Stat 190 Homework 3 Solutions

1. Problem 1 - 4.24 IPS

Part (a):
Examining the table, we see that
\[ P(A) = .09 + .20 = .29 \]

and
\[ P(B) = .09 + .05 + .04 = .18 \]

Part (b):
A^c is the event that the farm is 50 or more acres in size, and
\[ P(A^c) = 1 - .29 = .71 \]

Part (c):
\{A or B\} is the event that a farm is either less than 50 or more than 500 acres in size, and
\[ P(A or B) = .29 + .18 = .47 \]
(Since events A and B are disjoint, the probability of \{A or B\} is the sum \(P(A) + P(B)\).)

2. Problem 2 - 4.32 IPS

The probability that at least one light fails is equal to one minus the probability that none of the lights fail. Since the lights fail independently of one another, the probability that none of the lights fail is equal to the probability that a single light fails raised to the 20th power. In other words:
\[ (1 - .05)^{20} = (.95)^{20} \approx .5404 \]

3. Problem 3 - 4.36 IPS

Part (a):
Examining the table, we see that
\[ P(\text{under 65}) = .321 + .124 = .445 \]

and
\[ P(65 \text{ or older}) = 1 - .445 = .555 \]

Part (b):
Again the table shows that
\[ P(\text{tests done}) = .321 + .365 = .686 \]

and
\[ P(\text{tests not done}) = 1 - .686 = .314 \]

Part (c):
The table shows that
\[ P(A \text{ and } B) = .365 \]
but
\[ P(A)P(B) = (.555)(.686) \approx .3807 \]
Thus, $A$ and $B$ are not independent. If they were independent, we would expect that tests would be done on 38.07% of patients over 65. Since tests were actually done on 36.5% of patients over 65, we conclude that tests were done less frequently on older patients than if these events were independent.

4. Problem 4 · 4.46 IPS

Part (a):
The height of the curve should be $1/2$, since the area under the curve must be 1. The density curve is shown below:

Part (b):
$P(y \leq 1) = (.5)(1) = .5$

Part (c):
$P(.5 < y < 1.3) = (.5)(1.3 - .5) = .4$

Part (d):
$P(y \geq .8) = (.5)(1.2) = .6$

5. Problem 5 · 4.54 IPS

If your number is $abc$, then of the 1000 three-digit numbers, there are six—$abc, aeb, bac, bca, cab, cba$—for which you will win the box. Therefore, we win nothing with probability $994/1000 = .994$ and win $\$83.33$ with probability $6/1000 = .006$. The expected payoff on a $\$1$ bet is therefore

$(.0)(.994) + (\$83.33)(.006) = .50$
6. Problem 6 - 4.64 IPS

Observe that
\[ \mu_X = (\mu - \sigma)(.5) + (\mu + \sigma)(.5) = \mu \]
and
\[ \sigma^2_X = [\mu - (\mu - \sigma)]^2(.5) + [\mu - (\mu + \sigma)]^2(.5) = \sigma^2(.5) + \sigma^2(.5) = \sigma^2 \]

7. Problem 7 - 4.70 IPS

Part (a):
In this case
\[ \mu_Y - X = \mu_Y - \mu_X = 2.001 - 2.000 = .001 \text{ g} \]
and
\[ \sigma^2_{Y - X} = \sigma^2_Y + \sigma^2_X = .002^2 + .001^2 = .000005 \]
and so \( \sigma_{Y - X} \approx .002236 \text{ g} \).

Part (b):
We have
\[ \mu_Z = \frac{1}{2} \mu_X + \frac{1}{2} \mu_Y = 2.0005 \text{ g} \]
and
\[ \sigma^2_Z = \frac{1}{4} \sigma^2_X + \frac{1}{4} \sigma^2_Y = .00000125 \]
so \( \sigma_Z \approx .001118 \text{ g} \). \( Z \) is slightly more variable than \( Y \), since \( \sigma_Y < \sigma_Z \).