

# RAD229: Midterm Exam – 2017/2018 – With Solutions

October 30, 2017 - 80 Minutes

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

## General Instructions:

1. Write your name legibly on this page.
2. You may use notes including lectures, homework, solutions or Matlab script text on the course website.
3. You may **not** use Matlab or the Internet. (You may use a laptop **only** to read the above.)
4. Please answer questions on the exam, and show your final answer clearly.
5. Show your reasoning and work, as this will often earn you partial points.
6. You may request more paper if needed.

**Please do not proceed to the next page until the exam begins.**

The following formula from the class notes may be useful.

$$\begin{bmatrix} F_n^+ \\ F_n^- \\ Z_n \end{bmatrix}' = \begin{bmatrix} \cos^2(\alpha/2) & e^{2i\phi} \sin^2(\alpha/2) & -ie^{i\phi} \sin \alpha \\ e^{-2i\phi} \sin^2(\alpha/2) & \cos^2(\alpha/2) & ie^{-i\phi} \sin \alpha \\ -i/2e^{-i\phi} \sin \alpha & i/2e^{i\phi} \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} F_n^+ \\ F_n^- \\ Z_n \end{bmatrix}$$

## Short Answer Questions (20% of Points)

1. [5pts] Briefly state why we might use each of the following sequences:

a) RF-spoiled gradient echo

*Pure T1-weighted imaging in a short-TR sequence, often used for 3D contrast-enhanced MR. Speed is an advantage.*

b) Spin-echo train

*Robustness to off-resonance, and an echo train for PD or T2 weighting in an efficient acquisition.*

c) Inversion-Recovery Preparation

*T1 contrast including for suppression of fat (STIR) or fluids (FLAIR), both robust to off-resonance effects.*

2. [5pts] Consider two gradient-spoiled sequences with identical TR, TE and flip angle. In (i) the RF flip angle alternates  $\alpha_x, \alpha_y, \alpha_x, \alpha_y, \alpha_x, \dots$ . In (ii) the flip angle is constant  $\alpha_x, \alpha_x, \alpha_x, \alpha_x, \dots$ . Explain if the signals are the same or different, assuming the received signal is demodulated with the same phase as the current RF pulse.

*Yes. This is just a phase-cycling, so the bSSFP signal just shifts within the voxel, so the signals are identical.*

3. [10pts] A student is trying to measure SNR by assuming that a region of the image with 200pixels is homogeneous. In the *magnitude* image with a single receive coil, she measures a mean of 120, and standard deviation of 10.

a) Explain if she can reliably calculate SNR, and if so, what is the SNR?

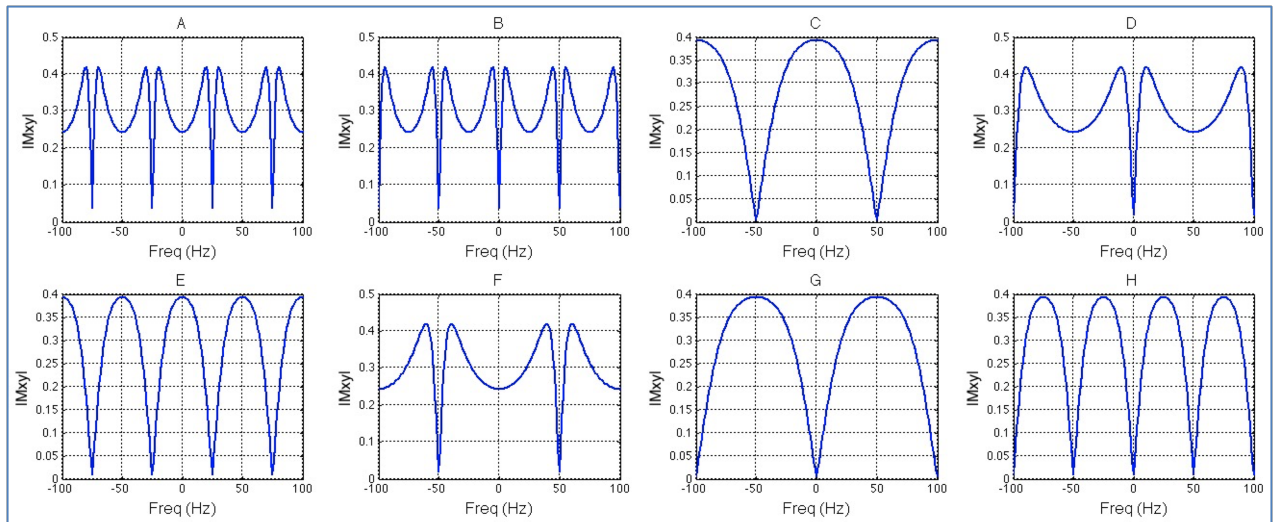
*Yes, since the SNR is more than about 4, Rician noise looks like Gaussian noise, so the SNR is simply  $120/10 = 12$ .*

b) If the *actual* signal in the region is not homogeneous, but has a Gaussian distribution with mean of 120 and  $\sigma^2 = 44$ , what would you expect her to measure as the SNR in (a)?

*The noise variance of  $10 \times 10 = 100$  adds, leading to an overall variance of 144, so the measure standard deviation is about 12 and measured SNR will be 10, and lower.*

## Medium Answer Questions (35% of Points)

4. [10pts] For  $T_2/T_1=0.7$ , **Explain** which of the following plots of signal magnitude vs frequency at  $TE=TR/2$  corresponds to each balanced SSFP sequence in (a) and (b) and **why**:



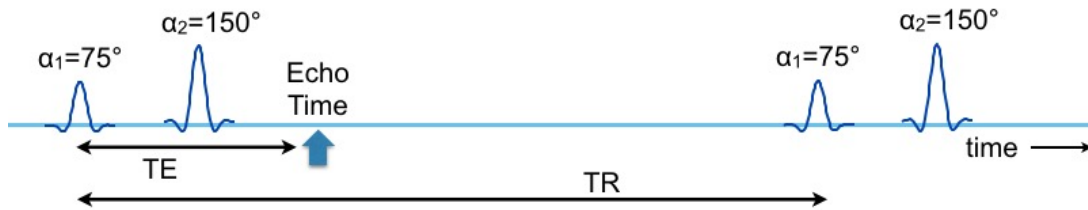
- a)  $30^\circ$  flip angle,  $TR=10\text{ms}$ , alternating sign of RF pulse.

*Sqrt(0.7) is about 0.85, so the ellipse is close to a circle.  $30^\circ$  will be above the equator, so there will be a dip in the profile at the passband center. The bands are  $1/TR$  or  $100\text{Hz}$  apart, and there is signal on-resonance since the RF sign is alternating.*

- b)  $100^\circ$  flip angle,  $TR = 20\text{ms}$ , constant sign of RF pulse.

*Sqrt(0.7) is about 0.85, so the ellipse is close to a circle.  $100^\circ$  will be below the equator, so there will be no dip in the profile at the passband center. The bands are  $1/TR$  or  $50\text{Hz}$  apart, and there is a null on-resonance since the RF sign is constant.*

5. [10pts] For a spin-echo sequence below, unfortunately the TR is too short to ignore transverse magnetization at the end of TR, and the flip angles are in error due to  $B_1$  variations. The RF rotations are about the  $m_y$  axis. You can ignore off-resonance for this problem.



- a) Write a matrix expression for the magnetization at the echo time shown, in terms of magnetization at the start of the sequence (which is a fraction of equilibrium magnetization  $m_0$ ). The expression can simply use rotation matrices ( $R_x(\alpha)$ ,  $R_y(\alpha)$ ,  $R_z(\alpha)$ ) and relaxation (you may define intermediate matrix functions or expressions as needed to simplify notation).

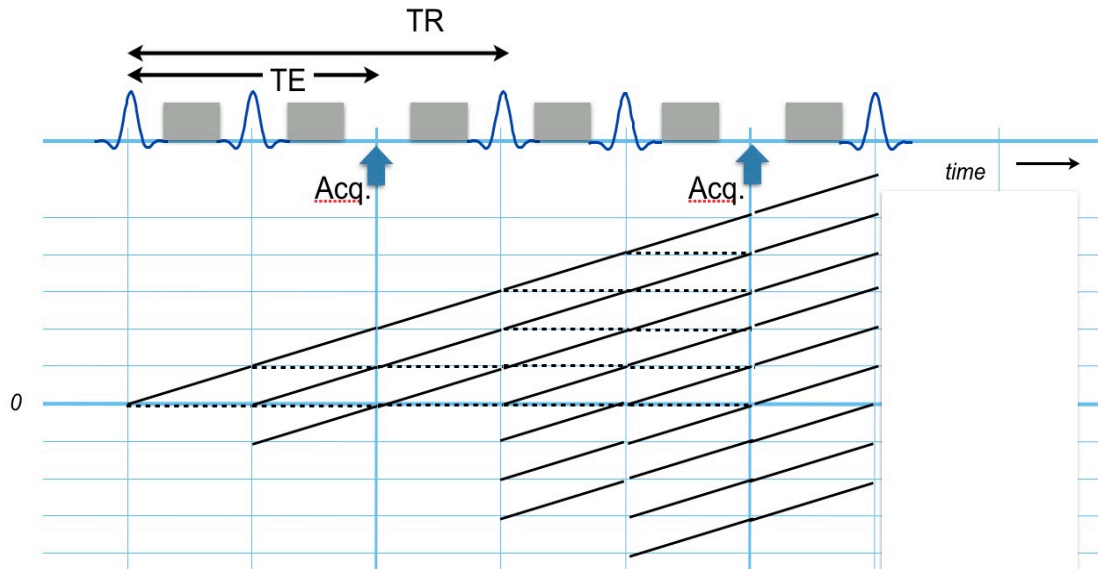
Define  $E_1(\tau) = \exp(-\tau/T_1)$ , and  $E_2(\tau) = \exp(-\tau/T_2)$ , then  $E_A(\tau) = \text{diag}[E_2(\tau) \ E_2(\tau) \ E_1(\tau)]$  and  $E_B(\tau) = (I - E_B(\tau)) \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ .

$$M' = E_B(TE/2) + R_y(150^\circ) E_B(TE/2) + E_A(TE/2) R_y(150^\circ) E_A(TE/2) R_y(75^\circ) M$$

- b) Write a similar matrix expression for the magnetization at the echo time in the steady state.

$$\begin{aligned} M_{SS} &= (I - A)^{-1} B \\ A_1 &= E_A(TR - TE) \\ B_1 &= E_B(TR - TE) \\ A_2 &= E_A(TE/2) \\ B_2 &= E_B(TE/2) \\ A &= A_2 R_y(150^\circ) A_2 R_y(75^\circ) A_1 \\ B &= B_2 + (R_y(150^\circ) (B_2 + R_y(75^\circ) B_1)) \end{aligned}$$

6. [15pts] Below is shown the “Missing Pulse SSFP” sequence. Every 3<sup>rd</sup> RF pulse is removed, and the signal is acquired at that time. All gradients (gray boxes) have the same area. Consider a low flip angle (not 90° or 180°!) for this question.



- a) Draw the phase coherence diagram (F and Z states), all the way to the 5<sup>th</sup> (final) RF pulse shown.

*See diagram – solid = F, dashed = Z. Each RF splits F into F+, F- and Z and vice versa. The phase coherence diagram is quite simple – note that at the first acq, the F<sub>2</sub> does not create Z<sub>2</sub>*

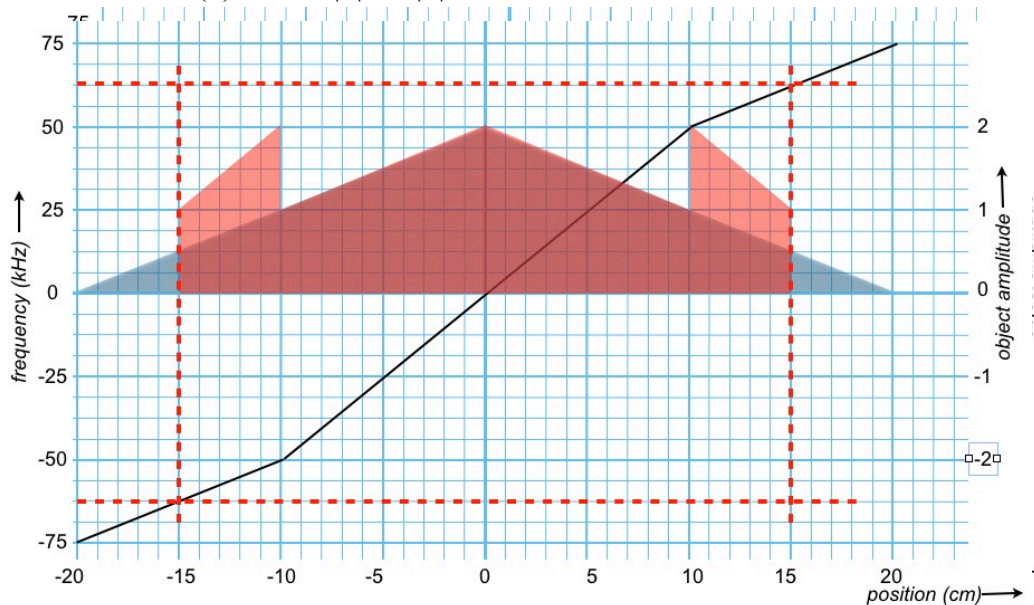
- b) Identify all distinct pathways that lead to signal at the second readout (Acq.) point?

*Must be F<sub>0</sub> state at the readout point. Note that you need an even number of transverse periods! These are states just after the RF (or missing RF):*

- $Z_0 - F_0 - F_1 - F_{-2} - F_{-1} - F_0$  (Excited on 2<sup>nd</sup> RF, 2 cycles then spin echo)
- $Z_0 - Z_0 - Z_0 - F_0 - F_{-1} - F_0$  (Not excited til 3<sup>rd</sup> RF, then spin echo)
- $F_0 - F_{-1} - F_0 - Z_1 - F_{-1} - F_0$  (First spin echo, stored by 3<sup>rd</sup> RF on z, stim echo 4<sup>th</sup> RF)
- $F_0 - Z_1 - Z_1 - Z_1 - F_{-1} - F_0$  (Stimulated echo (stored on z from 2<sup>nd</sup> to 4<sup>th</sup> RF)

## Long Answer (45% of Points)

7. [25pts] Consider the gradient with the following variation with position ( $x$ ), which is used as the readout gradient of a Cartesian acquisition with perfect gradients in other directions, an A/D filter bandwidth of  $\pm 62.5$  kHz and 250 samples. Note that  $(\gamma/2\pi)G_x$  is plotted, to make the math easier. The object is defined as  $m(x) = 2 - 0.1|x|$  for  $|x| < 20$  and 0 elsewhere.



- a) What is the *actual* FOV that is encoded?

*30cm – just draw across from  $\pm 62.5$  kHz. See lines above. There is no aliasing since the gradient is monotonic in this range.*

- b) If the image is reconstructed assuming the gradient strength at  $x=0$  applies everywhere, draw the reconstructed image as a function of  $x$ . (Please draw on the above plot!)

*Between -10 and 10cm it is the same. From 10 to 12.5cm, the signal drops from 2 to 1 (The signal from 10 to 15 is mapped into this smaller space, and is doubled because it is shrunk into  $\frac{1}{2}$  the space).*

- c) Describe how you should fix the image to match the shape of the proper image?

*Cut the parts outside 10cm in half, and stretch them out. You can crop the image to 25cm FOV or keep it at 30cm (both acceptable)*

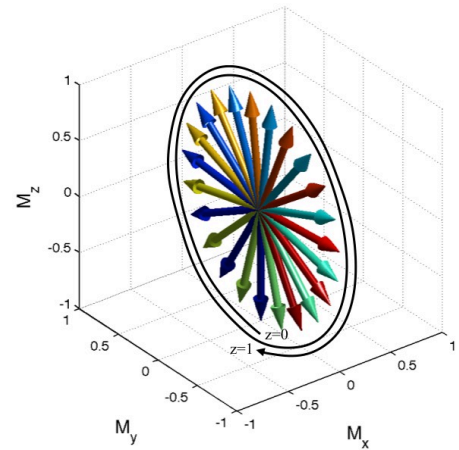
- d) Assuming (c) how does the resolution vary in the image?

*Nominally 250 pixels over 25cm, so 1mm in the center. But beyond  $\pm 10$ cm the resolution is 2mm.*

- e) Assuming (c) what is the relative SNR at the center and outermost pixels in  $x$ , assuming a uniform, single-channel receiver coil?

*The SNR doubles with pixel size, but signal at the edge is  $\frac{1}{4}$  as high, so SNR is double at the center what it is at 30cm. If you cropped the image to 25cm, then the signal at the edge is  $\frac{3}{8}$  as much so the SNR at the center is  $\frac{4}{3}$  higher at the center as at  $\pm 12.5$ cm.*

8. [20pts] Consider magnetization across a voxel that is dephased by exactly 2 cycles in the  $m_y - m_z$  plane, as shown below. Note the  $z=0$  and  $z=1$  positions across a voxel shown.



a) What are the EPG F and Z coefficients for this magnetization?

*Note that only state 2 will have non-zero coefficients.*

*From the definition on B2-9:*

$$M_{xy} = F_2 e^{4\pi i z} + F_{-2} e^{-4\pi i z} = i \sin(4\pi z). \quad (\text{purely imaginary!})$$

$$M_z = \text{Real}\{2Z_2 e^{4\pi i z}\} = -\cos(4\pi z)$$

$$F_2 = 0.5, F_{-2} = -0.5, Z_2 = -0.5$$

b) Rotate the magnetization by  $90^\circ$  in *either* direction around the  $\mathbf{m}_x$  axis and again specify the non-zero F and Z coefficients.

*This is a bit tricky – still in my-mz plane. This means  $F_2^+ = -F_{-2}^+$  or equivalently*

$$\text{Now } M_{xy} = \pm i \cos(4\pi z)$$

$$M_z = \pm \sin(4\pi z)$$

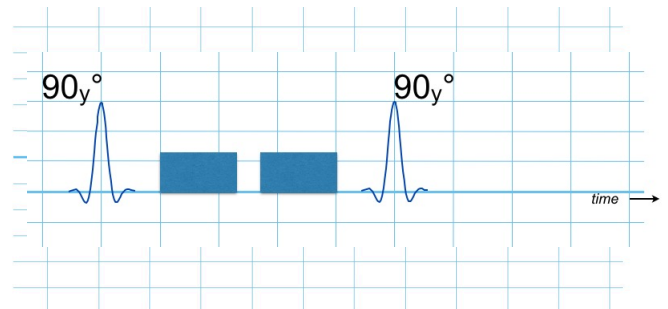
$$F_2 = 0.5i, F_{-2} = 0.5i \quad (F_2 = -0.5i), Z_2 = -0.5i$$

*Note – I intended for this to be a rotation around  $m_y$  which would have made this easier. Sorry!*

c) Draw a sequence that, starting at equilibrium, produces the magnetization in part (a).

*This is a  $90_y^\circ$  RF rotation, followed by 2 dephasing cycles (gradients) and a  $90_y^\circ$  pulse.*

*Replacing the RF pulses both with  $90_{-y}^\circ$  is acceptable – it will just reverse the dephasing but is close enough for the purpose of this test, where we're forgiving about signs(!).*



d) Draw a sequence that, starting with magnetization in part (a), produces the maximum possible measured signal.

*Looking at (c), just add another  $90_y^\circ$  pulse and you have a refocusing pulse, then 2 dephasing cycles and you have a spin echo.*

*It is conceivable to have a  $90_{-y}^\circ$  and negative gradients as well, but notice that essentially this is not a spin-echo, but a gradient echo, so although you'd get full credit, it is not the best answer.*

