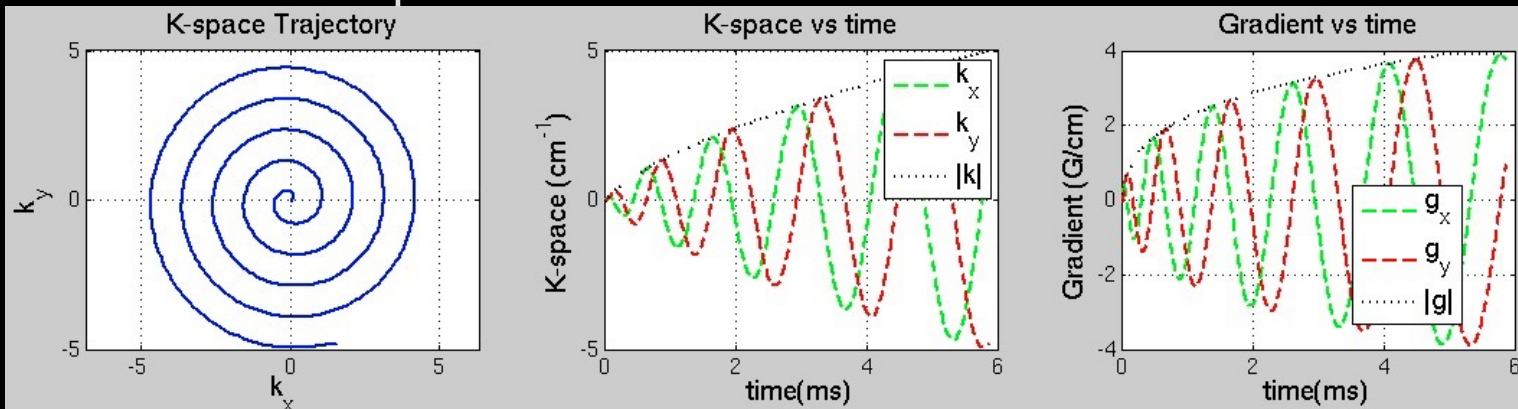


WHIRL vs Archimedean Spiral

WHIRL



Archimedean Spiral

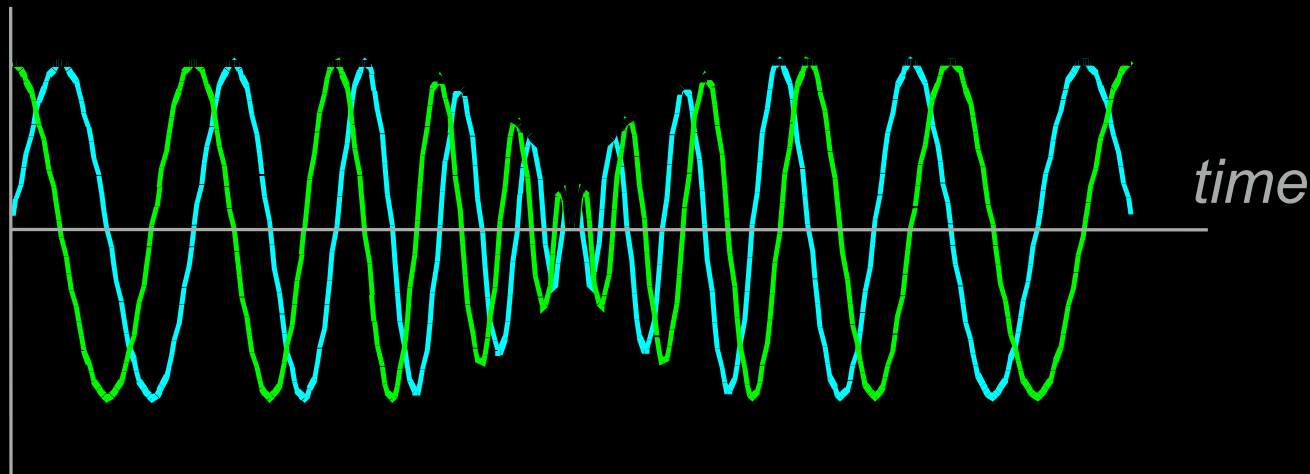
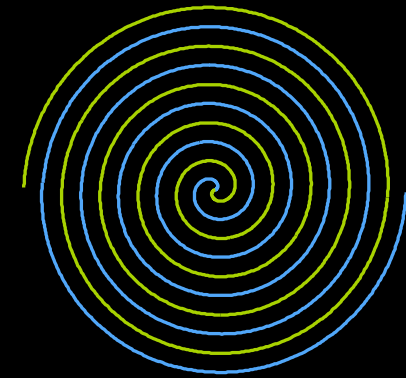


25 interleaves, 24cm FOV, 1mm resolution



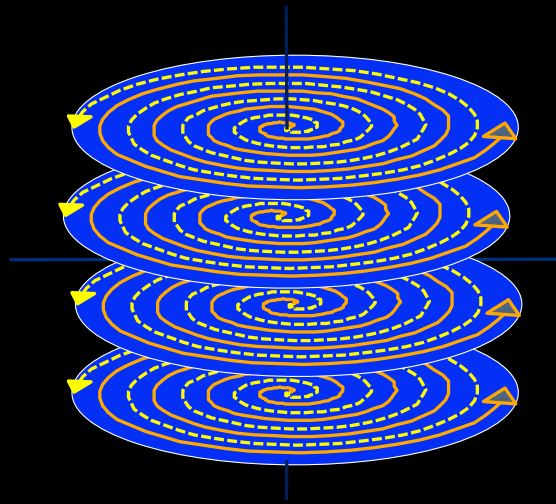
Spiral in - Spiral out

- Useful for delayed TE
- Perhaps also for spin echo
- **Simply add time-reversed gradient**

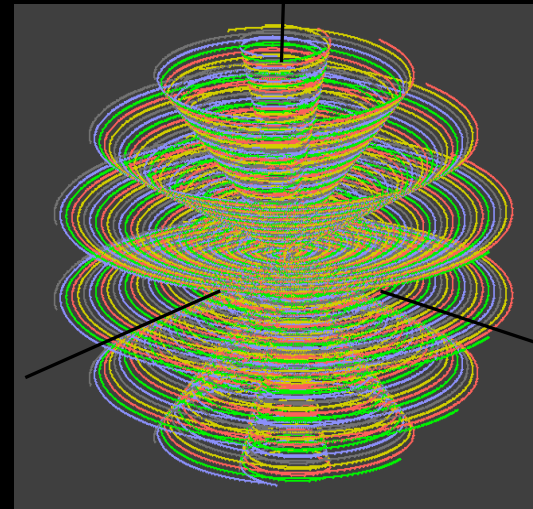


3D Methods: Spiral Stack, TPI, Cones

Stack of Spirals



Cones, Twisted-Projections



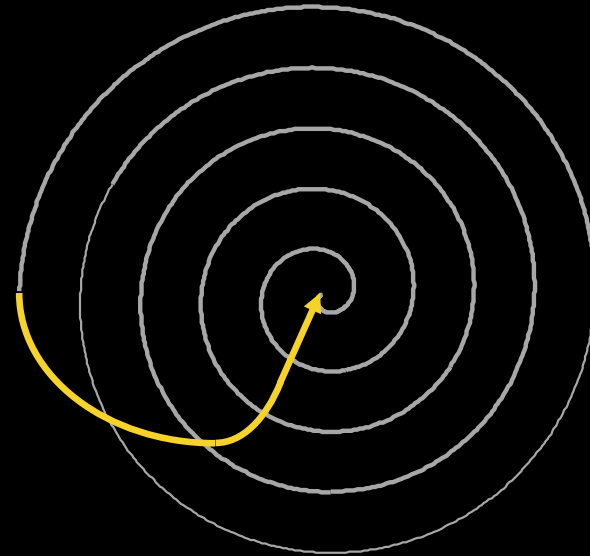
- *Many variations (spherical stack of spirals)*
- *Density-compensated cones, TPI*
- *3D design algorithms get very complicated*

Irrarrazabal 1995, Boada 1997, Gurney 2006



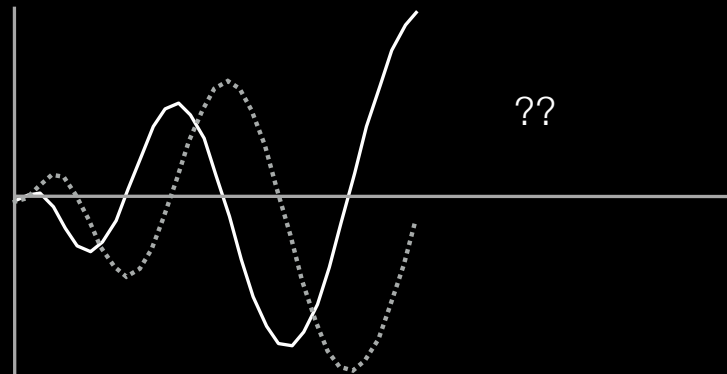
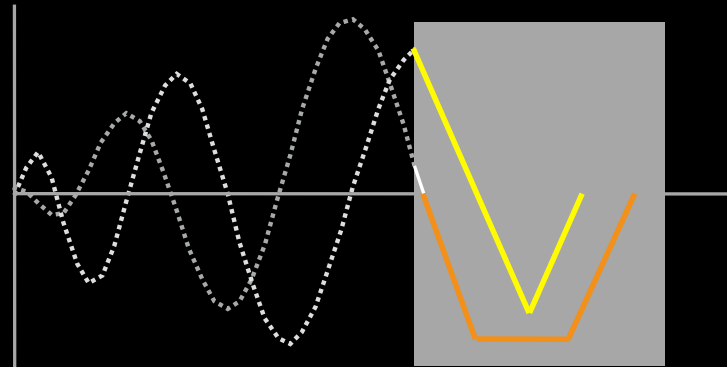
Rewinders and Prewinders

- “Preparatory gradients”
 - Spiral rewinders
 - Phase-encode/rewind
 - spoilers
- Consider 3D rotation again
- Arbitrary path
- Speed always helps efficiency



Approaches to “Rewinder” Design

- Goal: Bring G and k to zero quickly
- Just use trapezoids
- Problem:
 - How much “power” to use on each axis
 - Finite segment method solutions (Meyer 2001)
 - Convex Optimization (more later!)

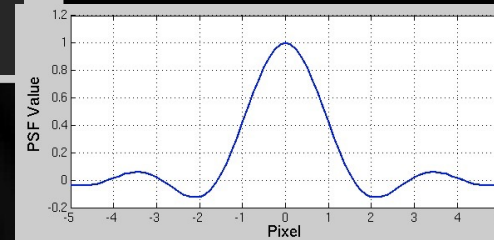
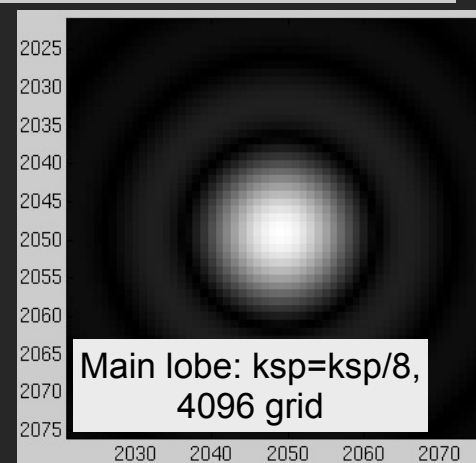
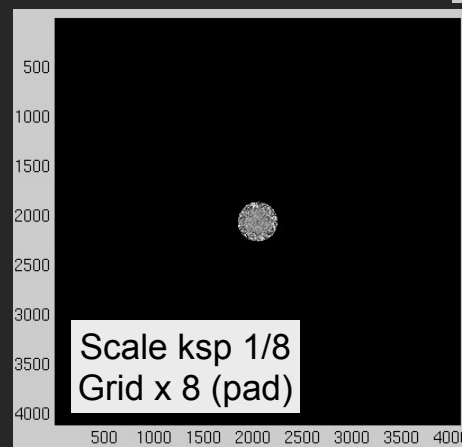
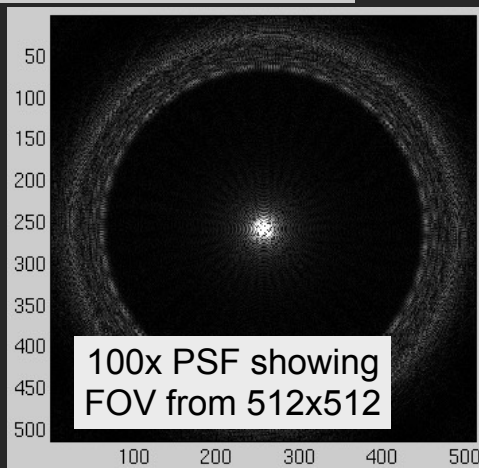
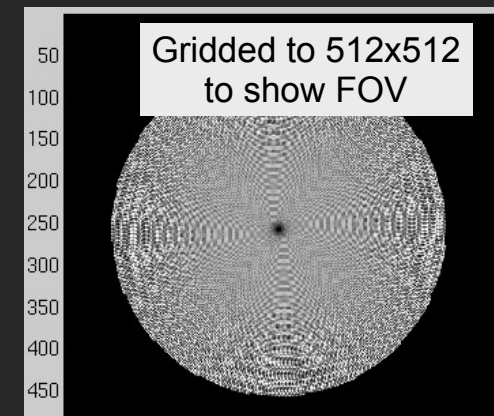
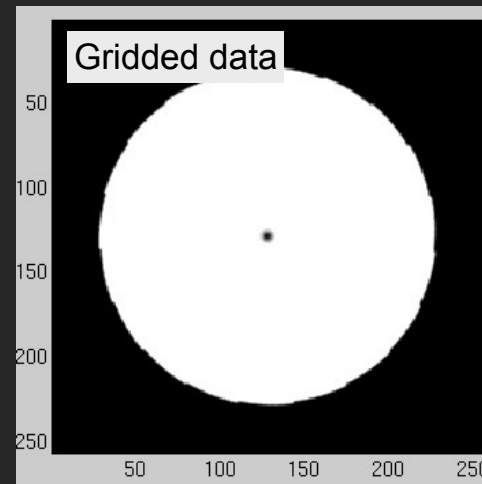
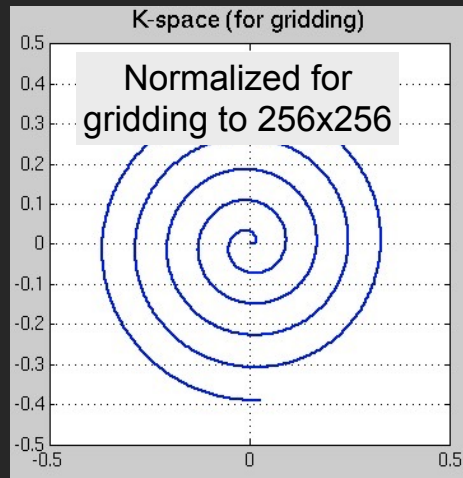
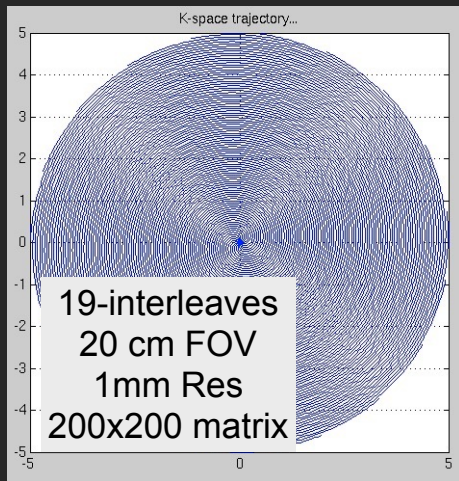


Spirals and Gradient/RF Delays

- Delays can have a few effects:
 - Mis-mapping of center of k-space, causes a low-frequency “cloud”
 - In outer k-space, delays cause rotation of the image
 - Other effects can be similar to radial
- Often measure actual gradient waveforms

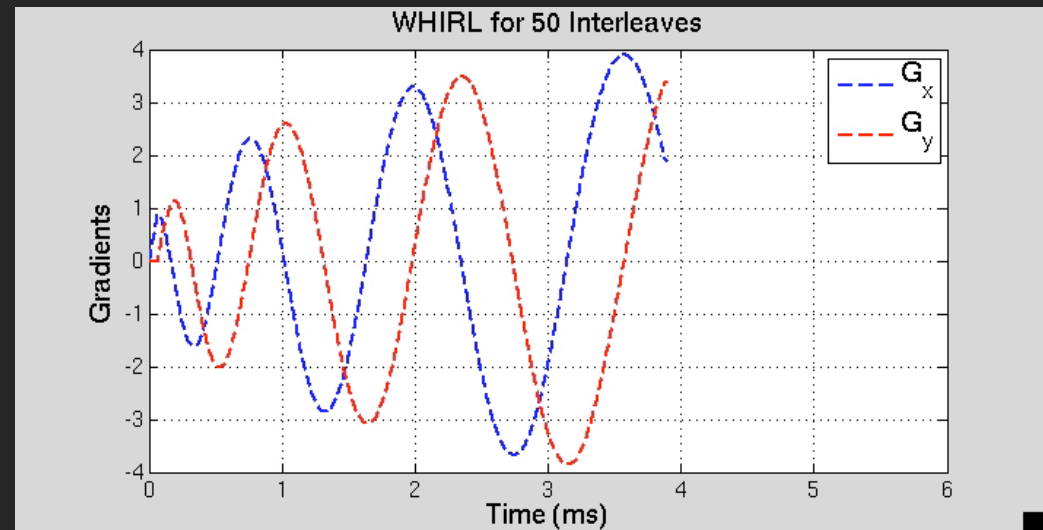


Gridding Example (FOV / Res)



Spiral Design Example

- Desire maximum 2π cycles between fat/water at 3T, and 1 mm resolution over 30cm FOV
- What is the maximum spiral duration?
- What is the minimum number of interleaves to achieve this?
 - $1/440\text{Hz} = 2.2\text{ ms}$
 - Loop:
 - Design `whirl(N,resolution,FOV)`
 - If duration of $g < 2.2\text{ms}$, decrease N, otherwise increase N (using Bisection)



Question 2: Spiral Design



Trajectory Comparison?

Trajectory Features	Cartesian	Radial Out	Projection	Spiral	EPI
Needs Gridding		X	X	X	Maybe
Flexible #shots				X	X
Low First Moments		X		X	
Off-Resonance Causes	Displacement	Blur	Blur+Ringing	Blur	Large Displacement
Sensitivity to Delays	Minimal	Loss	Loss/Artifact	Rotation	Ghosts



Spiral Summary

- Flexible duration/coverage trade-off
 - Center-out: $TE \sim 0$, Low first-moments
- Archimedean, TWIRL, WHIRL, variable-density
- PSF with circular aliasing, swirl-artifact outside
- Off-resonance sensitivity, correct in reconstruction
- Variations: Spiral in/out, 3D TPI, 3D Cones
- Rewinder design



A photograph of a large, multi-story building with a red-tiled roof and arched windows, likely a Stanford University building. The building is set against a dark, overcast sky. In the foreground, there is a green lawn and a paved path. The text is overlaid on the image.

Rad229 – MRI Signals and Sequences

Daniel Ennis & Brian Hargreaves

dbe@stanford.edu –or– bah@stanford.edu