Rad229 – MRI Signals and Sequences

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Lecture-15A — Magnetization Preparation II - Diffusion

Diffusion, Spins, and MRI Signals

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Learning Objectives

- Understand that the MRI signal is sensitive to diffusion.
- Appreciate the relationship between random walks, diffusion, and MRI signal attenuation.
- Describe the dependence of the MRI signal on the diffusion coefficient.
- Recall the spatial and temporal scale of diffusion in biological tissues.

Diffusion – Bloch Equations

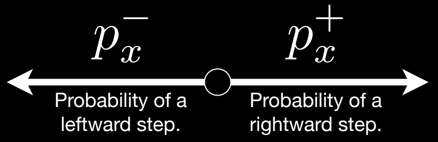
$$\frac{d\vec{\mathrm{M}}}{dt} = \underbrace{\vec{\mathrm{M}} \times \gamma\vec{\mathrm{B}}}_{\text{Precession}} - \underbrace{\frac{M_x\hat{\mathrm{i}} + M_y\hat{\mathrm{j}}}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_z - M_0)\,\hat{\mathrm{j}}}{T_1}}_{\text{Longitudinal Relaxation}} + \underbrace{\mathbf{D}\nabla^2\vec{\mathrm{M}}}_{\text{Diffusion}}$$

- Precession
  - Magnitude of $\overrightarrow{M}$ unchanged
  - Phase (rotation) of $\overrightarrow{M}$ changes with $\overrightarrow{B}$
- Relaxation
  - T_1 change are slow O(100ms)
  - T_2 changes are fast O(10ms)
  - Magnitude of M can be ~0 if T2<<T1
- Diffusion
  - Spins are thermodynamically driven to exchange positions.
    - This exchange is ***irreversible*** and can lead ***signal attenuation***.

Application? Changes in proton self-diffusion are an early indicator of cellular disruption.

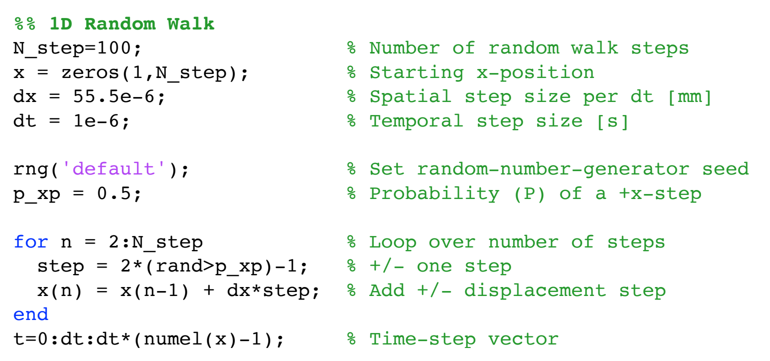
Diffusion – 1D Random Walk

$\Delta x \equiv \text{step length} \qquad \Delta t \equiv \text{time step}$

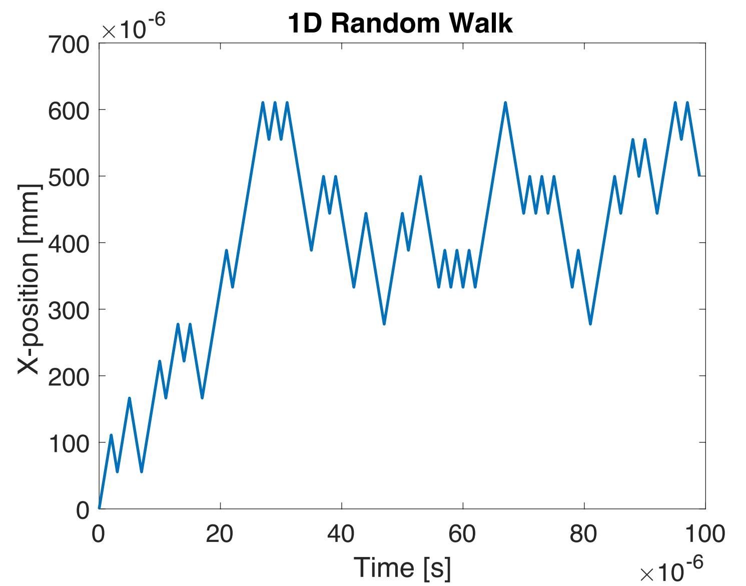


$$p_x^+ = p_x^- = \frac{1}{2}$$

¡Not a Gaussian!



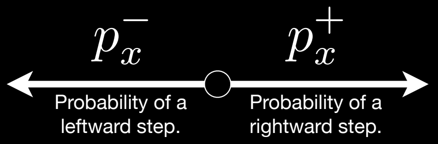
```
%% 1D Random Walk
N_step=100;                 % Number of random walk steps
x = zeros(1,N_step);        % Starting x-position
dx = 55.5e-6;               % Spatial step size per dt [mm]
dt = 1e-6;                  % Temporal step size [s]

rng('default');             % Set random-number-generator seed
p_xp = 0.5;                 % Probability (P) of a +x-step

for n = 2:N_step            % Loop over number of steps
  step = 2*(rand>p_xp)-1;   % +/- one step
  x(n) = x(n-1) + dx*step;  % Add +/- displacement step
end
t=0:dt:dt*(numel(x)-1);     % Time-step vector
```



Rad229_Random_Walk.m with 55.5μm steps for 100μs.

Diffusion – 1D Random Walk

$$\Delta x \equiv \text{step length} \qquad \Delta t \equiv \text{time step}$$



Probability of a leftward step.

Probability of a rightward step.

$$p_x^+ = p_x^- = \frac{1}{2}$$

¡Not a Gaussian!

$$P(x,t)$$

Probability distribution for a spin.

$$\downarrow$$

Some math...

$$\frac{\partial P(x,t)}{\partial t} = D\frac{\partial^2 P(x,t)}{\partial x^2}$$

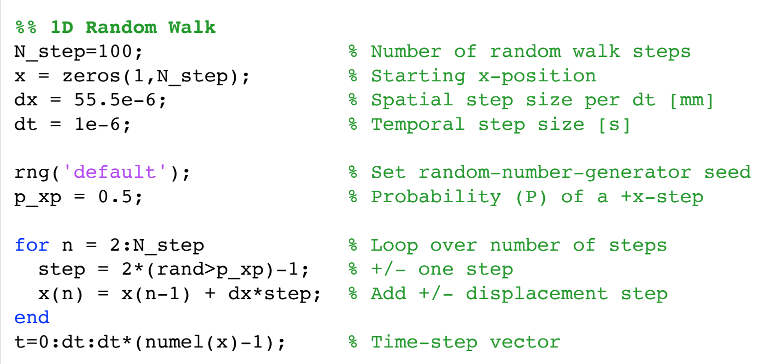
1-D Diffusion Eqn.

$$\downarrow$$

Solution from: x=0 @ t=0.

$$P(x,t) = \frac{1}{\sqrt{4\pi D t}} e^{-\frac{x^2}{4Dt}}$$

¡A Gaussian!



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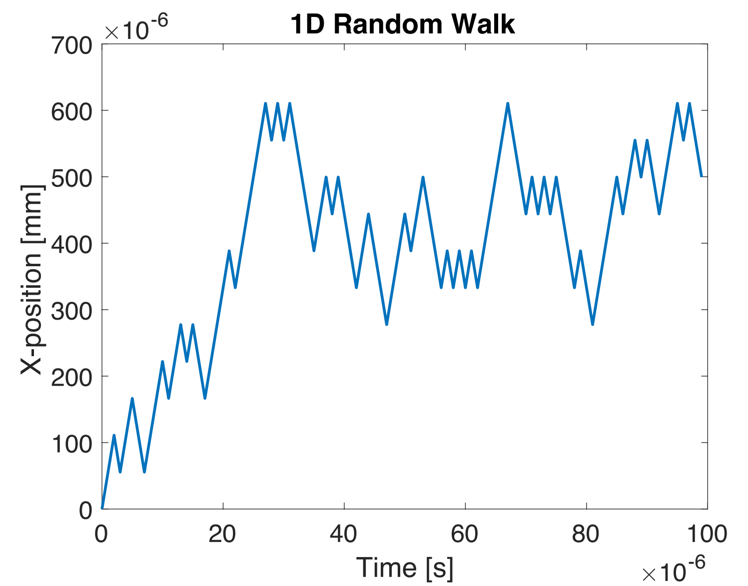
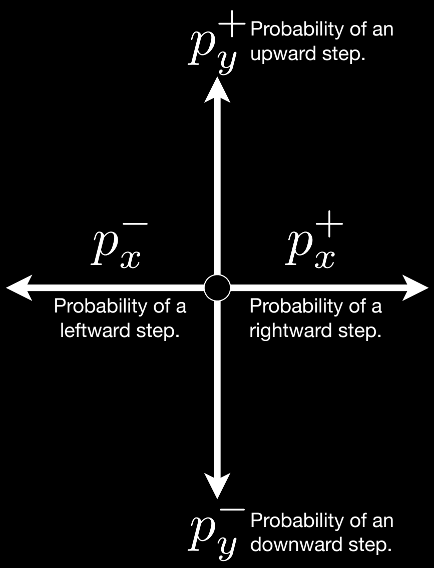


Rad229_Random_Walk.m with 55.5µm steps for 100µs.

Diffusion – 2D Random Walk


p_y^+ Probability of an upward step.

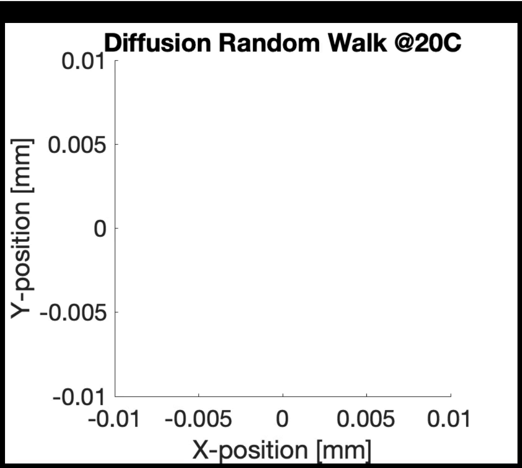
p_x^- Probability of a leftward step.

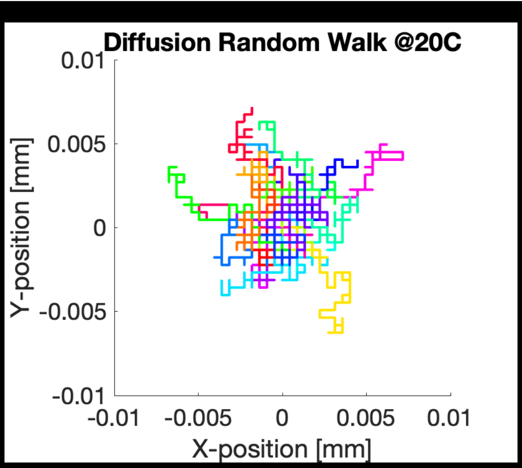
p_x^+ Probability of a rightward step.

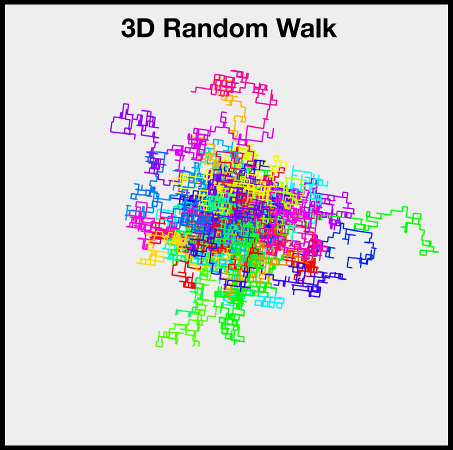
p_y^- Probability of an downward step.

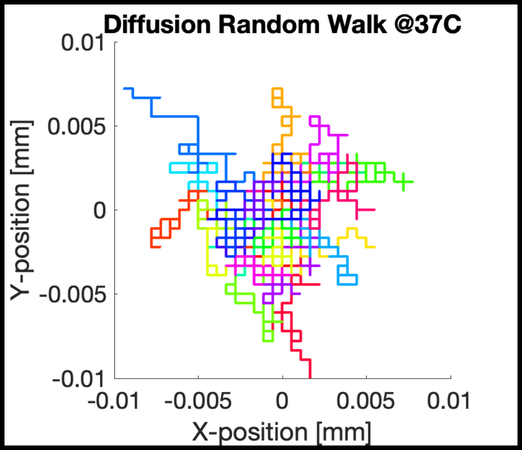
$$p_x^+ = p_x^- = p_y^+ = p_y^- = \frac{1}{4}$$

$$P(\vec{r},t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{\vec{r}^{\,2}}{4Dt}\right)$$


Diffusion Random Walk @20C


Diffusion Random Walk @20C

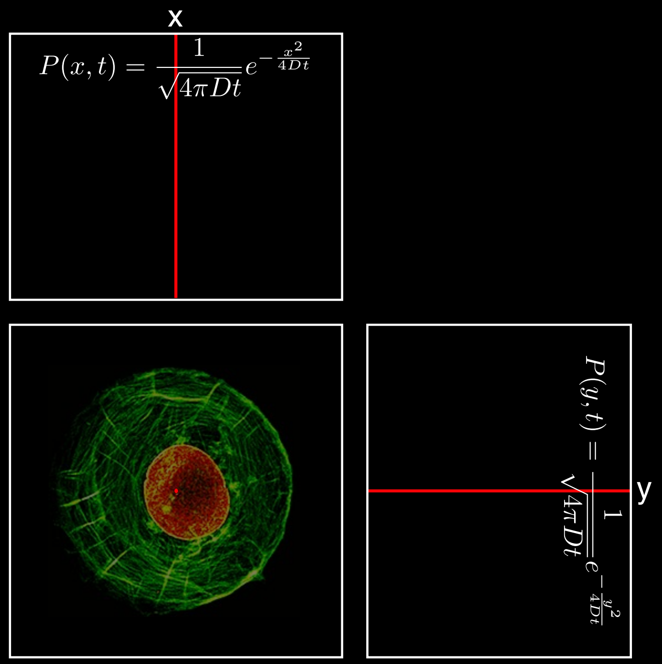

3D Random Walk


Diffusion Random Walk @37C

Rad229_Random_Walk.m with 100μs steps for 10ms.

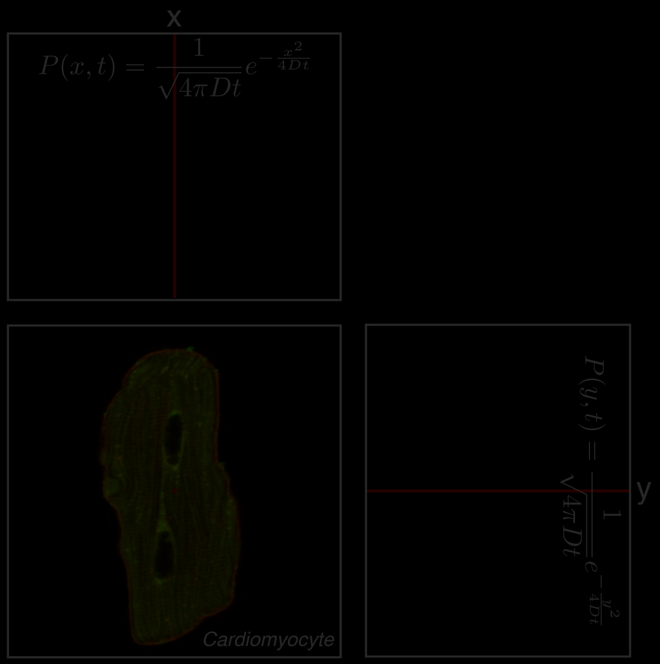
Diffusion – Free vs. Restricted

Free (Isotropic) Diffusion



$$P(x,t) = P(y,t)$$

Restricted (Anisotropic) Diffusion

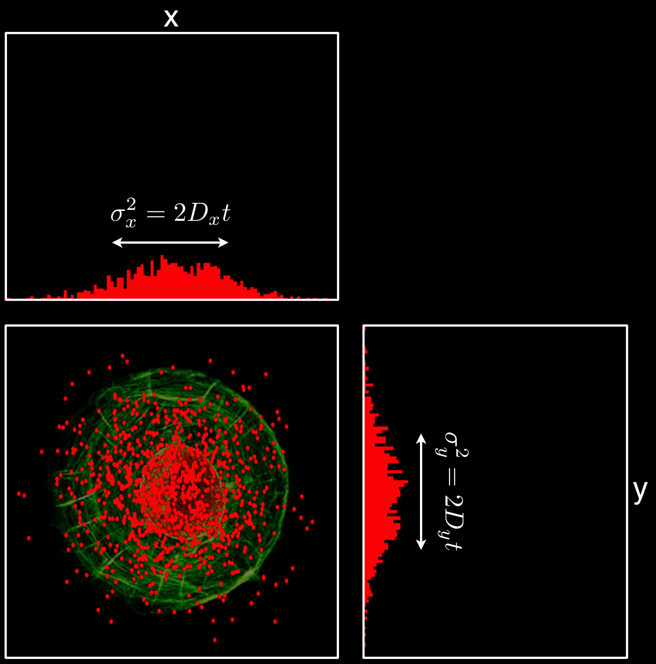


$$P(x,t) \neq P(y,t)$$

Diffusion in biological tissues can be free (isotropic) or restricted (anisotropic).

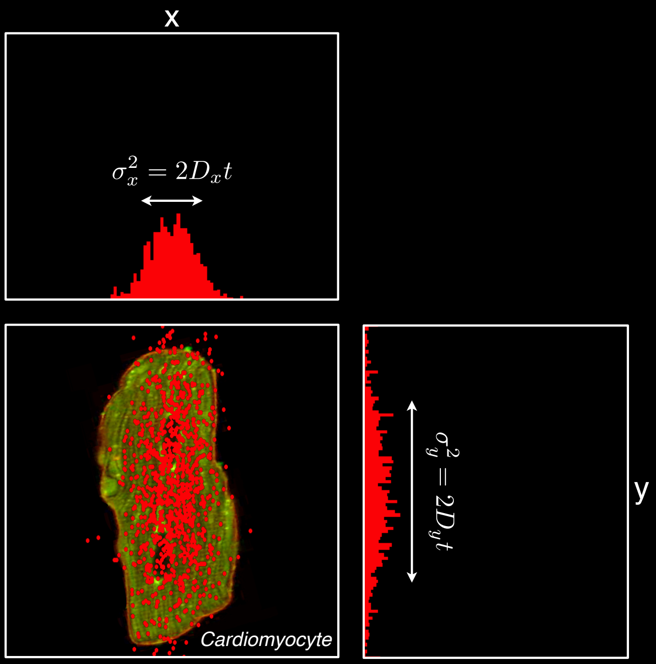
Diffusion – Free vs. Restricted

Free Diffusion



$$D_x = D_y$$

Restricted Diffusion



$$D_x < D_y$$

The variance of spin position is directly related to the diffusion coefficient.

Diffusion and Gradients

$$\underset{\substack{\uparrow\\ \text{Phase from a}\\ \text{Gradient}}}{\phi(t)} = \gamma \int_0^t \underset{\substack{\uparrow\\ \text{Applied}\\ \text{Gradient}}}{\vec{G}(\tau)} \cdot \underset{\substack{\uparrow\\ \text{Spin History}\\ \text{(Random Walk!)}}}{\vec{r}(\tau)} d\tau$$

Gradients impart a position-dependent phase on the spin.

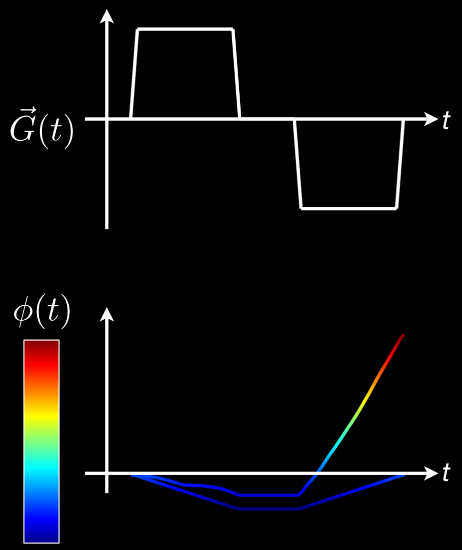
Diffusion and Gradients

$$\phi(t) = \gamma \int_0^t \vec{G}(\tau) \cdot \vec{r}(\tau) d\tau$$

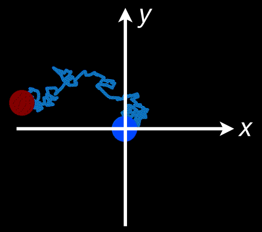
↑ Phase from a Gradient

↑ Applied Gradient

↑ Spin History (Random Walk!)



Stationary vs. Diffusing Spin



Stationary spins don’t accumulate phase, but diffusing spins have non-zero phase.

Movies courtesy of Kévin Moulin

Diffusion and Gradients

$$\phi(t) = \gamma \int_0^t \vec{G}(\tau) \cdot \vec{r}(\tau) d\tau$$

Phase from a Gradient

Applied Gradient

Spin History (Random Walk!)

What is the b-value?

Diffusion Weighted Signal

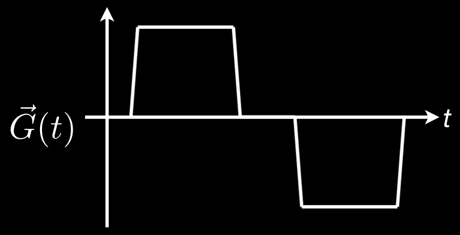
$$S = S_0 e^{-\langle \phi^2 \rangle} = S_0 e^{-bD}$$

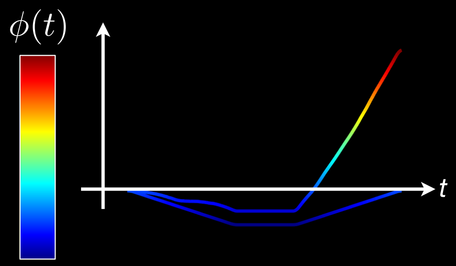
Signal without Diffusion Effects

Variance of the Phase Distribution

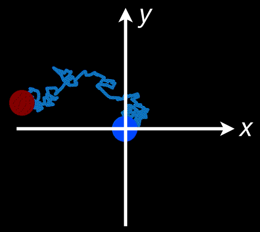
b-value

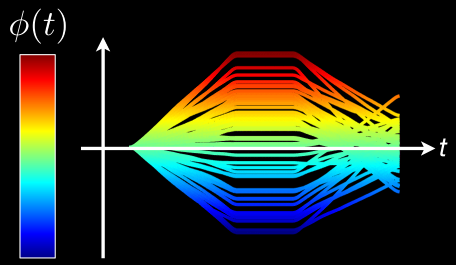
Diffusion Coefficient





Stationary vs. Diffusing Spin





Movies courtesy of Kévin Moulin

An ensemble of diffusing spins will accumulate different amounts of phase.

The larger the phase dispersion, The higher the diffusion coefficient.

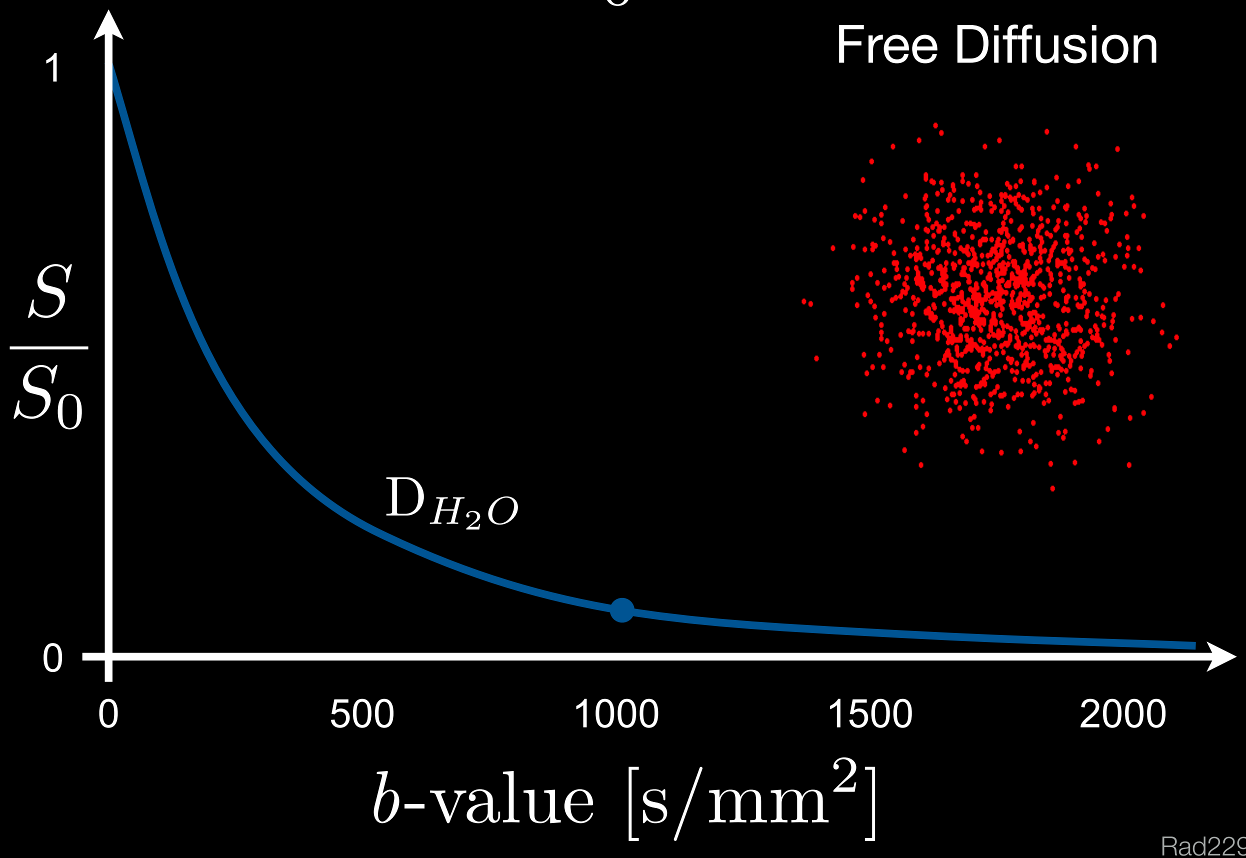
Longer and stronger diffusion gradients contribute more sensitivity to diffusion.

Diffusion – b-value [s/mm^2]

- $D_{H_2O} \approx 3.0x10^{-3}$ mm^2/s at 37°C
  - – After 50 ms
    - ~32% of spins ≥17μm in 50 ms
    - ~5% of spins reach >34 μm!
  - – For b-value = 1000 s/mm^2
    - Water signal attenuated 95%!

Example from LeBihan PLoS 2015

$$S = S_0 e^{-bD}$$



The signal attenuation depends on the diffusion coefficient and the b-value.

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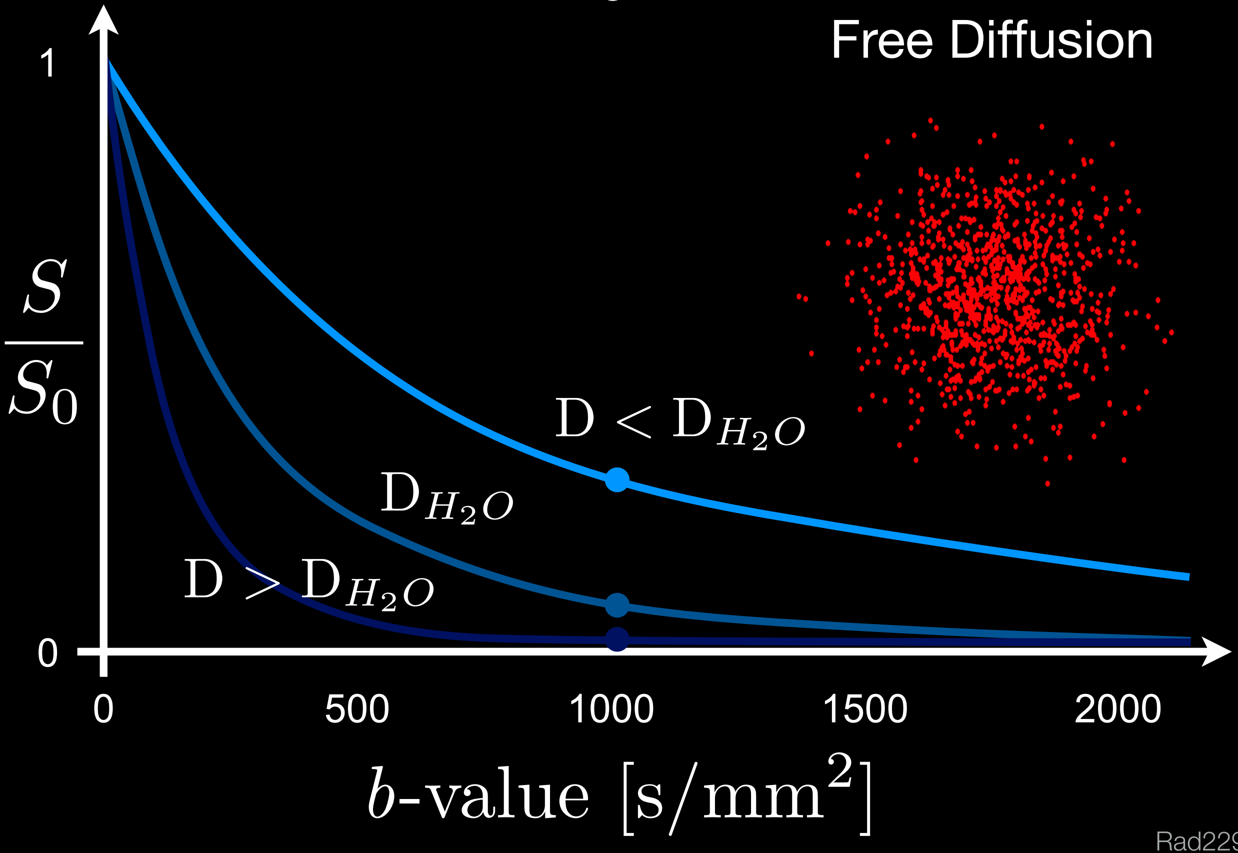
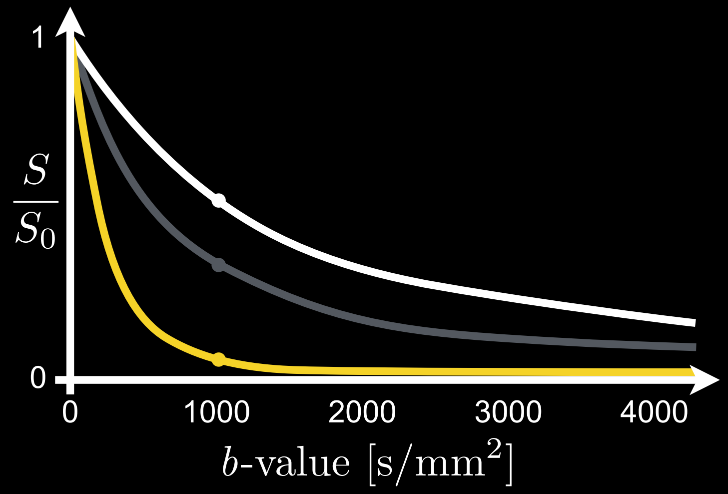


High (fast) diffusion coefficients contribute more signal de-phasing (signal drop).

Diffusion – b-value [s/mm^2]

- 1000 s/mm^2 is typical. Why?
  - For D=700 mm^2/s, S/S_0~50%.



Tissue	Diffusion Coefficient [10^{-6} mm^2/s]
White matter	670-800
Cortical grey matter	800-1000
Deep grey matter	700-850
CSF	3000-3400

https://radiopaedia.org/articles/apparent-diffusion-coefficient-1?

How do we acquire diffusion weighted images?

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